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**Non-Maxwellian electromagnetism in
astrophysics and in space applications at
low radio frequencies**

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Contents

Acknowledgments	v
1 NON-MAXWELLIAN THEORIES	1
Résumé	1
1.1 Maxwell theory	6
1.2 Alternatives to Maxwell's theory	15
1.2.1 Massive theories	19
1.2.1.1 Power deviation from Coulomb's law	20
1.2.1.2 de Broglie-Proca	21
1.2.1.3 Stueckelberg	24
1.2.1.4 Chern-Simons	28
1.2.2 Non-linear theories: classical non-linearities	29
1.2.2.1 Born-Infeld	29
1.2.2.2 Podolsky	32
1.2.3 Non-linear theories: quantum non-linearities	33
1.2.3.1 Heisenberg-Euler	33
2 APPLICATIONS OF NON-LINEAR THEORIES	37
Résumé	37
2.1 Field of an accelerated particle in the Born-Infeld theory	43
2.1.1 The regular field	43
2.1.2 The Rindler spacetime	44
2.1.3 The Maxwellian case	47
2.1.4 Field of an uniformly accelerating particle in the Born-Infeld electrodynamics	50
2.2 Electromagnetic shifts in the field of an Heisenberg-Euler dipole	55
2.2.1 Context	55
2.2.2 The effective metric formalism	57
2.2.3 Computation of the shift	59
2.2.4 EMS for the HE dipole	62
2.3 Conclusions	65

3	EFFECTIVE PHOTON MASS	69
	Résumé	69
3.1	Context	74
3.1.1	Chapter outline	77
3.2	Models for SUSY and Lorentz symmetry breaking	79
3.2.1	Classes 1 and 2: Carroll-Field-Jackiw Model	79
3.2.2	Classes 3 and 4: k_F -term model	83
3.3	Dispersion Relations	85
3.3.1	Class 1 and 2	85
3.3.1.1	Time delay	85
3.3.1.2	Dispersion relations for the field strengths	91
3.3.2	Class 3	99
3.3.3	Class 4	103
3.4	A de Broglie-Proca-like approach	105
3.5	Conclusions	107
4	OBSERVATIONAL LIMITS ON THE PHOTON MASS	111
	Résumé	111
4.1	Fast Radio Bursts	117
4.1.1	Near source dispersion	118
4.2	Time delay for FRB 150418	119
4.2.1	DM for the IGM	121
4.2.2	Time lag for a massive photon	122
4.3	Results	122
4.4	Future improvements	123
4.5	Other ways to set a photon mass limits	127
4.5.1	Gamma Ray Bursts	127
4.5.1.1	Two limits using GRBs	129
4.6	OLFAR: space based low frequencies investigation	132
4.6.1	A swarm of nanosatellites	134
4.6.1.1	Organization of the swarm	134
4.6.1.2	Orbit and launch	135
4.6.1.3	Power	136
4.6.1.4	Objective	137
4.6.1.5	OLFAR: limitations and scientific contributions	137
4.6.2	Scientific contribution	138
4.7	Conclusions	141
A	Units and dimensions	147
B	QED and Feynman diagrams	153
C	ΛCDM model	155
	Publication List and Curriculum Vitae et Studiorum	159
	List of Figures	163

<i>CONTENTS</i>	iii
List of Tables	165
Bibliography	166

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Chapter 1

NON-MAXWELLIAN THEORIES

Résumé

Ce Chapitre introductif a pour but de revoir à la fois l'électromagnétisme maxwellien ainsi que les diverses théories alternatives. Dans notre introduction, nous avons essayé d'être aussi pédagogique que possible de façon à ce que le lecteur puisse avoir des bases solides pour bien comprendre la suite de ce travail. Ceci est particulièrement important puisque, par la suite, nous traiterons de divers sujets, séparés dans différentes branches de la physique. Le lecteur n'est peut-être pas familier avec toutes. De ce fait, il est de la plus haute importance de fixer les concepts clés qui seront questionnés, testés et étudiés tout au long de ce manuscrit.

Nous débutons avec un court résumé des étapes cruciales qui ont mené à la compréhension des phénomènes électromagnétiques depuis les temps anciens jusqu'au milieu du 19ème siècle. Notre but n'est pas de donner un récit historique détaillé, par conséquent le lecteur ne devra pas s'attendre à trouver une discussion très approfondie. Nous avons simplement pour objectif d'exposer dans les grandes lignes le processus qui a mené aux équations de Maxwell, qui sont le fondement de la théorie électromagnétique largement acceptée de nos jours et le point de départ de notre discussion.

Nous obtenons les équations de Maxwell dans le contexte de la relativité spéciale. Cela nous permet d'introduire le formalisme covariant et des concepts tels que les invariances de Gauge et Lorentz-Poincaré. Nous adoptons une approche heuristique. En fait, nous ne donnons pas de justification a priori pour les équations. Ceci est cohérent avec ce qui s'est historiquement passé : les équations que Maxwell réunifia étaient en fait des lois empiriques, découvertes au fil des années. Cependant, nous discutons aussi d'une approche plus auto-consistante : l'approche lagrangienne. Nous consacrons suffisamment de temps à cette approche puisque les lagrangiens seront le point de départ pour chaque

théorie alternative dont nous discutons dans ce travail. Par conséquent, nous détaillons comment le Principe de Variation (aussi connu comme le Principe de Moindre Action) fonctionne, comment un lagrangien consistant et covariant s'écrit et comment les équations de Maxwell sont obtenues.

Après ce préluide, nous allons présenter les déviations possibles à partir de la théorie maxwellienne. Malgré des tentatives [1, 2], il n'existe pas, dans la littérature, de review complète des théories alternatives électromagnétiques. Par conséquent, il est important de trouver les concepts fondamentaux soigneusement résumés.

Dans cette partie, nous répondons à une question très importante. La théorie maxwellienne fonctionne très bien, comme cela a été prouvé par ses nombreux succès. Donc, pourquoi avons-nous besoin d'aller au-delà ? Les raisons sont diverses. Il est effectivement exact que les équations de Maxwell donnent une description très cohérente et précise de la manière dont les phénomènes électromagnétiques fonctionnent ; malgré cela, elles présentent aussi quelques problèmes. Premièrement, comme toutes les théories, celle de Maxwell n'a été testée qu'avec une précision finie. Plus spécifiquement, la loi de l'inverse au carré dans la loi de Coulomb a été sujette à de minutieuses recherches. Pendant ces tests, la force de Coulomb a été écrite comme

$$F = \frac{qQ}{r^{2+\alpha}} \quad (1.1)$$

où q et Q sont les deux charges et r la distance entre elles. Les expériences ont montré que le paramètre α is smaller than 10^{-6} [3–7], mais la possibilité existe qu'une infime déviation de la parfaite loi de l'inverse au carré ait lieu et la théorie communément acceptée doit changer.

Plus important, il y a des complications avec la théorie de Maxwell, particulièrement quand on veut l'adapter au régime quantique. Le problème le plus significatif rencontré dans la théorie de Maxwell est que, même dans le contexte classique, des infinis non physiques surviennent. Par exemple, l'énergie propre d'une charge ponctuelle comme l'électron est illimitée ; l'énergie propre d'une charge est définie comme l'énergie contenue dans son champ. Eq. 1.1 nous donne un potentiel de la forme $\sim \frac{1}{r}$, qui présente de toute évidence une singularité en $r = 0$. Par conséquent, l'énergie propre devient infinie. Cette particularité empire en passant au régime quantique. Il arrive qu'un photon, le porteur quantique de l'interaction électromagnétique, qui ne possède pas de masse, ait une énergie propre infinie. Cela vient du fait qu'en écrivant la version quantique de la théorie de Maxwell, il y a des contributions de processus selon lesquels les photons se séparent en une paire particule-antiparticule. Cette paire, par la suite, s'annihile redonnant un photon. Cependant, l'énergie de ce processus est infinie. Les deux problèmes ont été résolus, à travers la procédure connue sous le nom de renormalisation. Toutefois, cette procédure est très artificielle ; peut-être que des théories alternatives pourraient servir le même objectif, tout en étant plus intuitives.

Enfin, nous maintenons notre choix d'étudier les théories non-maxwelliennes

d'un point de vue plus épistémologique. Durant le dernier siècle, il y eut de grandes avancées concernant notre compréhension de toutes les forces fondamentales à l'exception de la force électromagnétique. Bien sûr, ce seul fait n'est pas suffisant pour critiquer la théorie de Maxwell. Cependant, si combiné avec les limites que nous venons de mentionner, cela nous force à nous demander s'il s'agit du mieux que l'on puisse faire. C'est une question que la physique fondamentale ne peut pas éviter de soulever, puisque la plupart de nos conclusions sur l'univers proviennent de l'étude des signaux électromagnétiques et de leur propagation.

Dans ce Chapitre, nous classifions les théories non-maxwelliennes en deux groupes :

1. Les théories massives : où la particule qui porte l'interaction électromagnétique est massive
2. Les théories non-linéaires : où les champs apparaissent non-linéaires dans le lagrangien

De plus, les théories non-linéaires peuvent être sous-divisées entre les théories où la non-linéarité apparaît déjà au niveau classique et les théories où la linéarité apparaît seulement dans le régime quantique.

Parmi les théories massives, nous discutons de celles de Broglie-Proca, Stueckelberg et Chern-Simons. Ces électromagnétismes sont décrits par les lagrangiens suivants [8–11]:

$$L_{Proca} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - A^\mu j_\mu \quad (1.2)$$

$$L_{Stueckelberg} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2 \left(A_\mu - \frac{\partial_\mu D}{m} \right)^2 - (\partial^\mu A_\mu + mD)^2 \quad (1.3)$$

$$L_{Chern-Simons} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \quad (1.4)$$

où $F^{\mu\nu}$ est le tenseur de Maxwell, A^μ est le 4-potential électromagnétique, j^μ est le courant, m est la masse du photon, D est un champ scalaire auxiliaire, κ est une constante de couplage et $\epsilon^{\mu\nu\rho}$ est le tenseur de Levi-Civita.

Les deux premières théories sont strictement reliées. Elles donnent toutes les deux lieu à un photon massif. En fait, la théorie de Stueckelberg peut être considérée comme une extension de celle de de Broglie-Proca. Cette dernière a la déplaisante caractéristique de ne pas être un invariant de gauge, ce qui signifie qu'un changement dans le potentiel de la forme (ϕ indique le potentiel scalaire électrique et $\vec{A}$ le potentiel vecteur magnétique)

$$\phi \rightarrow \phi' = \phi - \frac{\partial\Lambda}{\partial t} \quad (1.5)$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\Lambda \quad (1.6)$$

introduit un terme dépendant de Λ dans les équations du mouvement. Le formalisme de Stueckelberg est capable de résoudre ce désagrément sans toutefois modifier la forme des équations du mouvement.

La théorie de Chern-Simons est un peu étrange. Elle attribue une masse topologique au photon. Une masse topologique est purement inertielle, à la différence des masses communes. En d'autres termes, la masse usuelle se couple à la métrique et apporte une contribution autonome au tenseur énergie-moment. Au lieu de cela, une masse topologique ne se couple pas à la métrique et apporte donc une contribution au tenseur énergie-moment seulement via $E^2 = p^2 + m^2$. La théorie Chern-Simons est aussi un invariant de gauge. D'autre part, cette théorie peut seulement s'appliquer dans des espaces-temps avec un nombre impair de dimensions.

Des caractéristiques communes à toutes ces théories massives sont :

- la vitesse de la lumière est diffusée selon la relation

$$(\omega)^2 - |\vec{k}|^2 = m^2 \quad (1.7)$$

où ω est la fréquence angulaire et $\vec{k}$ le moment

- un photon massif présente trois polarisations contrairement au cas sans masse
- le potentiel électrique a une décroissance exponentielle

$$\phi_{m \neq 0} = \frac{e^{-mr}}{r} \quad (1.8)$$

Parmi les théories non-linéaires, nous traitons de deux qui présentent des non-linéarités classiques et une où la déviation reproduit les effets quantiques. Les deux théories classiques sont la théorie de Born-Infeld et la théorie de Podolsky. Leurs lagrangiens sont [12–14]:

$$L_{BI} = \sqrt{-g} \left(\sqrt{1 + F + G^2} - 1 \right) \quad (1.9)$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a^4}{4} (\partial^\lambda F^{\mu\nu}) \partial_\lambda F_{\mu\nu} \quad (1.10)$$

où $F = F^{\mu\nu} F_{\mu\nu}$, $G = \frac{1}{4} F_{\mu\nu} F^{\mu\nu*}$ ($F_{\mu\nu}^*$ étant $\epsilon_{\alpha\beta\mu\nu} F^{\alpha\beta}$), g est le déterminant de la métrique du espace-temps et a une constante.

La théorie de Born-Infeld est très importante puisqu'elle résout le problème des infinis non physiques pour des particules ponctuelles. La paire dynamique des équations de Maxwell dans le vide indique, dans ce cas,

$$\vec{\nabla} \cdot \vec{D} = 0 \quad (1.11)$$

$$\vec{\nabla} \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} \quad (1.12)$$

où

$$\vec{D} = \frac{\vec{E} - G\vec{B}}{\sqrt{1 + F - G^2}} \quad (1.13)$$

$$\vec{H} = \frac{\vec{B} - G\vec{E}}{\sqrt{1 + F - G^2}} \quad (1.14)$$

Elles ressemblent étroitement aux équations usuelles de Maxwell dans un milieu. Dans ce cas, c'est le vide qui agit comme milieu du fait des interactions non-linéaires avec le champ électromagnétique. Le problème des divergences est résolu parce que les champs $\vec{D}$ et $\vec{H}$ augmentent indéfiniment proche des sources, mais pas $\vec{E}$ et $\vec{B}$.

La théorie de Podolsky est très particulière parce qu'elle implique des termes avec des dérivées d'ordre supérieur. Cette théorie est classique, mais a été développée en prêtant attention à ce que ses conséquences quantiques pourraient être. En fait, ces nouveaux termes contenant des dérivées d'ordre supérieur du 4-potential aident le processus de renormalisation. L'électrodynamique de Podolsky produit deux modes photoniques, un maxwellien (A'_μ) et un dont le potentiel est exponentiellement décroissant (A''_μ):

$$\square A'_\mu = 4\pi j_\mu \quad (1.15)$$

$$\square A''_\mu + a^2 A''_\mu = 4\pi j_\mu \quad (1.16)$$

$$\phi_{A''_\mu} = \frac{e}{4\pi r} \left(1 - e^{-\frac{r}{a}} \right) \quad (1.17)$$

La seule théorie non-linéaire dont le but est de reproduire les effets quantiques que nous analysons est la théorie de Heisenberg-Euler. Le Lagrangien est [15]

$$L = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + e^2 \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \cdot \left\{ i \frac{\eta^2}{2} F^{\mu\nu} F_{\mu\nu}^* \cdot \frac{\cos \left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + i F^{\mu\nu} F_{\mu\nu}^*} \right] + c.c.}{\cos \left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + i F^{\mu\nu} F_{\mu\nu}^*} \right] - c.c.} + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6} \cdot F_{\mu\nu} F^{\mu\nu} \right\} \quad (1.18)$$

où $\mathfrak{E}_k \left(= \frac{m^2 c^3}{e\hbar} \right) \sim 10^{18} \frac{V}{m} \sim 4.4 \cdot 10^{13} G$ est le champ critique de la théorie et *c.c.* représente le complexe conjugué de l'expression précédente. C'est un lagrangien effectif qui reproduit avec précision les contributions du tree level de la théorie quantique pour tous les ordres, expliquant des phénomènes tels que la diffusion lumière-lumière et la polarisation du vide.

1.1 Maxwell theory

Electric and magnetic phenomena are known to mankind since ancient times. Even cavemen witnessed lightnings and were shocked in the water by catfishes. Of course, at the beginning, all these phenomena were interpreted in the light of superstition and associated with deities. But, as civilization developed, natural philosophers started to deal with electromagnetism in a more rational fashion. For instance, it is proved that Thales of Miletus, already in 600 BC, was aware of the fact that rubbing fur against some kind of materials these latter ended up attracting small specks of dust [16].

Not only electric phenomena were known. Also magnetism was discovered in the early days. In Central America was found an artifact dating back to the age when Olmecs lived in the region, around 1000 BC, that seems to testify that that civilization was aware of the magnetic properties of lodestone. As suggested by the American astronomer John Carlson [17], such evidence might lead to think that Olmec profited of such a knowledge to create geomagnetic compasses well before the Chinese, who has been always awarded the discovery more or less six centuries later, hence around the 4th century BC. Also the Greeks were aware that some materials naturally attracted iron powder. Aristotle credited Thales of Miletus with the first "scientific" study on the subject [18], for having in fact discussed how lodestone affected the distribution of iron powder. It has been reported that, later, Archimedes (in the 3rd century BC) tried to magnetize Syracusan army weapons in order to disarm the enemy more easily.

It was, in its embryonic form, the birth of the research on electromagnetism that lead to James Clerk Maxwell's theory, many centuries later.

We are all familiar with such a theory, but for future purposes it is useful to revise it. Our main goal is not to stress the implications of this theory. Instead, what we aim for is to trace back the roots of this theory. In doing so, we will be able to stress what the fundamentals of Maxwell's equations are. Such fundamentals, even if well sounded, like everything in physics, are not written in stone. Purpose of this thesis will be to investigate theoretically and experimentally what happens if we reject one or more of those assumptions.

Of course, during Middle Age the before mentioned knowledge on electromagnetism was enlarged. Electric and magnetic phenomena were thought not to be intertwined. Despite the remarkable work of Peter Peregrinus of Maricourt - who in the 13th century studied magnetism extensively, investigating the different magnetic properties of several substances and pivoting compass needles [19], Flavio Goja - who invented the dry compass [20], or Gerolamo Cardano - who, presumably for the first time, identified with precision the electric and magnetic phenomena, trying an interpretation in terms of forces [21], it is only in the 16th century that we have a significant advancement on the subject. This is due to the man who is, not by chance, called the "founder of electric science" [22] - William Gilbert. Gilbert in his comprehensive treatise *De Magnete* enlarged Cardano's work, invented the word *electricus* - from the Greek word for amber - and tested a long list of substances, showing that many could be electrified but many others could not, thereby preparing the

ground for the understanding of the difference between conductors and insulators [23]. Gilbert's work was then continued, in the 18th century, by Robert Boyle, who, apart from enlarging Gilbert's list of conductor materials, proved that the electric force acts across vacuum, therefore being independent of the air medium [22, 24]. Other milestones on the road to Maxwell's theory were posed by Stephen Gray, William Watson, Ewald von Kleist, Benjamin Franklin, Luigi Galvani and Alessandro Volta. Gray further convinced of the difference between conductors and insulators showing that a metallic wire conducts electricity whereas a silk one doesn't. Watson understood that the discharge of static electricity is equivalent to a current. Von Kleist received a severe shock from a bottle with a needle on its neck near a generator as he casually touched it and formulated the concept of capacitance [25]. Franklin contributions were twofold; he renewed the interest in the subject and enlarged the scope of the research by proving that lightnings and electricity induced by rubbing materials shared the same nature and also introduced (vaguely) the concept of potential. In fact Franklin performed a famous experiment tying a key to a kite and letting it soar during a thunderstorm. A lightning hit the kite and through the key, that was connected to what we now call a capacitance, charged it [26]. Galvani experimented on frogs noticing that when put in touch with a charged metal the frog muscles contracted; hence he supposed that some kind of electricity were present inside the animal: this link between biological chemistry and electricity was fundamental for the development of the chemical battery [22]. Volta discovered that particular chemical reactors could be used to create a difference in potential between the two extremes of the apparatus so that if a conductor were inserted between them a current could be observed; he then was able to create the first electric battery and to better explain the role of potential in the electric theory [22].

What we are really interested in, anyway, is the contribution due to Charles-Augustin Coulomb. Thanks to a specific torsion balance he discovered that the electric force between two charges is directly proportional to the intensity of the two charges involved and inversely proportional to the squared distance separating them:

$$\vec{F} \propto \frac{q_1 q_2}{r^2} \vec{n} \quad (1.19)$$

where $\vec{n}$ is the unitary vector indicate the direction between the two charges (taken with sign), q_1 and q_2 are the charges and r is the absolute value of the distance. This equation is known as Coulomb's Law.

To be historically correct, Coulomb was not the first to devise such a inverse square law for the electric force. Others, before him, reached the same conclusion. In particular John Robinson and Henry Cavendish measured the $\frac{1}{r^2}$ behavior of the electric force. The first one devised an apparatus that measured the deviation from the vertical of a metal sphere when electrified in vicinity of another identical one. With such an equipment he was able to estimate that the electric force should decrease as $\frac{1}{r^{2.06}}$ at maximum. Cavendish created an

instrument where two metallic spheres were placed one inside the other, put in contact with a wire and then disconnected. The scientist was interested in knowing if the inner sphere would have absorbed all the charge. Of course we know that it doesn't. Cavendish saw this too and was also able to exploit this phenomenon to determine, at the best of his experimental accuracy, what we call now the Coulomb's law: his precision was such that it could narrow down the maximum deviation from the perfect square behavior to only 0,02 [27, 28]. Unfortunately for them, they did not choose to publish their discoveries, and as a result the credit went to Coulomb, whose accuracy was actually much lower (only 5-6 per cent) in comparison to Cavendish. Coulomb used a torque pendulum for his experiments, therefore his results were direct and easy to understand [29].

The theory of electromagnetism was rapidly developed after Coulomb's work, also thanks to mathematical tools like Gauss's law¹ and the divergence theorem², both used to derive the well-celebrated Maxwell's equations.

After Coulomb, scientists lively studied electric and magnetic phenomena, understanding in a more quantitative fashion how linked the two were. Armed with Eq. 1.19 they abandoned static charges to devote their attention to moving ones. It is so that Kirchhoff found his laws regarding circuits, that Kelvin calculated the oscillatory nature of the discharge of a condenser circuit. But, more importantly, in 1819 Hans Christian Ørsted discovered that in placing a compass near a wire in which an electric current flowed the compass needle moved. This was the first quantitative evidence of the link between electric and magnetic effects. Not to mention the beginning of a new step toward what might as well be regarded, from a practical point of view, as one of the most important scientific revolutions in human history. Ørsted's finding helped André-Marie Ampère to develop his celebrated theory of electrodynamics, which is the germ of Maxwell's theory. Based on his observation, he understood that three principles ruled the attraction or repulsion of wires passed through by a current:

1. Two parallel portions of a circuit attract one another if the currents in them are flowing in the same direction, and repel one another if the currents flow in the opposite direction
2. Two portions of circuits crossing one another obliquely attract one another if both the currents flow either towards or from the point of crossing, and repel one another if one flows to and the other from that point
3. When an element of a circuit exerts a force on another element of a circuit, that force always tends to urge the second one in a direction at right angles to its own direction

¹Gauss's law states that the integral over a closed, oriented surface of the electric field is equal to 4π the charge enclosed inside the surface

²The divergence theorem states that for a continuous field $\vec{A}(\vec{x})$ defined in a volume V enclosed by the closed surface S holds $\oint_S \vec{A} \cdot \vec{n} da = \int_V \vec{\nabla} \cdot \vec{A} dV$ where da is the element surface, dV the element volume and $\vec{n}$ the unitary vector point by point normal to the surface and outwardly directed.

Proceeding on this path, Michael Faraday discovered the last piece of the puzzle. In fact it was Faraday himself to observe for the first time the electric induction. He put two solenoids in "contact" coiling their wires around a torus. One solenoid was inserted in a circuit in which a current flowed; the other was linked to a galvanometer. Doing so he noticed a current measured by the galvanometer. He attributed such a current to an inductive effect generated by the other solenoid [22].

Unfortunately for him, Faraday was not particularly well versed in mathematics so he was not able to go beyond the experimental relation he found. It was Maxwell that, in his impressive work - first published in 1864 in the paper *A Dynamical Theory of the Electromagnetic Field* [30] - summarized all the knowledge acquired at the time on the subject in his famous set of equations, that are still today the starting point for basically everything involving electric and magnetic forces:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1.20)$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \frac{1}{\mu_0} \vec{j} \quad (1.21)$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad (1.22)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (1.23)$$

where $\vec{E}$ is the electric field, $\vec{B}$ the magnetic field, ρ the charge density, $\vec{j}$ the electric current, ϵ_0 the vacuum electric permittivity, μ_0 the vacuum magnetic permeability and c the speed of light. Taking the divergence of Eq. 1.21 and using Eq. 1.20, the continuity equation ensues

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0 \quad (1.24)$$

Eq. 1.24 describes the charge conservation in a closed volume.

We have used the SI units for the Maxwell equations. In the following we will use, instead, the Planck units³. In the latter units, Maxwell's equations take the form:

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (1.25)$$

$$\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t} \quad (1.26)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.27)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (1.28)$$

The successes of Maxwell's theory are uncountable. It is the base for the study of every electric circuit that makes our electronic devices work. In addition, in a more speculative context Maxwell's theory has proven its solidity. Eqs.

³See the Appendix on units and dimensions

1.25-1.28 are in perfect agreement with Einstein's theory of relativity. Not only they are naturally derived within that mathematical context, but also predict the correct propagation for waves and respect the causal structure. If all this were not sufficient, the expected numerical value of the speed of light is correct⁴

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (1.29)$$

We now set Maxwell's electromagnetism in a relativistic covariant form. In a four dimensional Minkowski space with signature (1, 3) the infinitesimal interval is

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \quad (1.30)$$

and, hence, the metric is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (1.31)$$

The inverse metric is defined by the relation

$$g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu \quad (1.32)$$

where δ_μ^ν is the Kronecker's delta: $\delta_\mu^\nu = 0$ for $\mu \neq \nu$ and $\delta_\mu^\nu = 1$ for $\mu = \nu$. Eq. 1.30 is invariant under the Lorentz-Poincaré group. In fact the metric Eq. 1.31 defines a scalar product given by

$$(a \cdot b) = a^T g b = g_{\mu\nu} a^\mu b^\nu \quad (1.33)$$

where a and b are 4-vectors, g the metric matrix⁵ and the operator T indicates the algebraic operation of transposition. The matrices that conserve such a kind of scalar product belong to the 4x4 representation of the Lorentz-Poincaré group. In other words, to prove our statement it is sufficient to show that 4x4 matrices satisfying Eq. 1.33 generate the Lorentz-Poincaré group. Let's then suppose to have one of such matrices, called Λ . The effect of Λ on the position vector is given by

$$x' = \Lambda x \quad (1.36)$$

⁴On this relation see [31] and [32].

⁵Unlike what happens in the Euclidean case, here we must distinguish between *contravariant 4-vectors* that transform according to

$$A'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} A^\nu \quad (1.34)$$

and *covariant 4-vectors* that transform as

$$B'_\nu = \frac{\partial x^\mu}{\partial x'^\nu} B_\mu \quad (1.35)$$

One can pass from covariant to contravariant vectors using the metric tensor Eq. 1.31 and from contravariant to covariant using the inverse metric.

If Λ conserves the scalar product 1.33 it holds that

$$(x')^T g x' = x^T g x \quad (1.37)$$

Using Eq. 1.36 one has

$$(\Lambda x)^T g (\Lambda x) = (x)^T \Lambda^T g \Lambda x = x^T g x \quad (1.38)$$

for every x . Therefore

$$\Lambda^T g \Lambda = g \quad (1.39)$$

Eq 1.39 and 1.45 define unambiguously the Lorentz group⁶. Eq. 1.39 is the fundamental relation from which one can infer other important properties. For instance, taking the determinant of Eq. 1.39, one sees that

$$\det (\Lambda^T g \Lambda) = \det g \cdot (\det \Lambda)^2 = \det g \Rightarrow \det \Lambda = \pm 1 \quad (1.45)$$

because $\det g = -1$. To be precise this latter admits four subgroups. If $\det \Lambda$ is $+1$ then we are in the *proper* subgroup, otherwise in the *improper*. Also the sign of the 00-component of Λ is a discriminant that gives two distinct subgroups: if $\Lambda_0^0 > 0$ we are in the *ortochronus*, if $\Lambda_0^0 < 0$ the subgroup is called *non-ortochronus*. Physical applications require to work in the *proper ortochronus*

⁶Lorentz's transformations are the only non-singular coordinate transformation that leave Eq. 1.30 invariant. In fact, for any other variable changes the interval in the primed coordinates is

$$(ds')^2 = g_{\mu\nu} \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta = ds^2 \quad (1.40)$$

for all dx^α . Thus, we get

$$g_{\mu\nu} \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} = g_{\alpha\beta} \quad (1.41)$$

and deriving with respect to x^ϵ

$$0 = g_{\mu\nu} \frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\epsilon} \frac{\partial x'^\nu}{\partial x^\beta} + g_{\mu\nu} \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial^2 x'^\nu}{\partial x^\beta \partial x^\epsilon} \quad (1.42)$$

The latter can be rewritten as

$$\begin{aligned} 0 = g_{\mu\nu} & \left[\frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\epsilon} \frac{\partial x'^\nu}{\partial x^\beta} + \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial^2 x'^\nu}{\partial x^\beta \partial x^\epsilon} + \frac{\partial^2 x'^\mu}{\partial x^\epsilon \partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} + \frac{\partial x'^\mu}{\partial x^\epsilon} \frac{\partial^2 x'^\nu}{\partial x^\beta \partial x^\alpha} \right. \\ & \left. - \frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\beta} \frac{\partial x'^\nu}{\partial x^\epsilon} - \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial^2 x'^\nu}{\partial x^\epsilon \partial x^\beta} \right] \end{aligned} \quad (1.43)$$

After simplification we finally obtain

$$0 = 2g_{\mu\nu} \frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\epsilon} \frac{\partial x'^\nu}{\partial x^\beta} \quad (1.44)$$

But we have assumed the transformation to be non-singular so $\frac{\partial x'^\nu}{\partial x^\beta} \neq 0$ and, of course, also $g_{\mu\nu}$ is non-singular. So $\frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\epsilon} = 0$ which means that the transformation must be linear, or, in other terms, a Lorentz transformation.

subgroup. For a boost transformation such a matrix reads

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta_1 & -\gamma\beta_2 & -\gamma\beta_3 \\ -\gamma\beta_1 & 1 + \frac{(\gamma-1)\beta_1^2}{\beta^2} & \frac{(\gamma-1)\beta_1\beta_2}{\beta^2} & \frac{(\gamma-1)\beta_1\beta_3}{\beta^2} \\ -\gamma\beta_2 & \frac{(\gamma-1)\beta_1\beta_2}{\beta^2} & 1 + \frac{(\gamma-1)\beta_2^2}{\beta^2} & \frac{(\gamma-1)\beta_2\beta_3}{\beta^2} \\ -\gamma\beta_3 & \frac{(\gamma-1)\beta_1\beta_3}{\beta^2} & \frac{(\gamma-1)\beta_2\beta_3}{\beta^2} & 1 + \frac{(\gamma-1)\beta_3^2}{\beta^2} \end{pmatrix} \quad (1.46)$$

where $\vec{\beta} = \vec{v}$ (it would be $\frac{\vec{v}}{c}$, but the reader must recall we set $c = 1$; the consequence is that the value of $\vec{v}$ ranges from 0 to 1 because the base unit for the velocity is c which is exactly $\frac{l_P}{t_P}$ as one can see from Tab. A.3) with i component β_i , $\beta = |\vec{\beta}|$ and $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$.

In order for the principle of relativity to be satisfied, nature laws must be written in terms of entities whose transformation properties respect the Lorentz group.

We write 1.25-1.28 in a covariant form, i.e. in such a way that they are formally invariant under Lorentz transformations. We introduce a second rank, completely antisymmetric tensor, called Maxwell's tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \quad (1.47)$$

Thus, Eqs. 1.25-1.26 can be written as

$$\partial_\nu F^{\nu\mu} = 4\pi j^\mu \quad (1.48)$$

where we have defined the 4-vector $j^\mu = (\rho, \vec{j})$, called the 4-current. It is really a 4-vector since the 4-divergence is formally given by

$$\partial_\mu Y^\mu = \frac{\partial Y^0}{\partial x^0} + \vec{\nabla} \cdot \vec{Y} \quad (1.49)$$

that applied to j^μ is just the charge conservation law Eq. 1.24 derived by Maxwell's equation, which holds true. We will see that there is a more refined way to see that the 4-current is a conserved 4-vector.

Instead Eqs. 1.27-1.28 change into

$$\partial_\mu f^{\mu\nu} = 0 \quad (1.50)$$

where

$$f^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad (1.51)$$

is the the dual Maxwell tensor and $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol that takes the value +1 if $\mu\nu\alpha\beta$ is an even permutation of 0123, -1 if it is an odd permutation and 0 otherwise. Eq. 1.50 is equivalent to the system of four equations given by

$$\partial^\alpha F^{\beta\gamma} + \partial^\beta F^{\gamma\alpha} + \partial^\gamma F^{\alpha\beta} = 0 \quad (1.52)$$

that is the Ricci Curbastro-Bianchi identity for a rank-2 tensor.

Proposed in this fashion, all this seems very casual and depending on the particular choice we made in Eq. 1.47. But Maxwell's tensor was not chosen arbitrarily. There is a deeper reason for the form it takes. The 4-potential is given by

$$A^\mu = (\phi, \vec{A}) \quad (1.53)$$

where ϕ is the usual electric scalar potential and $\vec{A}$ is the magnetic vector potential, well defined in virtue of Eq. 1.28.

Thanks to Maxwell's equations, the fields are related to such potentials via the following relations:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi \quad (1.54)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (1.55)$$

Given a 4-vector such as Eq. 1.53, the most general way to construct a second rank, completely antisymmetric tensor is

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad (1.56)$$

Using Eqs. 1.54-1.55, the tensor 1.47 is obtained. For instance, for the x direction, the electric field component is

$$E_x = -\frac{\partial A_x}{\partial t} - \frac{\partial \phi}{\partial x} = -(\partial^0 A^1 - \partial^1 A^0) \quad (1.57)$$

Even if we justify the form we chose for Maxwell's tensor, we cannot be completely satisfied with how the theory was derived. We would like it to follow from general principles. One of the most fascinating aspects of Maxwell's theory is that such a derivation is possible and in agreement with Einstein's relativity.

To achieve such a goal, we construct an action that, after having applied the Variation Principle, gives us as Euler-Lagrange equation exactly Eq. 1.25-1.26. So we must build the Lagrangian for electromagnetic fields interacting with sources. In fact, by definition,

$$S = \int \mathcal{L} dt = \int \left(\int L d^3x \right) dt \quad (1.58)$$

where S indicates the action, $\mathcal{L}$ is the Lagrangian and L represent the Lagrangian density. Taken Maxwell's tensor as in Eq.1.56, we could write the Lagrangian density

$$L_{free}(A^\mu, \partial^\mu A^\nu) = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \quad (1.59)$$

for the electromagnetic fields. In fact, in order to have a Lorentz invariant action, it is clear, from Eq. 1.58, that the Lagrangian density must be a Lorentz scalar. Further, we expect the Lagrangian density to be quadratic in $\partial^\mu A^\nu$, just like in the dynamical case the Lagrangian is quadratic in the velocities. Given these premises, $F^{\mu\nu}$ must be used to build up L^7 . The only Lorentz scalars one can construct out of it are $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}f^{\mu\nu}$. However, the latter is a scalar only under proper Lorentz transformation, but behaves like a pseudoscalar if time inversion is present. So the only reasonable, physical choice is the one given in Eq. 1.59, where the numerical proportionality constant has been chosen such that at the end of the calculation one obtains the correct factors in Maxwell's equations.

Of course this Lagrangian density describes only electromagnetic fields. To obtain the whole theory we must consider how fields couple with sources. These latter are represented by the 4-current. And the easiest Lorentz scalar that one can think of is

$$L_{int}(A^\mu) = -A_\mu j^\mu \quad (1.60)$$

Finally, the full Lagrangian density is

$$L(A^\mu, \partial^\mu A^\nu) = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - A_\mu j^\mu \quad (1.61)$$

which is the full Lagrangian density for the theory.

According to the Variation Principle, the equations of motion are given by setting the first order variation of the action to zero. Considering that we have a Lagrangian density depending on A^μ and $\partial^\mu A^\nu$ only, we obtain

$$\begin{aligned} \delta S &= \int \delta L d^4x \\ &= \int \left(\frac{\partial L}{\partial A_\mu} \delta A_\mu + \frac{\partial L}{\partial (\partial_\mu A_\nu)} \delta (\partial_\mu A_\nu) \right) d^4x \\ &= \int \left(\frac{\partial L}{\partial A_\mu} \delta A_\mu - \partial_\mu \left[\frac{\partial L}{\partial (\partial_\mu A_\nu)} \right] \delta A_\mu \right) d^4x \\ &\quad + \frac{\partial L}{\partial (\partial_\mu A_\nu)} \delta A_\mu \Big|_{-\infty}^{+\infty} \end{aligned}$$

But the variation vanishes on the boundaries, so

$$\frac{\partial \mathcal{L}}{\partial A_\nu} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} \right] = 0 \quad (1.62)$$

With the Lagrangian density Eq. 1.61 one has

$$\frac{\partial L}{\partial A_\mu} = -\frac{1}{c} j^\mu \quad (1.63)$$

⁷Eqs. 1.27-1.28 are directly derived from Eq. 1.52. The field tensor is defined via the commutator of the two covariant derivatives acting on the 4-potential. Thus, the field tensor satisfies the Ricci Curvature-Bianchi identity by construction.

$$\frac{\partial L}{\partial [\partial_\mu A_\nu]} = \frac{1}{4\pi} F^{\mu\nu} \quad (1.64)$$

and hence

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\nu \quad (1.65)$$

which is exactly Eq. 1.48.

Also Eq. 1.50 can be easily obtained. In fact, the potentials have been chosen according to Eqs. 1.54-1.55 which are a direct consequences of 1.27-1.28. We write the left hand-side term of Eq. 1.50 explicitly

$$\begin{aligned} \partial_\mu f^{\mu\nu} &= \frac{1}{2} \partial_\mu \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \\ &= \partial_\mu \epsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta \\ &= \epsilon^{\mu\nu\alpha\beta} \partial_\mu \partial_\alpha A_\beta \end{aligned} \quad (1.66)$$

The operator $\partial_\mu \partial_\alpha$ is symmetric in its indexes, whereas $\epsilon^{\mu\nu\alpha\beta}$ is antisymmetric. Thereby, contraction on these indexes makes the right hand side vanish, implying that Eq. 1.50 is satisfied, like we wanted to prove.

Similarly, also the continuity equation can be obtained in a covariant formalism. If one takes Eq. 1.48 and performs the 4-derivative one has

$$\frac{1}{4\pi} \partial_\mu \partial_\nu F^{\mu\nu} = \partial_\mu j^\mu \quad (1.67)$$

Again, the operator $\partial_\mu \partial_\nu$ is symmetric whereas $F^{\mu\nu}$ is antisymmetric. The contraction between them is vanishing and one arrives to the expression

$$\partial_\mu j^\mu = 0 \quad (1.68)$$

which is exactly Eq. 1.24 thanks to Eq. 1.49.

1.2 Alternatives to Maxwell's theory

If we have such a successful theory that can also be derived from general principles, one could wonder why we are interested in finding alternatives. The answer to this question is crucial and can basically be summarized as follow.

On the one hand, we have to remind ourselves the large, but yet limited range in which Coulomb's law has been tested. In the previous section we quoted the remarkable achievements reached by the early experimenters in this field. For many purposes, that kind of precision is vastly sufficient - to prove once more the caliber of those scientists. As technology has progressed, more and more stringent tests have been performed, in order to know up to what precision we can count on Coulomb's law. It is possible that outside the regime in which classical electrodynamics has been tested, deviations could emerge from it. Part of the following work will deal with calculating observable quantities precisely for phenomena supposedly in critical regions.

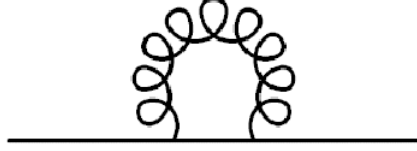


Figure 1.1: Feynman diagram showing the one loop QED contribution to the electron self-energy. Going from left to right, the diagram shows the electron evolving freely, producing a virtual photon at the first node. The photon is re-absorbed at the second node and, finally, the electron continues its propagation.

On the other hand, we have a more impellent reason to try and modify Maxwell's electromagnetism. The 20th century was marked by the discovery of quantum mechanics, the comprehension tool for molecular, atomic and sub-atomic physics. In its relativistic form - the so called quantum field theory - quantum mechanics has the pretense of dealing with continuous fields too, not only with particles. Unfortunately a lot of difficulties arise, namely unphysical infinities. Maybe the most remarkable example is the electron self-energy, known even before quantum electrodynamics (QED hereafter). In fact, the self-energy of a charge must equal that contained in its electrostatic field. Since the electron is supposed point-like we incur in the pole at $r = 0$ in Coulomb's potential. Being the energy is proportional to the integral of the field squared, one obtains an infinite value. This problem is not solved in the quantum theory. Feynman's diagrams beyond tree level show that the first order correction, where the creation and subsequent annihilation of a virtual photon along the propagation of the electron occurs, gives a divergent additive contribution to the scattering amplitude (Fig. 1.1)⁸.

Even worse, in QED also photons have infinity self energy, something that has no classical counterpart. In fact, diagrams like the ones in Fig. 1.2 appear, where we see a process of creation and annihilation of a virtual pair of particles. As in the electron case, the divergent contribution to the scattering amplitude is additive. Such a divergence is corrected by replacing the massless photon with a massive boson. During the renormalization process, the mass of this boson is then required to vanish. In such a way the photon itself is recovered and the gauge invariance of the resulting scattering amplitude is assured. Even if these problems are not enough to question the basis of the Maxwellian theory

⁸See Appendix for an heuristic introduction to the Feynman diagrams

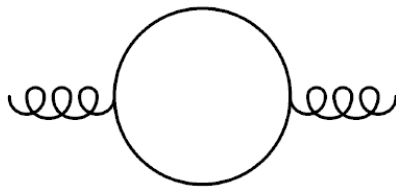


Figure 1.2: Feynman diagram showing the one loop QED contribution to the photon self energy. Reading from left to right, the diagram shows the photon propagating and split into a particle-antiparticle pair at the first node. The pair annihilates at the second node reforming the photon that continues its propagation.

and its results, it is legitimate to ask ourselves if we can do better than this. Usually, the counterargument used in order to stick with Maxwell's theory is the following. Experiments show that the classical theory of electromagnetism fails in the realm of atomic or subatomic physics. For instance, Maxwell's theory doesn't predict any non-zero ground state for the kinetic energy of an electron orbiting a nucleus, nor quantized transitions among the energy levels in atoms. However, the QED gives extremely precise predictions that match with experimental data up to an incredible accuracy.

This is true, of course, but not conclusive. In fact, a very orthodox theoretical physicist might argue that the success of Dirac equation spurs from that of Schrödinger's. This latter origin as the author itself admits, is more fortuitous than profound. We should never forget that QED is a phenomenological theory, i.e. it aims at explaining the observed results for scattering experiments. It is far from actually explaining everything one is keen to know concerning charged particles and their dynamics. It is here appropriate to remind that Feynman himself said that we know very little about what actually happens during a transition between two states in a quantum theory. We can only create a theoretical apparatus that is able to accurately reproduce the output given the input parameters.

On a more philosophical tone, we can observe that during the 20th century our understanding in all fundamental interactions but electromagnetism has evolved drastically. Instead, we stick with a theory that is more than a hundred years old when it comes to EM phenomena. This, of course, is not a detriment in itself. If that theory were proven true, it would be, on the contrary, a great praise. Fundamental physics, though, is called to asked this question: is Maxwell's theory the best we can do? Sadly, despite the many alternatives present in literature and that we will cite in this work, a comprehensive answer

has not been given yet. The main reason behind this lack in our knowledge is that - most probably given by the trust most of the scientific community poses in the experimental accuracy of the QED - the problem has never been tackled systematically. Even if major physicists have come up with alternatives, these were only considered when there was an *ad hoc* need for them. Proof of our statements is the fact that, to this day, there is still no comprehensive review of all non-Maxwellian theories that have been put out in the last centuries, underlining the similarities, differences and the constraints of validity.

The formulation of a parametrized post-Maxwellian scheme is a research target of its own. Conversely to the theories of gravitation for which the parametrized post-Newtonian (PNN) formalism acts as reference to describe specific deviations from general relativity, it does not exist anything of the sort for electromagnetism. Our aim will be thus limited: we want to briefly present and organize the most important of these theories. With these, we aim at investigating the subject of light propagation, mostly in the astrophysical context, to see if it is possible to obtain significant differences with Maxwell's theory. If such is the case, that would bring new contributions to fields such Dark Energy, Dark Matter and, with some more work, Cosmology. More generally, it would change our way of understanding the way light propagates. This would be crucial because, even if gravitational waves have been recently revealed by the VIRGO/LIGO collaboration, it will take some time for the gravitational window to be exploited to explore the universe. Until then the greatest part of information we get will be in the form of the electromagnetic signals we receive from stars and galaxies. A deeper or modified understanding of how light propagates might put in question some of the aspects we consider certain.

We can classify the alternatives to Maxwell's theory in two large groups:

1. MASSIVE THEORIES: where the particle that carries the electromagnetic (EM hereafter) interaction is not taken to be massless
2. NON-LINEAR THEORIES: where the fields appear non-linearly in the Lagrangian.

From a physical point of view, a difference between these two classes is that the first implies a long range modification of the EM interaction, which is, however, held valid for shorter ranges. Instead, the second case implies a modification of the physics for all possible ranges.

In addition, non-linear theories can be further divided into theories where the non-linearity appears already at classical level and theories where the linearity occurs only in the quantum regime. In the former case, we have a purely classical theory, whereas, in the second one, we obtain an effective theory. This latter aims at reproducing the QED results without passing through the quantum formalism and, therefore, avoiding the non-rigorous apparatus of renormalization. It must be noted that the first class, by its nature, is intrinsically quantum. Indeed, only in such a context we can meaningfully talk about an interaction carrier. However, that does not prevent from taking the classical limit, as we shall see.

1.2.1 Massive theories

We first analyze some general features that are common to all EM massive theories.

A non-zero photon mass implies a free propagating wave described by the following expressions⁹:

$$\vec{E} = E_0 \exp \left[-i \left(\omega t - \vec{k} \cdot \vec{x} \right) \right] \quad (1.74)$$

$$\vec{B} = B_0 \exp \left[-i \left(\omega t - \vec{k} \cdot \vec{x} \right) \right] \quad (1.75)$$

$$(\omega)^2 - |\vec{k}|^2 = m^2 \quad (1.76)$$

where $\vec{k}$ is the wave vector, ω the angular frequency and m represents the photon mass in terms of units of wavenumbers or, equivalently, of inverse length. The last expression, is, of course, what makes massive theories differ with respect to the classical EM theory. In the latter light propagation speed is a constant. Instead Eq. 1.76 tells us that we have a true dispersion relation. For the group velocity of a wave packet, we have

$$v_g = \frac{d\omega}{d|\vec{k}|} = \frac{|\vec{k}|}{\omega} = \frac{|\vec{k}|}{\sqrt{|\vec{k}|^2 + m^2}} = \frac{\sqrt{\omega^2 - m^2}}{\omega} \quad (1.77)$$

A second remarkable feature is that a massive photon corresponds to a state with three independent polarizations. This is somewhat expected, since the photon must behave like every other massive particle. In fact, only massless particles have only transverse polarizations since a longitudinal one would mean

⁹To obtain these solution, we take Eq. 1.27 and perform its curl:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\partial \vec{\nabla} \times \vec{B}}{\partial t} \quad (1.69)$$

where we have assumed the continuity of the fields to commute the derivation with respect to t and the curl operation. The left-hand side is know, from the algebra, to be

$$-\nabla^2 \cdot \vec{E} + \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) \quad (1.70)$$

In the right-hand side we use Eq. 1.26 obtaining

$$-\frac{\partial^2 \vec{E}}{\partial t^2} \quad (1.71)$$

In absence of free charges $\vec{\nabla} \cdot \vec{E} = 0$, Thus,

$$-\nabla^2 \cdot \vec{E} = -\frac{\partial^2 \vec{E}}{\partial t^2} \quad (1.72)$$

$$\Rightarrow \nabla^2 \cdot \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (1.73)$$

The solution to Eq. 1.73 is Eq. 1.74. For the magnetic field, it is possible to proceed very similarly.

that they can change their speed along the direction of propagation, which is impossible because they must travel at the speed of light. Why has that mode not been detected yet, then? If we place ourselves in the reference frame where the massive photon is at rest, such a particle has

$$\left| \vec{k} \right| = 0 \quad (1.78)$$

Of course, the plane transverse to $\left| \vec{k} \right|$ is undefined. However, in the rest frame we know that the photon electric energy $\vec{E}^2$ is proportional to the photon intensity. Lorentz transformations provide the parallel ($\parallel$) and perpendicular ($\perp$) components of the field in a different frame. In that case, one gets:

$$E_{0\perp} = \frac{\omega}{m} E_{0\perp rest} \quad (1.79)$$

$$E_{0\parallel} = E_{0\parallel rest} \quad (1.80)$$

where we have exploited that

$$K_{\perp} = \omega = \gamma m \Rightarrow \gamma = \frac{\omega}{m} \quad (1.81)$$

$K_{\perp}$ being the energy of the photon. Measuring the transverse mode from a non-comoving reference frame, the mode is suppressed by a factor $\frac{m}{\omega}$. If we wanted to measure the perpendicular polarization with an experiment, the field would be received by an antenna. The power absorbed by the antenna is proportional to the square of the field; thus, the longitudinal photons are suppressed by the square of such a factor. We use the verb to suppress because the photon mass, if it has one, is very small, and thus, $\frac{m}{\omega}$ is very small as well.

Last but not least, if the field is static, we have that

$$\omega = \sqrt{\left| \vec{k} \right|^2 + m^2} = 0 \quad (1.82)$$

$$\Rightarrow \left| \vec{k} \right| = im \quad (1.83)$$

This means that the potential of a static charge is the of the Yukawa type, with an exponential decay over the range m^{-1}

$$V_{m \neq 0} = \frac{e^{-mr}}{r} \quad (1.84)$$

In the following subsection, we examine some remarkable massive theories which still have a bearing in contemporary research.

1.2.1.1 Power deviation from Coulomb's law

In the beginning of this Chapter, we have already mentioned that Coulomb (and others) hypothesized and proved experimentally that the electric interactions between two charges has the same behavior of Newton's mechanics. They

experimentally determined the largest data compatible deviation from the inverse square law. This deviation is expressed as an algebraically added index α to the second inverse power (we remind here that proportionality constant $k = \frac{1}{4\pi\epsilon_0}$ has been set to 1 because of the Planck units we have chosen)

$$F = \frac{qQ}{r^{2+\alpha}} \quad (1.85)$$

Of course, such an approach is only roughly correct; however, the precision on such experimental tests can be used to determine initial limits on the photon mass. It is therefore worth mentioning other experiments that have gone in these directions after those mentioned in [27, 28]. In 1936 Plimpton and Lawton [3] improved the Cavendish experiment. They used two concentric conducting spheres, grounded them and then charged the outer one at 3000V. A galvanometer was connected to the two spheres, visible through a conducting window and measured the difference in the potential. The ensuing value for α was smaller than 10^{-6} .

Others attempted to improve, always in laboratory settings, on this number. Among those, we can recall Cochran and Franken [7], Williams, Faller and Hill [?, 4], Bartlett, Goldhagen and Phillips [6]. These experiments improved on the accuracy by up to three order of magnitude, without substantially changing the setup, but reducing the noise due to the mechanical components.

1.2.1.2 de Broglie-Proca

In 1936, Alexandru Proca, in collaboration with his Doctorate Thesis supervisor Luis de Broglie, put unwillingly forward what is, to date, the most relevant advancement in massive, non-Maxwellian theories [9, 11, 33–37]. It is a consistent modification of the classical theory that gives rise to a non-zero photon mass, preserving, at the same time, the explicit Lorentz invariance of electrodynamics. The idea is very simple: the EM Lagrangian is modified by adding a term given by the contraction of the field associated to the photon with itself and multiplied by a factor that is proportional to the mass of the particle. Of course, this is the modern way of writing such Lagrangian, which is heavily influenced by our know-how in QFT¹⁰. Originally, Proca didn't use contravariant vectors, nor described the photon as a vector potential. We have:

$$L_P = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - \frac{\mu^2}{2} A_\mu A^\mu - A^\mu j_\mu \quad (1.86)$$

where μ is the photon mass.

From the Lagrangian, we write the modified version of Maxwell's equations

¹⁰QFT is the acronym for Quantum Field Theory. It is the theory that helps us treat the quantum physics in a relativistic way. The aforementioned QED is the branch of QFT dealing with the electric phenomena.

for the de Broglie-Proca (dBP hereafter) theory. They read

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho - \mu^2\phi \quad (1.87)$$

$$\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t} - \mu^2 \vec{A} \quad (1.88)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (1.89)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.90)$$

where ϕ is the scalar electric potential, $\vec{A}$ the magnetic potential. We recall that a mass has dimension of the inverse of a length. In fact, restoring SI units $\mu = \frac{mc}{\hbar}$ (c being the Maxwellian speed of light and $\hbar$ the reduced Plank constant) and it represents the inverse of the reduced Compton wave length for the photon rest mass. It is important to notice that only the Gauss and Ampère laws are affected by the modification. This is understandable because, as we have underlined previously, the remaining couple of Maxwell's equation come from the Ricci Curbastro-Bianchi identities, i.e. from the geometrical properties of the manifold we are working on. Instead, the dynamic ones are changed. However, the fact that the second couple of Maxwell's equations are left unchanged is important because it tells us that the definitions of the fields in terms of the potential are left unchanged too.

The three most important consequences of the dBP theory are:

1. the potential of a static point charge decreases exponentially
2. the field equations imply a dispersion of the group velocity with the frequency
3. dBP theory breaks a fundamental symmetry of classical EM: the gauge symmetry.

Let's start by discussing the first aspect. From Eq. 1.87 we obtain the potential

$$\phi(r) = q \frac{e^{-\mu r}}{r} \quad (1.91)$$

where q is the point charge positioned at the origin of the reference frame. We see that the scaling factor in the exponential is the reduced Compton wave length μ^{-1} . To be historically correct, we must say that Proca and de Broglie never drew this conclusion. They were more interested in the dispersion effect. It was due to the effort of Yukawa [38], Schrödinger [39] and Bass [40] that such a suppression of the potential was found.

We can see the light dispersion as follows. Eqs. 1.87-1.88 we have that the potentials are

$$A_\mu \sim \exp \left[- \left(\vec{k} \cdot \vec{x} - \omega t \right) \right] \quad (1.92)$$

The wave equation is given by

$$(\square - \mu^2) A_\mu = -j_\mu \quad (1.93)$$

where $\square$ is the D'Alembertian operator and j_μ is the 4-current $(\rho, \vec{j})$. For a vanishing current, then

$$k^2 = \omega^2 - \mu^2 \quad (1.94)$$

Therefore the phase and group velocities become

$$u = \frac{\omega}{k} = \left(1 - \frac{\mu^2}{\omega^2}\right)^{-\frac{1}{2}} \sim 1 + \frac{\mu^2}{2\omega^2} \quad (1.95)$$

$$v_g = \frac{\partial \omega}{\partial k} = \left(1 - \frac{\mu^2}{\omega^2}\right)^{\frac{1}{2}} \sim 1 - \frac{\mu^2}{2\omega^2} \quad (1.96)$$

We clearly see then, that because of the photon mass, there is a dependance on the frequency, which, in first approximation, can be taken as going with the inverse square. Furthermore, the phase and group velocity differ. This relevant feature is used to test the photon mass, namely measuring the time delay in the arrival of EM signals, coming, e.g., from pulsars and Fast Radio Bursts (FRBs). From Eq. 1.96 we evince the time delay. Considering two wave packets with frequencies

$$\omega_1 > \omega_2 \gg \mu \quad (1.97)$$

The difference in velocities is

$$\begin{aligned} \Delta v_g &= v_{g1} - v_{g2} = \mu^2 \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) + O \left[\left(\frac{\mu^2}{\omega_1^2} \right)^4 \right] \\ &= \frac{\mu^2}{8\pi^2} (\lambda_2^2 - \lambda_1^2) + O[(\mu\lambda_1)^4] \end{aligned} \quad (1.98)$$

If both waves travel a distance L , then the time delay is

$$\Delta t = \frac{L}{v_{g1}} - \frac{L}{v_{g2}} \sim \frac{L}{8\pi^2} (\lambda_2^2 - \lambda_1^2) \mu^2 \quad (1.99)$$

once we neglect the higher order terms.

Coming to the gauge symmetry breaking, we are aware of the importance that symmetries play. The fundamental ones (time and space translation, rotation etc...) are a profound, mathematical way of expressing the conservation of quantities that we observe through experiments. Gauge symmetry, however, is very peculiar. The gauge symmetry tells us that we have a freedom in the way we choose the potentials. The peculiarity of such a symmetry is that the potentials are not measurable quantities. Hence gauge symmetry is not associated to a physical conserved quantity. However, it is associated to a conservation law. In Maxwell, we know that a transformation of the form

$$\phi \rightarrow \phi' = \phi - \frac{\partial \Lambda}{\partial t} \quad (1.100)$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla} \Lambda \quad (1.101)$$

where Λ is a function of the event x^μ leaves the theory invariant. The electric and magnetic fields of dBP and Maxwell theories are not influenced by the changes of Eqs.1.100,1.101. However, the explicit mass term in Eqs. 1.87-1.88 couple directly to the potentials, introducing a term that depends on Λ in the equation of motions. The problem is usually circumvented recurring to the local charge conservation principle. In fact, we know that the four current must satisfy the continuity equation

$$\vec{\nabla} \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0 \quad (1.102)$$

This one is satisfied, for the dBP theory, only if the a specific condition is met

$$\vec{\nabla} \cdot \vec{A} + \frac{\partial V}{\partial t} = 0 \quad (1.103)$$

It is, of course, the famous Lorentz condition $\partial_\mu A^\mu$ that we have already encountered. In this context however, it has a completely different meaning. In Maxwell's theory it is a choice we make to decouple the wave equations. Instead, in this context it is a necessary subsidiary condition that breaks the explicit gauge invariance of the theory! This claim is justified because only functions that satisfy the equation

$$\left[\nabla^2 - \frac{\partial^2}{\partial t^2} \right] \Lambda = 0 \quad (1.104)$$

It would therefore seem that requiring the gauge invariance of an EM theory corresponds to asking for the photon mass to be zero. As will see later in the text, this is not so straightforward. There are some experimental limits to the photon mass which have been put forward over the years, as we will now discuss.

In the early 20's, de Broglie assessed - without explanations - the mass upper limit to be $10^{-53} kg$ [9,10]. Later, he used dispersion to set an upper limit of $10^{-42} kg$ (emended of a computation error [11,41]). Nowadays the best dispersion limit is in $3 \times 10^{-49} kg$ [42]. Exploiting the magnetic field of Jupiter and Earth, resulted into a limit of $10^{-51} kg$ [43,44]. Ryutov used the solar wind to lower the limit at $10^{-52} kg$ at a distance of 1AU [45,46] and at $10^{-54} kg$ at 40 AU [46]. The latter is the one currently accepted by the Particle Data Group [47]. Conversely to the model-derived bounds set in [43,44], the Ampère law has been tested experimentally using the data from the ESA mission Cluster. The authors derived an upper limit ranging between $1.4 \times 10^{-49} kg$ and $3.4 \times 10^{-51} kg$ at 1AU, depending on the assessed value of the potentials [48], yielding a mass smaller than $10^{-49} kg$. Through the modeling of the galactic magnetic field limits as constraining as $10^{-63} kg$ have been set [49–51], however, because of how heavily these result depend on the model their validity has been put in doubt by [48].

1.2.1.3 Stueckelberg

As we have mentioned in the above paragraph, in 1957 Stueckelberg [8] found a way to let a massive photon and gauge symmetry coexist. At first, it was

thought that it was merely a mathematical trick to hide the problem. Only after, it was understood that such a theory had a deeper significance. This was achieved mainly through the impact that it had in other fields and, in particular, in QFT. The Stueckelberg mechanism casts a new light also in the understanding that we have of the link between gauge invariance and masslessness.

Stueckelberg's idea was to introduce a new scalar field D in the Lagrangian. This field had a fixed magnitude and carried an electric charge q . In vacuum, the Lagrangian looks as:

$$L_S = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left[\left| -\partial_t D + iq\vec{A}b \right|^2 - \left| \vec{\nabla} D - iq\vec{A}b \right|^2 \right] \quad (1.105)$$

We fix the coupling on the field b so that the additional term in Eq. 1.105 becomes

$$-\frac{1}{2}\mu^2 A_\mu A^\mu \quad (1.106)$$

where the “photon mass” is given by

$$\mu = qD \quad (1.107)$$

In SI units, the last expression would read $\mu = \frac{qD}{\sqrt{\epsilon_0 \hbar c}}$. In the Stueckelberg case, we have obtained a massive term without any restriction on the kind of gauge we must choose. We only chose how to couple the field D , by setting its phase variation to zero.

The lesson to be learnt is that it is not gauge invariance that requires a vanishing photon mass. It is instead the minimal dynamics obeying gauge invariance (that is Maxwell's action) that requires the masslessness of the carrier. However, if there is a coupling of said carrier with another field (D in our case) it is possible to respect gauge invariance and accommodate a massive photon.

We can see some steps in the development of Stueckelberg's theory. A first modification to dBP theory leads to the Lagrangian

$$L = -\partial_\mu A_\nu \partial^\mu A^\nu + m^2 A_\mu A^\mu \quad (1.108)$$

In this case Lorenz gauge fixing arises naturally from equation of motion, whereas it had to be imposed *ad hoc* in the dBP theory. But choosing the form in Eq. 1.108 affects the Hamiltonian (given in [52]) positive definiteness

$$H = -\sum_{\lambda=0}^3 (\partial_\lambda A_\mu) (\partial^\lambda A^\mu) - m^2 A_\mu A^\mu \quad (1.109)$$

Because of the metric signature, A_0 has a negative value and hence H is not positive definite.

In QED this problem is solved imposing the Gupta-Bleuler condition¹¹ [53]

$$\partial^\mu A_\mu^{(-)} |phys\rangle = 0 \quad (1.111)$$

¹¹In QED, a photon is described by a wavefunction defined by the eigenvalues corresponding

where $|phys\rangle$ is the ket for any physical state and $A_\mu^{(-)}$ is the part of the field A_μ containing only annihilation operators¹². Unfortunately, this trick cannot

to the 4-momentum and 4-polarization. Using the common bra-ket notation, the state of the photon is labeled $|k_\mu, \epsilon_\mu\rangle$, where ϵ_μ is indeed the 4-polarization vector. Since the 4-momentum of the photon is null-vector ($k^\mu k_\mu = 0$) and its time component must be positive because it represents the energy, k_μ is uniquely determined by its spatial components. The expectation value for a transition between two quantum states with different momenta and polarizations is

$$\langle \vec{k}_1, \epsilon_\mu | \vec{k}_2, \epsilon_\nu \rangle = -\eta_{\mu\nu} \frac{1}{2|\vec{k}_1|} \delta(\vec{k}_1 - \vec{k}_2) \quad (1.110)$$

The problem with this expression is that it gives positive norms for spatial polarizations and negative ones for time-like polarizations. Since the previous expression is linked to the probability of finding a photon in certain state, this is unacceptable. Furthermore, we have already shown that the photon - if massless - should have two polarizations, not four as it is here the case. One more polarization is hidden in the gauge covariance. It can be set by $k_\mu \epsilon^\mu = 0$ but it gives an unphysical state because it is a mode that is parallel to the 4-momentum. Even so, one should impose a condition that eliminates the two unphysical polarization. Unfortunately, as shown in [53], this cannot be done in a Lorentz covariant way.

The solution to this apparent dilemma is in a redefinition of the space of the physical states in which we work. The new space will be the quotient of the full space by its unphysical components. The result of this operation is that the gauge choice will not be the Lorentz one anymore ($\partial_\mu A^\mu = 0$), but $\partial^\mu A_\mu |phys\rangle$ where we have labeled $|phys\rangle$ the element of the new space.

¹²Creation and annihilation operators are mathematical operators that, acting on a physical state of a system, create or annihilate a particle. We will sketch here how the mechanism works for the case at hand. We warn the reader that, however, our purpose is not to give a full and formal explanation of how the second quantization works.

We have repeatedly said that Maxwell's theory is gauge invariant. Therefore we can choose a gauge to quantize it. Let's pick the Coulomb gauge ($\vec{\nabla} \cdot \vec{A} = 0$). The Fourier expansion of the vector potential confined in a cubic box of volume V reads

$$\vec{A}(r, t) = \sum_{\vec{k}} \sum_{\alpha=-1,1} \left(\epsilon^\alpha(\vec{k}) a_k^\alpha(t) e^{i\vec{k} \cdot \vec{r}} + \bar{\epsilon}^\alpha(\vec{k}) \bar{a}_k^\alpha(t) e^{-i\vec{k} \cdot \vec{r}} \right) \quad (1.112)$$

where ϵ^α is the vector of the two possible polarizations and the sign $-$ indicates the complex conjugate.

The idea of the quantization is to treat the functions $a_k^\alpha(t)$ and $\bar{a}_k^\alpha(t)$ as operators. So

$$a_k^\alpha(t) \rightarrow \sqrt{\frac{1}{2\omega V}} a^\alpha(\vec{k}) \quad (1.113)$$

$$\bar{a}_k^\alpha(t) \rightarrow \sqrt{\frac{1}{2\omega V}} a^{\dagger\alpha}(\vec{k}) \quad (1.114)$$

Technically speaking, the operators are elements of an algebra which is characterized by the following commutation relations

$$[a^\alpha(\vec{k}), a^\beta(\vec{k}')] = [a^{\dagger\alpha}(\vec{k}), a^{\dagger\beta}(\vec{k}')] = 0 \quad (1.115)$$

$$[a^\alpha(\vec{k}), a^{\dagger\beta}(\vec{k}')] = \delta_{\vec{k}, \vec{k}'} \delta_{\alpha, \beta} \quad (1.116)$$

where the symbol $[A, B] = AB - BA$. So the quantized expression for the vector potential is

$$\vec{A}(r, t) = \sqrt{\frac{1}{2\omega V}} \sum_{\vec{k}} \sum_{\alpha=-1,1} \left(\epsilon^\alpha(\vec{k}) a^\alpha(\vec{k}) e^{i\vec{k} \cdot \vec{r}} + \bar{\epsilon}^\alpha(\vec{k}) a^{\dagger\alpha}(\vec{k}) e^{-i\vec{k} \cdot \vec{r}} \right) \quad (1.117)$$

We do realize that this is not a complete and precise explanation of why this is correct, but, once again, the purpose of this text is not to summarize our knowledge on QED.

be applied in the case of the Stueckelberg formalism because it conflicts with commutation relations. The commutation relation gives

$$[\partial^\mu A_\mu(x), \partial^\nu A_\nu(y)] = i\partial^2 \Delta_m(x-y) \quad (1.118)$$

where Δ is the propagator and the subscript m has been added to remind that we are in the massive theory¹³. Actually, in the QED case $\partial^2 \Delta_0 = 0$ and Gupta-Bleuler mechanism works. Instead for the massive vector field, $\partial^2 \Delta_m = -m^2 \Delta_m$ and hence the inconsistency.

Stueckelberg solved the problem by introducing an auxiliary field D , the so called Stueckelberg field. The D field obeys the same equations of motion as A_μ :

$$(\partial_\mu^2 + m^2) D = 0 \quad (1.120)$$

$$[D(x), D(y)] = i\Delta_m(x-y) \quad (1.121)$$

The Gupta-Bleuler condition is replaced by

$$(\partial^\mu A_\mu + mD)^{(-)} |phys\rangle = 0 \quad (1.122)$$

and in this is consistent with Eq. 1.121.

The Stueckelberg Lagrangian turns into:

$$L = -\partial_\mu A_\nu \partial^\mu A^\nu + m^2 A^\mu A_\mu + \partial_\mu D \partial^\mu D - m^2 D^2 \quad (1.123)$$

This expression is gauge invariant. On top of that benefits of the absence of higher order derivatives in the commutation relation, which would render the theory more divergent at higher energies. The gauge fixing condition arises naturally from the equation of motion

$$\partial_\mu \partial^\mu A^\nu + m^2 A^\nu = 0 \quad (1.124)$$

$$\partial_\mu \partial^\mu D + m^2 D = 0 \quad (1.125)$$

and reads

$$(\partial^2 + m^2) \Lambda(x) = 0 \quad (1.126)$$

Eq. 1.123 can be also put in a nicer form at the cost of losing the manifest gauge invariance

$$L = -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m^2 \left(A_\mu - \frac{\partial_\mu D}{m} \right)^2 - (\partial^\mu A_\mu + mD)^2 \quad (1.127)$$

¹³The propagator is defined as

$$i\Delta(x-y) = \langle 0 | [\phi(x), \phi(y)] | 0 \rangle \quad (1.119)$$

where $\phi(x)$ is a field and $|0\rangle$ indicates the ground state of the system.

1.2.1.4 Chern-Simons

Chern-Simons theory is a peculiar but very important theory that assigns a topological mass to the photon. A topological mass is purely inertial, unlike the common kind of mass. In other words, the usual mass couples to the metric and brings an autonomous contribution to the energy-momentum tensor. Instead, a topological mass does not couple to the metric and therefore brings a contribution to the energy-momentum tensor only via $E^2 = p^2 + m^2$. The Chern-Simons theory is also gauge invariant. However, it has the peculiarity of being written only for a 2+1 spacetime, that is with only two spatial dimension. Why do we need such a reduced space?

The reason is rather simple. In a 3+1 spacetime, Maxwell's Lagrangian is the most simple, Lorentz invariant and holds also in a 2+1 space, with some minimal differences. The magnetic field is not a vector anymore, but a pseudo-scalar. In fact, for a 2D vector potential, the magnetic field is defined by $B = \epsilon^{ij} \partial_i A_j$, without a free index that gives it its vectorial nature. The electric field, remains a vector, but only with two components. Hence, the completely antisymmetric Maxwell tensor has only three free components.

However, the interest of reducing by one the dimension of our spacetime lies in the generation of a new Lorentz and gauge invariant term, which is also local. Such a term reads

$$L_{CS} = \frac{\kappa}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \quad (1.128)$$

where $\epsilon^{\mu\nu\rho}$ is the Levi-Civita symbol.

The attentive reader might argue that such a Lagrangian, explicitly depending on A_μ is not necessarily gauge invariant. That's true, but, if we perform a local gauge transformation, the modification to Eq. 1.128 is only a total derivative term

$$\frac{\kappa}{2} \partial_\mu (\lambda \epsilon^{\mu\nu\rho} \partial_\nu A_\rho) \quad (1.129)$$

Such a term can be easily excluded because it doesn't contribute to the action, being just a boundary term. Also the equations of motion reflect this fact and are gauge invariant

$$\frac{\kappa}{2} \epsilon^{\mu\nu\rho} F_{\nu\rho} = j^\mu \quad (1.130)$$

We see that the Chern-Simons theory is of the first order. Therefore, its canonical structure is essentially different from Maxwell's one. Also, a Chern-Simons term cannot be written in a 3+1 spacetime because there is always an odd number of indexes in the structure $A \partial A$, whereas there indexes in the Levi-Civita symbol would be even. Actually, Chern-Simons terms can be obtained in every odd-dimensioned spacetime.

Unlike Maxwell's theory, Chern-Simons doesn't predict interesting solution in absence of a source. Under such assumption, the equation of motion reduces to $F_{\mu\nu} = 0$. However, its interesting features arise when this new term is coupled with the usual Maxwell's Lagrangian. We obtain then the so called

Maxwell-Chern-Simons Lagrangian [14]

$$L_{MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \quad (1.131)$$

Via least action principle, we obtain the Euler-Lagrangian equations

$$\partial_\mu F^{\mu\nu} + \frac{\kappa}{2}\epsilon^{\nu\alpha\beta}F_{\alpha\beta} = 0 \quad (1.132)$$

If we write Eq. 1.132 for the dual of the Maxwell tensor, we obtain

$$[\partial_\mu\partial^\mu + \kappa^2]\tilde{F}^\nu = 0 \quad (1.133)$$

where $\tilde{F}^\nu = \frac{1}{2}\epsilon^{\nu\alpha\beta}F_{\alpha\beta}$. From this equation we can easily see that these equations of motion describe a single degree of freedom with mass κ . Furthermore, gauge invariance is manifest because the operator is trivially invariant and so is $\tilde{F}^\nu$, which satisfies the Bianchi identity too.

1.2.2 Non-linear theories: classical non-linearities

We have already pointed out the distinction between theories that break linearity already at classical level those aiming to reproduce quantum phenomena in an effective fashion. We will therefore follow this point of view in our presentation.

1.2.2.1 Born-Infeld

For the appearance of the electron and photon self-energies, Born and Infeld [12] looked for an extension of Maxwell's theory. In their renowned paper, they proposed what is maybe the most popular and well explored non-linear theory. The success of this try is somewhat recent and finds its roots in the similarity to the Lagrangian emerging from string theory for the gravitational case. The string theory is an attempt to create a quantum theory of gravity. Its fundamental idea is that matter, radiation and, under certain hypothesis, also space and time are just the manifestation of fundamental, multidimensional entities called strings. In this context, it is possible to derive an action for the gravitational field that is very similar in form to the one we will present here below [54]

We will devote the following Chapter to a work of ours, which aims to study the radiation in the Born-Infeld (BI hereafter) theory. Therefore we will limit ourselves to outlining the most important features of such an appealing approach.

As we were recalling, the driving force behind BI attempt is a “principle of finiteness”, according to which a satisfying theory cannot allow physical quantities to diverge.

In order to better understand the ideas behind such a theory it is appropriate to stress a more “philosophical” reason that drove its formulation. In fact, as

the authors themselves underline, there are two mutually exclusive ways to deal with the relation between matter and EM field.

The first one, called unitarian, assumes that there exists only one physical entity, the EM field. Particles are considered singularities of the field and their mass is a derived concept (electromagnetic mass).

The second, currently accepted, is the dualistic standpoint. Field and particles are distinct entities. Particles are the sources of the field and influenced by it, but their distinguishing feature is inertia measured by their mass.

Born and Infeld were perfectly aware that there are well sound reasons for such a preference among physicists:

1. the absence of a fully developed unitarian theory
2. relativistic mass dependance on velocity is not characteristic of EM mass;
3. the Copenhagen School interpretation¹⁴ of Quantum Mechanics (QM) achieved great successes.

Nevertheless, Born and Infeld were convinced of the philosophical superiority of their approach.

They adapted Einstein's approach to relativity. Therein, Einstein replaces the Newtonian kinetic energy $\frac{1}{2}mv^2$ with the relativistic expression $mc^2 \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$ (we restore for this equation only the constant c in order to make the comparison more clear). Born and Infeld transfer this to the context of EM. The usual Lagrangian is, in components,

$$L = \frac{1}{2} (B^2 - E^2) \quad (1.134)$$

is arbitrarily recast into

$$L = b^2 \left(\sqrt{1 + \frac{(B^2 - E^2)}{b^2}} - 1 \right) \quad (1.135)$$

We can remark here that the parameter b plays the role of the upper limit for the electric field. If the magnetic field vanishes and $\vec{E}$ exceeds b in modulus the square root becomes imaginary and the theory is not applicable.

¹⁴The Copenhagen interpretation of QM is the most widely spread and accepted conceptual understanding of QM. It was devised between 1925 and 1957 by many famous physicists like Niels Bohr and Werner Heisenberg. The fundamental assertion of this interpretation is that a physical system defines its properties only through the act of measurement. QM is only able to predict the probability that a specific measure on said system yields a certain result. It is the act of measuring in itself that reduces this set of probabilities to the one value the instruments can register. This property is known as wavefunction collapse.

Opposed to this point of view there is the de Broglie-Bohm interpretation. According to de Broglie and Bohm, a QM system is described not only by the wavefunction but also by a "configuration". The first evolves in time normally, according the Schrödinger's equation. The second is ruled by a different equation called guiding equation. The de Broglie-Bohm interpretation postulates that an actual configuration of the system exists even when no measurement is performed. As opposed to the Copenhagen School, this theory is deterministic.

Einstein's action is invariant under a broader group of transformations with respect to Newton's action. That is why relativity explains a wider range of phenomena. Likewise, Eq. 1.135 substitutes Eq. 1.134 because it is invariant under the group of general transformations, not only the group of special relativity.

In a manifestly Lorentz invariant form, BI Lagrangian reads

$$L_{BI} = \sqrt{-g} \left(\sqrt{1 + F + G^2} - 1 \right) \quad (1.136)$$

where

$$G = \frac{1}{4} F_{\mu\nu} F^{\mu\nu*} \quad (1.137)$$

$$F_{\mu\nu}^* = \epsilon'_{\alpha\beta\mu\nu} F^{\alpha\beta} \quad (1.138)$$

g being the spacetime metric determinant and $\epsilon'_{\alpha\beta\mu\nu} = \frac{1}{2\sqrt{-g}}$ times the normal Levi-Civita tensor.

Via the least action principle, we can obtain the modified Maxwell's equations. They read in vacuum

$$\vec{\nabla} \cdot \vec{D} = 0 \quad (1.139)$$

$$\vec{\nabla} \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} \quad (1.140)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.141)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (1.142)$$

where

$$\vec{D} = \frac{\vec{E} - G\vec{B}}{\sqrt{1 + F - G^2}} \quad (1.143)$$

$$\vec{H} = \frac{\vec{B} - G\vec{E}}{\sqrt{1 + F - G^2}} \quad (1.144)$$

We can immediately see that the the pair of Maxwell's equations coming from the Ricci Curbastro-Bianchi identities are left unchanged. Thus

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (1.145)$$

$$\vec{E} = \vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \quad (1.146)$$

The other pair however, has a very common form. We can recognize that, formally, they don't differ from the usual Maxwell's equations, but written in a medium. This means that the electromagnetic wave behaves as if it traveled in a medium. Because of the non-linearities the vacuum behaves like a medium whose permeability and permittivity are functions of the fields themselves.

We presented here the most general form of BI Lagrangian. In any case, even if the the full BI Lagrangian contains the G^2 term, in practical applications

(including BI original one) this term is neglected because it is of the fourth order in b and hence relevant only in a neighborhood of the particle.

Such a theory is indeed able to solve the problem of the electron self energy. We refer to Chapter 2 for details. The key message is that the physical fields ($\vec{E}$ and $\vec{B}$) are finite everywhere. Only the unphysical $\vec{D}$ and $\vec{H}$ do actually diverge at the point where the particle is located. These fields are considered unphysical because they are only mathematical tools we introduce to simplify the form of the equations. They do not represent fields that can actually be measured. In an experiment we always measure the fields $\vec{E}$ and $\vec{B}$ only.

1.2.2.2 Podolsky

It is known that Higher Derivatives Theories (HDT) have the property of improving renormalizability [55], smearing ultraviolet divergences¹⁵, and of making the self-energies of point particles finite thanks to an electric field regular at the origin.

Among HDTs the most relevant one was formulated by Podolsky, which is the only linear, Lorentz and gauge invariant generalization of Maxwell's theory. Its Lagrangian reads [13]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a^4}{4}(\partial^\lambda F^{\mu\nu})\partial_\lambda F_{\mu\nu} + j^\mu A_\mu \quad (1.147)$$

By variational principle one obtains the following equations of motion

$$a^4\partial_\mu\partial_\lambda[\partial^\lambda F^{\mu\nu}] + \partial_\mu F^{\mu\nu} + j^\nu = 0 \quad (1.148)$$

which can be written in vector form as

$$a^4\partial_\mu\partial^\mu(\vec{\nabla}\cdot\vec{E}) + \vec{\nabla}\cdot\vec{E} - \rho = 0 \quad (1.149)$$

$$a^4\partial_\mu\partial^\mu\left[\frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B}\right] + \frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B} + \vec{j} = 0 \quad (1.150)$$

If we take into account the Lorenz gauge fixing $\partial_\mu A^\mu = 0$ we have

$$(1 - a^2\Box)\Box A_\mu = 4\pi j_\mu \quad (1.151)$$

It has been shown that this equation admits solutions of the form

$$A_\mu = A'_\mu - A''_\mu \quad (1.152)$$

¹⁵We are in presence of a ultraviolet divergence if an integral or a phenomenon diverges for very high energies or for very short distances. Generally, this divergences arise in developing a given theory perturbatively and can be erased via a suitable renormalization procedure. If the procedure is not successful, it means that the perturbative expansion is not well defined. The name follows from the first known example of ultraviolet divergence: the black body radiation. According to the 19th century thermodynamics, scientists expected a black body to produce much more ultraviolet radiation. Experiments showed the opposite, leading to surpassing classical mechanics in favor of the new developing quantum mechanics.

where

$$\square A'_\mu = 4\pi j_\mu \quad (1.153)$$

$$\square A''_\mu + b^2 A''_\mu = 4\pi j_\mu \quad (1.154)$$

A'_μ can be interpreted as the usual massless mode associated with the common photon, whereas A''_μ is to be considered a massive mode that automatically makes a cutoff a arise. Over the years Podolski's models has been accepted as an effective field theory and a sets the limit where it ceases to be valid and the classical regime is restored, as we will see in a just few lines.

Solving Eq. 1.151 for the static case the electric potential ϕ

$$\phi = \frac{e}{4\pi r} \left(1 - e^{-\frac{r}{a}} \right) \quad (1.155)$$

so we see that a can also be considered a cutoff. It is obvious that for $r \gg a$ Eq.1.155 becomes the Coulomb potential again, whereas if $r \rightarrow 0$ then $\phi \rightarrow \frac{e}{4\pi a}$, which is a finite quantity.

Here we can better understand in what sense a sets the validity limit of Podolski's model seen as an EFT: deviation from Maxwell's laws can be thought as quantum effects that cease for distances $r \gg a$. As anticipated at $r = a$ the classical regime is restored.

1.2.3 Non-linear theories: quantum non-linearities

1.2.3.1 Heisenberg-Euler

Also the Heisenberg-Euler theory [15] will be fully disclosed in Chapter 2. Here we will only outline the foundations of this theory.

Heisenberg-Euler Theory (HE) finds its root in 1928 Dirac's theory of the electron [56]. In 1931 Dirac associated the "holes" in its sea with positive charged electrons [57]. His idea found experimental support in the discovery of positron, as he discussed in 1933 [58].

As soon as the theory found sound basis, scientists started investigating it. Halpern and Debye pointed out independently that the new theory implied light-light scattering [15,59–65]. Heisenberg decided that his student Euler could study this topic in his doctorate thesis [15,64].

During his studies Euler published a paper with another of Heisenberg's students, Kockel. In that paper, they pointed out that quantum vacuum could be seen as a medium [66]; an idea that roots back to the BI theory. Heisenberg's idea was for his student to explain vacuum polarization with modifications to Maxwell's laws. Euler and Kockel computed the leading order modification to Maxwell's Lagrangian and interpreted it as vacuum polarization in the sense of defining a polarization vector and magnetic permeability compatible with the correction they had found. This has a strong resemblance, as they pointed out themselves, to BI theory, but in the QM framework.

In modern terms what they did was studying QED vacuum polarization in constant background and obtaining non-linear corrections in power of the field strength.

The real improvement, though, was made in 1936 when Heisenberg and Euler published an extended result [15]. In this paper a closed form for the Lagrangian was presented, a form incorporating all perturbative orders. It reads [67, 68]

$$L = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + e^2 \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \cdot \left\{ i\frac{\eta^2}{2} F^{\mu\nu} F_{\mu\nu}^* \right. \quad (1.156)$$

$$\left. \frac{\cos \left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F_{\mu\nu}^*} \right]}{\cos \left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F_{\mu\nu}^*} \right] - c.c.} + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6} \cdot F_{\mu\nu}F^{\mu\nu} \right\} \quad (1.157)$$

where the symbol *c.c.* stands for the complex conjugate of the previous expression. Again a critical field appears

$$\mathfrak{E}_k \left(= \frac{m^2 c^3}{e\hbar} \right) \sim 10^{18} \frac{V}{m} \sim 4.4 \cdot 10^{13} G \quad (1.158)$$

where, in brackets, we have put the SI units expression. This field had already been associated to the field strength at which real electron-positron pairs would be produced from vacuum. Or, with the words they would have used, the strength at which Dirac sea electron would tunnel into continuum. Unlike the BI critical field, this one does not set an upper limit to the values the fields can take. On the contrary, the phenomena that the HE theory describes are greatly suppressed under this threshold.

Heisenberg and Euler understood that quantum vacuum instability under the action of a classical electric field would give physical effects. Not only: they pointed out that magnetic and electric fields had different consequences. The former alone responded to particle and antiparticle wave function differently, the latter caused, if strong enough, physically measurable pair production.

They were able to compute the rate of such phenomenon, seeing it as tunneling effect. The result is

$$\exp \left[-\frac{\mathfrak{E}_k}{|E|} \right] \quad (1.159)$$

Heisenberg and Euler identified what the terms in Eq. 1.157 referred to. In particular the ones with minus signs inside the bracket. The first one is the subtraction of the infinite free field effective action. The second is related to Maxwell's usual charge renormalization.

Expanding Eq. 1.157 for strong fields we get

$$\begin{aligned} \mathcal{L} \sim & F + \frac{2\alpha^2}{45m^4} (4f^2 + 7G^2) + \frac{32\pi\alpha^3}{315m^8} (8F^3 + 13FG^2) \\ & + \frac{128\pi^2\alpha^4}{945m^{12}} (446F^4 + 201F^2G^2 + 38G^4) \end{aligned} \quad (1.160)$$

As we will see, this form is particularly helpful for what we pursue in the next Chapter.

Chapter 2

APPLICATIONS OF NON-LINEAR THEORIES

This Chapter will be devoted to the study of some innovative applications of non-linear theories that we have devised over the course of this doctorate thesis. We will propose here two examples, one dealing with the field originated by an accelerating charge in the Born-Infeld (BI) theory and the other concerning the frequency shift an Heisenberg-Euler (HE) photon experiences going through an overcritical magnetic field.

Résumé

Dans ce Chapitre, nous focalisons notre attention sur deux théories, celles de Born-Infeld (BI) et de Heisenberg-Euler. Pour les deux, nous présentons quelques caractéristiques supplémentaires et les appliquons dans un cas spécifique. Malheureusement, pour l'application de BI nous avons rencontré des difficultés informatiques qui étaient insurmontables, étant donné les contraintes de temps.

Comme mentionné dans la description de la théorie BI, une de ses caractéristiques intéressantes est qu'elle aide à lisser les champs à l'origine, empêchant l'apparition de divergences dont il est difficile de s'occuper. Bien sûr, une telle caractéristique serait d'une grande aide pour résoudre le problème de la force propre pour une charge accélérée. Dans les électrodynamiques, on divise souvent le problème en deux classes. Dans l'une, on considère que la configuration de la charge est connue et l'on trouve le champ. Dans l'autre, les champs sont connus et l'on étudie le mouvement des charges. Bien qu'elle soit adaptée puisque l'énergie de la radiation est fréquemment négligeable, l'approche reste une approximation et la solution du vrai problème nécessiterait un traitement des deux aspects en même temps. Une formulation formelle et consistante pour un tel cas n'existe pas pour le moment. Dans la physique classique, l'approche d'Abraham-Lorentz est souvent adoptée. Malheureusement, à cause de la singu-

larité à l'origine, cela produirait une énergie propre infinie. Un « rayon classique » doit donc être associé aux charges ponctuelles comme l'électron, dans le but d'éviter ce problème. Une fois ces prémices établis, il devient évident qu'une théorie régulière serait utile. Jusqu'à ce jour il n'existe aucune solution pour une charge rayonnante dans la théorie de Born-Infeld. Donc, nous avons essayé de faire des avancées dans cette direction en calculant le champ d'une charge en mouvement avec une accélération constante.

Notre tentative nécessite l'introduction de l'espace-temps de Rindler. Cet espace-temps est celui vu par un observateur solidaire avec une particule relativiste en constante accélération [69]. En fait, il est possible de trouver un changement de coordonnées non-linéaires qui conserve l'élément de ligne de Minkowski et décrit le mouvement non-inertiel. Écrit dans le référentiel qui bouge avec la charge mais dans les coordonnées de Minkowski, l'équation dynamique pour une particule se déplaçant avec une accélération constante α est

$$x^2 - t^2 = \frac{1}{\alpha^2} \quad (2.1)$$

Sans perte de généralité, nous nous sommes restreints à seulement une dimension spatiale. Les coordonnées de Rindler sont ensuite définies comme

$$\begin{cases} t = \frac{\sqrt{2\alpha\zeta}}{\alpha} \sinh(\alpha\tau) & x = \frac{\sqrt{2\alpha\zeta}}{\alpha} \cosh(\alpha\tau) & \text{for } \zeta > 0 \\ t = \frac{\alpha}{\sqrt{-2\alpha\zeta}} \sinh(\alpha\tau) & x = \frac{\alpha}{\sqrt{-2\alpha\zeta}} \cosh(\alpha\tau) & \text{for } \zeta < 0 \end{cases} \quad (2.2)$$

Avec la redéfinition supplémentaire des coordonnées

$$\zeta = \frac{\alpha}{2} X^2 \quad (2.3)$$

L'intervalle devient (en incluant les deux dimensions spatiales que nous avons négligées)

$$ds^2 = (\alpha X)^2 d\tau^2 - dX^2 - dY^2 - dZ^2 \quad (2.4)$$

where $Y = y$ and $Z = z$.

L'avantage de passer aux coordonnées de Rindler est qu'il est possible de résoudre le problème électromagnétique dans un référentiel instantanément inertiel. On peut utiliser toute l'expérience que l'on a en résolvant les équations différentielles en coordonnées non-linéaires et qui vient de la Relativité Générale. Une fois que l'on a la solution, c'est suffisant pour réaliser la transformation inverse pour obtenir les champs dans le référentiel non-comobile. On écrit les équations de Maxwell dans les coordonnées de l'élément de ligne Eq. 2.4

$$\vec{\nabla} \times \left[(1 + gx) \vec{E} \right] = 0 \quad (2.5)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (2.6)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2.7)$$

$$\frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \times \left[(1 + gx) \vec{B} \right] = -4\pi (1 + gx) \vec{j} \quad (2.8)$$

Par conséquent, en termes de potentiel, le champ électrique est

$$\vec{E} = \frac{\vec{\nabla}\Phi}{(1+gx)} \quad (2.9)$$

On doit insérer ce résultat dans la relation constitutive de Born-Infeld et ensuite prendre la divergence pour trouver la dynamique de la charge. Donc l'équation du mouvement est

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla}\Phi}{(1+gx)} \cdot \frac{1}{\sqrt{1 - \frac{\vec{\nabla}\Phi^2}{b^2(1+gx)}}} \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.10)$$

À ce stade, nous poursuivons pour produire une expansion perturbative autour de la solution de Maxwell, puisque les corrections de Born-Infeld sont très petites du fait qu'elles soient supprimées par le champ b qui est très grand. L'équation différentielle pour la perturbation φ est

$$\frac{\nabla^2\varphi}{\chi} - \frac{\partial_x\varphi}{\chi^2} = \frac{1}{g^2} \cdot \vec{\nabla} \cdot \left[-\frac{1}{2} \frac{\vec{\nabla}\Phi_0^2 \vec{\nabla}\Phi_0}{b^2\chi^3} \right] \quad (2.11)$$

L'équation différentielle pour la perturbation $\chi = \frac{1+gx}{g}$. En principe, la solution peut être trouvée par convolution. En fait, la fonction de Green de l'opérateur homogène est connue et est [70]

$$G(\vec{X}, \vec{X}_0) = \frac{\chi^2 + (y-y_0)^2 + (z-z_0)^2 + \chi_0^2}{4\pi\sqrt{[\chi^2 + (y-y_0)^2 + (z-z_0)^2 + \chi_0^2]^2 - 4\xi^2\xi_0^2}} \quad (2.12)$$

Donc la perturbation serait donnée par

$$I(\vec{X}) = -\frac{256q^3}{4\pi b^2 g^2} \int \left[\frac{\chi^2 + (y-b)^2 + (z-c)^2 + a^2}{\sqrt{[\chi^2 + (y-b)^2 + (z-c)^2 + a^2]^2 - 4\chi^2 a^2}} \cdot \frac{a\chi_0^6}{\left\{ [a^2 + (b-y_0)^2 + (c-z_0)^2 + \chi_0^2]^2 - 4a^2\chi_0^2 \right\}^{\frac{7}{2}}} \right] dadbdc \quad (2.13)$$

En plus de la divergence classique sur la ligne d'univers, une divergence additionnelle survient où les variables d'intégration prennent les valeurs au point où l'on calcule le champ. La première divergence est attendue et normalement éliminée par convolution. La deuxième, cependant, n'a pas d'équivalent classique.

La complexité de l'intégrale est telle que nous n'étions pas capable de la résoudre, aussi bien de façon analytique qu'informatique. Nous avons focalisé nos essais principalement sur la partie analytique, en cherchant aussi des solutions venant de l'utilisation des fonctions hypergéométriques. Seulement après avoir réalisé que de tels essais ne mènent nulle part, nous avons essayé d'approcher le problème du point de vue computationnel. À cause d'un manque d'expertise et de temps, aucun progrès considérable n'a abouti à cet égard. L'auteur et ses collaborateurs ont l'intention de continuer à travailler sur ces problèmes dans le futur. Toutefois, nous voulons souligner que de tels problèmes mathématiques peuvent être le résultat du fait que nous ayons développé la solution de BI: la série entière doit être convergente, mais, à notre connaissance, il n'existe aucune littérature nous assurant que chaque terme est fini.

La deuxième moitié de ce Chapitre est dédiée à l'étude de la propagation d'un photon provenant d'un champ dipolaire supercritique d'Heisenberg-Euler. La plupart des applications connues de la théorie d'Heisenberg-Euler viennent des expériences faites en laboratoire [71–79]. Cependant, la récente découverte des étoiles à neutrons dotées d'un champ magnétique supercritique appelé magnétars nous encourage à regarder à plus d'applications possibles dans le contexte astrophysique. Comme montré dans [80, 81], la propagation des excitations à haute énergie d'une théorie électromagnétique non-linéaire sur un fond fixé est gouvernée par une métrique efficace qui dépend de la métrique du fond de l'espace-temps, de la configuration du fond du champ électromagnétique, et des détails de la dynamique non-linéaire respectés par le champ électromagnétique (voir [82] pour une review). Par la suite, nous donnerons le décalage de la fréquence électromagnétique pour le cas du champ magnétique supercritique d'Heisenberg-Euler et discuterons des implications sur la propagation d'un photon. Le décalage est dû à un échange d'énergie entre les perturbations (dans ce cas, des photons) et le fond. Cette formulation peut être considérée comme un premier pas dans le processus de modélisation du décalage électromagnétique pour un magnétar. Nous analyserons la dépendance angulaire du décalage électromagnétique pour chacune des deux polarisations du photon, et montrerons qu'il peut y avoir un décalage vers le rouge ou vers le bleu, selon la polarisation du photon.

La première étape est de comprendre comment le formalisme de la métrique efficace fonctionne. Un photon se propageant dans un fond d'Heisenberg-Euler interagirait avec ce dernier via les non-linéarités et donc dévierait de la géodésique de l'espace-temps de Minkowski. Cet effet est équivalent à considérer un photon maxwellien se propageant dans un espace-temps différent dans lequel les non-linéarités sont prises en compte par la métrique. Dans ce nouvel espace-temps, la propagation du photon est gouvernée par l'équation géodésique, qui se révèle plus facile à résoudre que les équations de Maxwell modifiées. Pour un lagrangien dépendant de deux scalaires de Lorentz $F = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ et $G = \frac{1}{8}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$, cette nouvelle métrique est [83]

$$^{(\pm)}\tilde{g}^{\mu\nu} = L_F\eta^{\mu\nu} - 4[(L_{FF} + \Omega_{\pm}L_{FG})F_{\lambda}^{\mu}F^{\lambda\nu} + (L_{FG} + \Omega_{\pm}L_{GG})F_{\lambda}^{\mu}F^{\lambda\nu}] \quad (2.14)$$

où

$$\Omega_{\pm} = \frac{-\Omega_2 \pm \sqrt{\Delta}}{2\Omega_1} \quad (2.15)$$

$$\Omega_1 = -L_F L_{FG} + 2F L_{FG} L_{GG} + G (L_{GG}^2 - L_{FG}^2) \quad (2.16)$$

$$\Omega_2 = (L_F + 2G L_{FG}) (L_{GG} - L_{FF}) + 2F (L_{FF} L_{GG} + L_{FG}^2) \quad (2.17)$$

$$\Omega_3 = L_F L_{FG} + 2F L_{FF} L_{GG} + G (L_{FG}^2 - L_{FF}^2) \quad (2.18)$$

Nous avons utilisé le symbole L_X pour indiquer la dérivée partielle du lagrangien par rapport à X , $\frac{\partial L}{\partial X}$. Dans notre cas, nous avons un dipôle magnétique pur. Par conséquent, le champ électrique est identiquement nul et G est zéro. Donc, [84]

$$^{(1)}\tilde{g}^{\mu\nu} = [L_F \eta^{\mu\nu} - 4L_{FF} F_{\alpha}^{\mu} F^{\alpha\nu}]_0 \quad (2.19)$$

$$^{(2)}\tilde{g}^{\mu\nu} = [(L_F - 2F L_{GG}) \eta^{\mu\nu} - 4L_{GG} F_{\alpha}^{\mu} F^{\alpha\nu}]_0 \quad (2.20)$$

où l'indice zéro indique que la quantité a été évaluée sur le fond.

Le calcul du décalage électromagnétique doit être traité avec grande prudence. Nous montrons que p_0 n'est pas une quantité conservée le long de la géodésique dans l'espace défini par la métrique efficace. La nouvelle quantité conservée est donnée par

$$\tilde{p}_K = \tilde{g}^{\mu\nu} p_{\mu} K_{\nu} \quad (2.21)$$

et se relie à p_0 via

$$p_0 = g^{00} (\tilde{g}^{00})^{-1} \tilde{p}_K \quad (2.22)$$

D'après la définition de la fréquence en relativité – marquant R (E) le point de réception (émission)

$$1 + z = \frac{g^{00}(E) u_0(E) p_0(E)}{g^{00}(R) u_0(R) p_0(R)} \quad (2.23)$$

nous obtenons

$$1 + z = \frac{\tilde{g}^{00}(R)}{\tilde{g}^{00}(E)} \quad (2.24)$$

Nous remarquons que cette expression s'annule correctement si la théorie est linéaire comme $\tilde{g}^{\mu\nu} = g^{\mu\nu}$. De plus, nous soulignons que Eq. 2.24 corrige les résultats présents dans la littérature [85, 86]. Pour calculer le décalage électromagnétique dans l'espace-temps du dipôle de HE, nous fixons le point de réception à l'infini spatial, où la métrique se réduit au fond plat. Par conséquent, $\tilde{g}^{00}(R) = g^{00}(R) = 1$, et l'on obtient

$$^{(1)}z = L_F(E) - 1 \quad (2.25)$$

$$^{(2)}z = [L_F(E) - 2F(E) L_{GG}(E)] - 1 \quad (2.26)$$

Pour terminer notre calcul, nous avons besoin des expressions pour les dérivées lagrangiennes dans le cas de la théorie d'Heisenberg-Euler. Elles sont données dans [84] et dépendent du paramètre κ donné par

$$\kappa = \frac{B}{B_c} \quad (2.27)$$

B étant le module du champ magnétique. Puisque les dérivées doivent être calculées sur le fond et que ces expressions sont déjà du premier ordre en α , nous avons simplement besoin du module d'ordre zéro du champ dipolaire qui est donné par

$$B_0 = B_c \lambda \sqrt{1 + 3u^2} \quad (2.28)$$

où

$$u = \cos \theta \quad (2.29)$$

$$\lambda = \frac{m}{r_E^3 B_c} \quad (2.30)$$

isi nous utilisons des coordonnées sphériques et fixons le dipôle le long de l'axe z .

La dépendance du décalage avec l'angle et la force du champ est finalement obtenue en écrivant κ comme

$$\kappa = \frac{B_0}{B_c} = \lambda \sqrt{1 + 3u^2} \quad (2.31)$$

Et en insérant cette expression dans les dérivées du lagrangien. En traçant le décalage en termes de la variable κ , quantique. Il est important de remarquer que le décalage est négatif pour la première polarisation (donc un décalage vers le rouge) et positif (donc c'est en fait un décalage vers le bleu) et supérieur pour la seconde.

Des graphes de z en fonction de $\cos \theta$ pour différentes valeurs de λ montrent que pour les deux polarisations le décalage augmente (en valeur absolue) avec le champ, mais que $^{(1)}z$ a un maximum à l'équateur et un minimum aux pôles, $^{(2)}z$ se comporte de façon opposée.

Ce nouvel effet peut être important au moins dans deux domaines. Le premier est l'astrophysique des magnétars. Pour donner une description complète de tels objets, la gravitation et la rotation ne devraient pas être négligées, mais l'importance du décalage peut être vue dans un cadre plus simple, prenant en compte que le décalage s'ajoute au décalage gravitationnel vers le rouge (voir [85] pour la preuve de cette affirmation dans le cas du décalage électromagnétique vers le rouge). Pour une étoile de deux masses solaires et un rayon de 10^4 km (valeurs typiques pour un magnétar), le décalage gravitationnel vers le rouge (calculé en utilisant la métrique de Schwarzschild-Droste [87]) serait approximativement de 0.2. Donc, pour des champs de fond suffisamment forts, le décalage peut atteindre des valeurs qui sont de l'ordre du décalage gravitationnel vers le rouge (pour une des polarisations).

Cet effet mènerait à une variation du décalage des photons venant de la surface d'une étoile et donc dans le rapport de la masse et du rayon.

2.1 Field of an accelerated particle in the Born-Infeld theory

2.1.1 The regular field

For a pointlike charge, like an electron, the Maxwellian potential (and hence the field) is singular at the location of the source. Thereby, the self-energy of the charge is divergent at the particle position. This corresponds to stating that an infinite energy is required to bring the particle from spatial infinity to its actual location. In the preceding Chapter, we have already explained why such infinities represent a physical problem and a conceptual setback to any theory.

While introducing the BI theory, we have oftentimes repeated that the authors purpose was, among others, to generate a divergence free theory for the electron [12]. We show afterward how the physical fields are indeed regular at the source.

Recalling Eqs. 1.139-1.142, we remind that these BI equations correspond to those of Maxwell in a medium. Indeed BI electromagnetism in vacuum might be interpreted as Maxwell electromagnetism in a medium. This comparison is however, purely formal. Unlike what happens in Maxwellian electromagnetism, whenever permittivity or permeability differ from the vacuum values, the measurable fields in the BI theory are not $\vec{D}$ and $\vec{H}$. The real fields are $\vec{E}$ and $\vec{B}$, for which singularities - as we shall see - are removed. In other words, the BI vacuum doesn't actually have the properties of a medium. It is just a representation of the chosen approach.

Put otherwise, the BI theory regularizes the physical fields $\vec{E}$ and $\vec{B}$, while the remained divergences of $\vec{D}$ and $\vec{H}$ are shown to not troublesome. Unfortunately, given the high degree of complexity of BI non-linearity, a full solution to the dynamical problem does not, to the best of our knowledge, exist. This motivates the efforts outlined in this Chapter while it justifies the lack of a result. However, BI electromagnetism has been successfully applied to several specific cases. For example, extensive work has been done in the case of a planar solution [88–90]; there have been also efforts related to waveguides [91], Coulomb scattering [92], black holes thermodynamics [93] and Kaluza-Klein theories [94].

The regularity of the BI solutions can be appreciated in a very simple case. For the electrostatic case (hence $\vec{B} = 0$), assuming central symmetry Eq. 1.139 reduces to

$$\frac{d}{dr}(rD_r) = 0 \Rightarrow D_r = \frac{e}{r} \quad (2.32)$$

From the definition of the $\vec{D}$ field

$$D_r = -\frac{\phi'_r}{\sqrt{1 - \frac{\phi_r'^2}{b^2}}} \quad (2.33)$$

To derive this equations we have used Eq. 1.143 with G equal to zero because we don't have magnetic field and have written the electric field in as the time

derivative of the potential $E_r = -\phi'_r$. Therefore the differential equation for ϕ gives the result

$$\phi(r) = \frac{e}{r_0} f\left(\frac{r}{r_0}\right) \quad (2.34)$$

where

$$r_0 = \sqrt{\frac{e}{b}} \quad (2.35)$$

$$f(x) = \int_x^\infty \frac{dy}{\sqrt{1+y^4}} \quad (2.36)$$

e being the elementary charge and b the upper limit to the field as seen in Sec. 1.2.2.1. This is the new Coulomb potential. For $x \gg 0$ the potential is conform to the usual expression. But we will see that this expression stays finite everywhere.

Substituting $x = \tan \frac{1}{2}\beta$

$$f(x) = \frac{1}{2} \int_{\bar{\beta}(x)}^\pi \frac{d\beta}{\sqrt{1 - \frac{1}{2} \sin^2 \beta}} = \frac{1}{2} F\left(\frac{1}{\sqrt{2}}, \bar{\beta}\right) \quad (2.37)$$

$$\bar{\beta} = 2 \arctan x \quad (2.38)$$

where $F(k, \bar{\beta})$ is the Jacobian elliptic integral. Therefore, in $x = 0$ the function f takes the value

$$f(0) = \frac{1}{2} F\left(\frac{1}{\sqrt{2}}, \frac{\pi}{2}\right) = 1.8541 \Rightarrow \phi(0) = 1.8541 \frac{e}{r_0} \quad (2.39)$$

implying that ϕ is finite everywhere. Thus D suffers of a singularity but ϕ and E do not. Passing to Cartesian coordinates the field has a discontinuity in the origin, but not a singularity. This, of course, solves the problem of the electron self-energy.

As previously stated, a full solution to the radiation problem is missing, let alone an expression for the radiation reaction. However, it is reasonable to assume that the property of regularity is carried to the general solution. Thereby investing some effort in analyzing radiation is justified. Indeed, the divergence at the source of Maxwell's potential is the root of problems in the EM self-force problem. If it were possible to have a solution to the BI theory, one could treat the self-force problem in a much simpler way and make experimental comparisons either confirm or disprove BI approach.

2.1.2 The Rindler spacetime

Before proceeding with our analysis, we pause here to introduce a tool that will be used in the following paragraph. We have announced that we are interested in studying the radiation of an accelerated charged particle. Hence, we need to put ourselves in a suitable framework to describe the motion of a relativistic

accelerated pointlike source. Thus, we choose a suitable framework, that of Rindler [69], to construct the motion of a relativistically accelerated pointlike source. We recollect the features of this known framework that are salient for our analysis.

In special relativity, the 4-acceleration α^μ is defined as the derivative with respect to the proper time τ of the 4-velocity $u^\mu = (\gamma, \gamma\vec{v})$

$$\alpha^\mu = \gamma \left(\frac{d\gamma}{dt}, \frac{d(\gamma\vec{v})}{dt} \right) \quad (2.40)$$

where $\gamma = \frac{dt}{d\tau}$. Obviously, the proper acceleration is that observed by a comoving observer, for whom $\gamma = 1$. In this case, the first term in the 4-acceleration is vanishing and the second becomes, as usual, the second derivative of the position with respect to proper time. Calling α_{RF}^μ the rest frame 4-acceleration, we study the motion of the charge if the α_{RF}^μ is constant.

For simplicity, and without loss of generality, we reduce to one spatial component, by fixing the direction of motion along $\hat{x}$. The two remaining spatial components remain unaffected by the motion. If the proper acceleration is a constant, we have

$$\alpha = \frac{d(\gamma v)}{dt} = \frac{d}{dt} \left[v (1 - v^2)^{\frac{1}{2}} \right] = \text{const} \quad (2.41)$$

where v is the space velocity along $\hat{x}$. Again, without loss of generality, we can set $\alpha > 0$, *i.e.* demanding that the particle accelerates in the positive x direction. We use the separation of variables method in Eq. 2.41 along with the initial condition $v(t=0)$, and get

$$v(t) = \frac{\alpha t}{\sqrt{1 + (\alpha t)^2}} \quad (2.42)$$

But $v(t) = \frac{dx(t)}{dt}$ and the trajectory expansion is

$$x(t) = \alpha \int_0^t \frac{dt' t'}{\sqrt{1 + (\alpha t')^2}} = \frac{1}{\alpha} \sqrt{1 + (\alpha t)^2} \quad (2.43)$$

Of course, the position of the particle, can be expressed in terms of the acceleration via the position

$$x(0) = \frac{1}{\alpha} \quad (2.44)$$

Being the particle still in the comoving rest frame, the equation of motion acquires a hyperboloid form, displayed in Fig. 2.1

$$x^2 - t^2 = \frac{1}{\alpha^2} \quad (2.45)$$

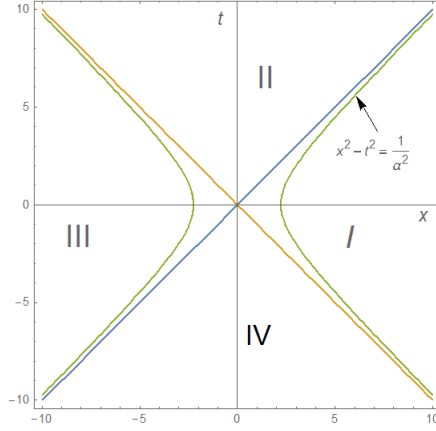


Figure 2.1: Spacetime in Rindler coordinates. Region I is the Rindler wedge. Region III the anti-Rindler wedge. Regions II and IV are the Milne and anti-Milne wedges respectively.

A plot of the arising situation can be found in Fig.

Under such description, the particle we are on starts from spatial positive infinity when we are infinitely in the past; we then get to $t = 0$ at $x = \frac{1}{\alpha}$ to finally return to spatial infinity in an infinitely distant future (Region I). The hyperbolas are constrained, of course, outside the lightcone, so the dynamics is confined into the so called Rindler's wedge. This latter is obviously spanned by a bundle of hyperbolas with various values of α . It is interesting to point out that the lightcone is actually an horizon for the particle. In fact, no causal signal can reach a point situated above the line $t = x$. Likewise, no signal originated under the horizon $t = -x$ can reach our particle.

Even if we will not deal with the other wedges and their properties a brief presentation is fair. Region II, called Milne region, contains all point separated by a time like interval in the future. It represent an ensemble of particles that move along x with different constant velocities and that cross the origin. Region III is called anti-Rindler wedge and has the same property of the Rindler wedge, with the minor difference that the acceleration is in the negative direction of the axis. Region IV is instead the anti-Milne space, which is spanned by all the worldlines of particles coming from the past and approaching the origin with different but constant velocities.

In Region I, we can see that, the coordinate transformation

$$\begin{cases} t = \frac{\sqrt{2\alpha\zeta}}{\alpha} \sinh(\alpha\tau) & x = \frac{\sqrt{2\alpha\zeta}}{\alpha} \cosh(\alpha\tau) & \text{for } \zeta > 0 \\ t = \frac{\alpha}{\sqrt{-2\alpha\zeta}} \sinh(\alpha\tau) & x = \frac{\alpha}{\sqrt{-2\alpha\zeta}} \cosh(\alpha\tau) & \text{for } \zeta < 0 \end{cases} \quad (2.46)$$

satisfies Eq. 2.45. In terms of τ and ζ , the Minkowski metric reads

$$ds^2 = 2\alpha\zeta d\tau^2 - \frac{d\zeta^2}{2\alpha\zeta} - d\rho^2 - \rho^2 d\theta^2 \quad (2.47)$$

where we have passed to planar coordinates for the two directions that are not involved in the motion, defining $\rho = \sqrt{y^2 + z^2}$ and $\theta = \arctan\left(\frac{y}{x}\right)$. For future use it is convenient to adopt another coordinate that is

$$\zeta = \frac{\alpha}{2} X^2 \quad (2.48)$$

and thus

$$d\zeta = \alpha X dX \quad (2.49)$$

Inserting Eqs. 2.48, 2.49 in Eq. 2.47 the metric interval is

$$\begin{aligned} ds^2 &= 2\alpha \frac{\alpha}{2} X^2 d\tau^2 - \frac{\alpha^2 X^2 dX^2}{2\alpha^2 \frac{X^2}{2}} - d\rho^2 - \rho^2 d\theta^2 \\ &= (\alpha X)^2 d\tau^2 - dX^2 - dY^2 - dZ^2 \end{aligned} \quad (2.50)$$

where $Y = y$ and $Z = z$.

2.1.3 The Maxwellian case

Stepping back and looking at what the Rindler formalism implies, after all the coordinate changes, we finally view the dynamics of a relativistic, uniformly accelerating particle as if it were at rest. Such perspective has been formerly applied to the Maxwellian case with the intent of assessing and comparing the difficulty of the computations [70, 95–100]. It is worth summarizing here how these computations work and the main results in order to prepare us for the BI case.

First the Maxwell tensor needs to be defined in the Rindler coordinates. The tensor must be transformed according to the new metric, therefore

$$F^{\hat{0}\hat{i}} = \sqrt{g_{00}|g_{ii}|} F^{0i} \quad (2.51)$$

$$F^{\hat{i}\hat{j}} = \sqrt{|g_{ii}||g_{jj}|} F^{ij} \quad (2.52)$$

where we have denoted with hatted indexes the Rindler components, and the metric $g_{\mu\nu}$ is, following Eq. 2.50

$$g_{\mu\nu} = \begin{pmatrix} \alpha X & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (2.53)$$

For evaluating the fields measured by the Rindler observer, we recur to the covariant force and have

$$f^i = qE^i = qF_\nu^\mu \frac{dx^\nu}{ds} = qF_0^i \frac{dx^0}{ds} = -q \frac{F_0^i}{\alpha X} = -q\alpha X F^{i0} = qF^{\hat{0}\hat{i}} \quad (2.54)$$

which turns the electric field explicitly as

$$E^i = F^{\hat{0}i} = \alpha X F^{0i} \quad (2.55)$$

and, similarly, for the magnetic field

$$B^1 = F^{\hat{2}\hat{3}} = F^{23} \quad (2.56)$$

and cyclic permutations.

This result must be inserted in the covariant Maxwell equations given by

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\mu\nu}) = 4\pi j^\nu \quad (2.57)$$

where we have denoted with g the determinant of the metric and

$$j^\mu = \frac{e}{\sqrt{-g}} \delta^3 (\vec{X} - \vec{X}_0) \frac{dX^\mu}{d\tau} \quad (2.58)$$

is the 4-current, e the charge and $\vec{X}_0 = (\frac{1}{\alpha}, 0, 0)$ is the position of the particle in the accelerated frame.

In [100] the authors pass to the coordinates used in Eq. 2.47 and write the wave equation for A_0 . This reads

$$\rho \frac{\partial^2 A_0}{\partial \zeta^2} + \frac{1}{2\alpha\zeta} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial A_0}{\partial \rho} \right) = -4\pi e \delta \left(\zeta - \frac{1}{2\alpha} \right) \delta(\rho - 0) \delta(\theta - 0) \quad (2.59)$$

The solution of this equation is far from trivial. However, the authors find a brilliant way to get by the problem. They study a charge placed outside the Schwarzschild-Droste black hole horizon. Using polar coordinates they obtain the differential equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial A_0}{\partial r} \right) + \frac{1}{(1 - \frac{2M}{r})} \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial A_0}{\partial \theta} \right) = -4\pi e \delta(r - r') \delta(\cos\theta - 1) \quad (2.60)$$

having chosen to place the charge at $r = r'$ and $\theta = 0$ and having labeled with M the black hole mass.

The solution to this equation is known [101] and reads

$$A_0 = \frac{e [(r - M)(r' - M) - M^2 \cos\theta]}{rr' \sqrt{(r - M)^2 + (r' - M)^2 - 2(r - M)(r' - M) \cos\theta - M^2 \sin^2\theta}} \quad (2.61)$$

If we put ourselves very close to the horizon

$$r = 2M + \zeta \quad (2.62)$$

with

$$\zeta \ll 2M \quad (2.63)$$

The following relation holds

$$1 - \frac{2M}{r} \sim \frac{\zeta}{2M} = 2\alpha\zeta \quad (2.64)$$

where $\alpha = \frac{1}{4M}$ has been named to underline the similarity to the Rindler case. In fact, recalling that the Schwarzschild-Droste metric is given by [102, 103]

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} - r^2 (d\theta^2 + \sin^2\theta d\phi^2) \quad (2.65)$$

we get

$$r^2 (d\theta^2 + \sin^2\theta d\phi^2) \sim d\rho^2 + \rho^2 d\phi^2 \quad (2.66)$$

In this last step, we have restricted ourselves to a small region along the z axis and set the origin at $\theta = 0$ and $r \sim 2M$. The combination of these two conditions yields

$$\rho \sim 2M \quad (2.67)$$

$$\sin\theta \sim 2M\theta \quad (2.68)$$

Putting together Eqs. 2.65- 2.66, Eq. 2.47 is derived. Applying these approximations to Eq. 2.61, the scalar potential of a relativistic charge uniformly accelerating reads

$$A_0 = \frac{\alpha e \left(\zeta + \frac{1}{2\alpha} + \frac{\alpha\rho^2}{2} \right)}{\sqrt{\left(\zeta - \frac{1}{2\alpha} + \frac{\alpha\rho^2}{2} \right)^2 + 2\alpha\rho^2 \frac{1}{2\alpha}}} \quad (2.69)$$

Before moving on, we remark how sensible the approximation we used is. Choosing r very close to $2M$ means to put the charge in the proximity of the horizon. Restricting the measurements to a region very close to this point, we essentially flatten out the horizon which can be, from a practical standpoint, considered a plane for an accelerating observer. Less constraining is the choice of the origin, which can actually be put arbitrarily in spacetime.

From the potential in such coordinates, the electric field reads

$$E^\zeta = \frac{(2\alpha e\zeta_0) \sqrt{2\alpha\zeta} \left(\zeta - \zeta_0 + \frac{\alpha\rho^2}{2} \right)}{\left[\left(\zeta - \zeta_0 + \frac{\alpha\rho^2}{2} \right)^2 + 2\alpha\rho^2\zeta_0 \right]^{\frac{3}{2}}} \quad (2.70)$$

$$E^\rho = \frac{(2\alpha e\zeta_0) \sqrt{2\alpha\zeta} \rho}{\left[\left(\zeta - \zeta_0 + \frac{\alpha\rho^2}{2} \right)^2 + 2\alpha\rho^2\zeta_0 \right]^{\frac{3}{2}}} \quad (2.71)$$

$$E^\phi = 0 \quad (2.72)$$

with $\zeta_0 = \frac{1}{2\alpha}$.

After long but simple algebra, the field in polar coordinates is given by

$$E^X = \frac{4e}{\alpha^2} \frac{X^2 - \frac{1}{\alpha^2} - \rho^2}{\left[\left(X^2 + \rho^2 - \frac{1}{\alpha^2} \right) - 4 \frac{1}{\alpha^2} X^2 \right]^{\frac{3}{2}}} \quad (2.73)$$

$$E^\rho = \frac{8e}{\alpha^2} \frac{\rho X}{\left[\left(X^2 + \rho^2 - \frac{1}{\alpha^2} \right) - 4 \frac{1}{\alpha^2} X^2 \right]^{\frac{3}{2}}} \quad (2.74)$$

$$E^\phi = 0 \quad (2.75)$$

For more physical insight, one more coordinate transformation turns useful. If we choose X as coordinate for the axis along which the particle moves, we have that at $t = 0$ the charge is at $\frac{1}{\alpha}$. This fact somewhat breaks the formal symmetry of the problem. Hence it is natural to define a simple translation where $\xi = X - \frac{1}{\alpha}$. The metric becomes

$$ds^2 = (1 + \alpha\xi)^2 d\tau^2 - dX^2 - dY^2 - dZ^2 \quad (2.76)$$

and the fields assume the following form

$$E^\xi = \frac{e\xi}{r^3} \left(1 + \frac{1}{2} \frac{\alpha r^2}{\xi} \right) \left(1 + \alpha\xi + \frac{1}{4} \alpha^2 r^2 \right)^{-\frac{3}{2}} \quad (2.77)$$

$$E^\rho = \frac{e\rho}{r^3} (1 + \alpha\xi) \left(1 + \frac{1}{2} \frac{\alpha r^2}{\xi} \right) \left(1 + \alpha\xi + \frac{1}{4} \alpha^2 r^2 \right)^{-\frac{3}{2}} \\ \left(1 + \alpha\xi - \frac{\alpha}{2} \frac{r^2}{\xi} \right)^{-1} \quad (2.78)$$

$$E^\phi = 0 \quad (2.79)$$

where $r^2 = \xi^2 - \rho^2$.

These last expressions are more telling since in the limit $\alpha r \ll 1$ we obtain the usual Coulomb's field.

2.1.4 Field of an uniformly accelerating particle in the Born-Infeld electrodynamics

Is there an expression for the field in case of the BI non-linear theory? Of course, non-linearities are destined to generate complex computations. In particular, we will see that, unlike to what happens in the Maxwellian case, a perturbative expansion will be required.

We choose to work in the metric given by Eq.2.76, which we rewrite as

$$ds^2 = (1 + gx)^2 d\tau - dx^2 - dy^2 - dz^2 \quad (2.80)$$

where we have used non-capital letters for convenience and renamed the acceleration g to better distinguish the current case from the previous one. The Maxwell equations adapted to the Rindler spacetime is our point of departure. They are known in literature [70, 99] and are obtained combining Eqs. 2.57, 2.51 and 2.52, once adapted to the coordinates at hand and having used the Ricci-Curbastro-Bianchi identity. They read

$$\vec{\nabla} \times \left[(1 + gx) \vec{E} \right] = 0 \quad (2.81)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (2.82)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2.83)$$

$$\frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \times \left[(1 + gx) \vec{B} \right] = -4\pi (1 + gx) \vec{j} \quad (2.84)$$

in which ρ is the charge density and $\vec{j}$ is the spatial electric current.

From Eq. 2.81, the relation between the electric field and the scalar potential ($\Phi = A_0$) is

$$(1 + gx) \vec{E} = \vec{\nabla} \Phi \quad (2.85)$$

hence, in Rindler's spacetime

$$\vec{E} = \frac{\vec{\nabla} \Phi}{(1 + gx)} \quad (2.86)$$

In BI theory, Eqs. 2.81, 2.82 still hold if we substitute the vector $\vec{E}$ in Eq. 2.82 with the vector $\vec{D}$ which can be expressed in term of $\vec{E}$ by the following constitutive relation [89, 90]

$$\vec{D} = \frac{\vec{E}}{\sqrt{1 - \frac{|\vec{E}|^2}{b^2}}} \quad (2.87)$$

Since Eq. 2.81 remains valid, Eq. 2.86 holds; therefore $\vec{D}$ can be written in terms of Φ like

$$\vec{D} = \frac{\frac{\vec{\nabla} \Phi}{(1 + gx)}}{\sqrt{1 - \frac{\left| \frac{\vec{\nabla} \Phi}{(1 + gx)} \right|^2}} = \frac{\vec{\nabla} \Phi}{(1 + gx)} \cdot \frac{1}{\sqrt{1 - \frac{\vec{\nabla} \Phi^2}{b^2 (1 + gx)^2}}} \quad (2.88)$$

If we choose a pointlike charge as source, plugging Eq. 2.88 in Eq. 2.82 the

divergence of $\vec{D}$ equals the point-like charge at x_0

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla}\Phi}{(1+gx)} \cdot \frac{1}{\sqrt{1 - \frac{\vec{\nabla}\Phi^2}{b^2(1+gx)}}} \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.89)$$

where q is the charge of the particle absolute value, and the minus on the left-hand side (LHS) supposes an electron. The BI potential Φ is decomposed in the sum of the Maxwell's potential Φ_0 plus a small perturbation φ

$$\Phi = \Phi_0 + \varphi \quad (2.90)$$

We assume that

$$\Phi_0 \gg \varphi \quad (2.91)$$

$$\frac{\vec{\nabla}\Phi^2}{b^2} \ll 1 \quad (2.92)$$

We want to expand the square root in the right-hand side (RHS) of Eq. 2.88, using the binomial series. Therefore we obtain

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla}\Phi}{(1+gx)} \cdot \left(1 - \frac{\vec{\nabla}\Phi^2}{b^2(1+gx)^2} \right)^{-\frac{1}{2}} \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.93)$$

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla}\Phi}{(1+gx)} \cdot \left(1 + \frac{1}{2} \frac{\vec{\nabla}\Phi^2}{b^2(1+gx)^2} \right) \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.94)$$

The series

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \quad (2.95)$$

converges for

1. $|x| < 1$ and $\forall \alpha \in \mathbb{C}$
2. $x = -1 \iff Re(\alpha) > 0$ or $\alpha = 0$
3. $|x| = 1 \wedge x \neq -1 \Rightarrow Re(\alpha) > -1$
4. $|x| > 1$ and α is negative,

we are either in case 1 or 4, so convergence is assured.

Now we expand the complete potential into its Mawellian part plus the variation thanks to Eq. 2.90. We get

$$\vec{\nabla} \cdot \left[\left(-\frac{\vec{\nabla}\Phi_0}{(1+gx)} - \frac{\vec{\nabla}\varphi}{(1+gx)} \right) \cdot \left(1 + \frac{1}{2} \frac{\vec{\nabla}\Phi^2}{b^2(1+gx)^2} \right) \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.96)$$

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla}\Phi_0}{(1+gx)} - \frac{\vec{\nabla}\varphi}{(1+gx)} - \frac{1}{2} \frac{\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi^2}{b^2(1+gx)^3} \right] = q\delta(\vec{x} - \vec{x}_0) \quad (2.97)$$

Formally, we are confronted with the operator $\hat{O}$ acting on the complete field Φ . This operator is composed by a Maxwellian part $\hat{O}_M$ plus a correction $\hat{O}'$; by definition, we have

$$\hat{O}_M \Phi_0 = -q\delta \quad (2.98)$$

and, therefore,

$$(\hat{O}_M + \hat{O}')(\Phi_0 + \varphi) = -q\delta \quad (2.99)$$

$$\hat{O}_M \Phi_0 + \hat{O}_M \varphi + \hat{O}' \Phi_0 + \hat{O}' \varphi = -q\delta \quad (2.100)$$

$$\hat{O}_M \varphi + \hat{O}' \Phi_0 + \hat{O}' \varphi = 0 \quad (2.101)$$

Likewise, in Eq. 2.97, we can cancel the first term in the LHS with the RHS, getting

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \varphi}{(1+gx)} - \frac{1}{2} \frac{\vec{\nabla} \Phi \vec{\nabla} \Phi^2}{b^2 (1+gx)^3} \right] = 0 \quad (2.102)$$

Now we can further expand Φ in the second term in the LHS and obtain

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \varphi}{(1+gx)} - \frac{1}{2} \left(\vec{\nabla} \Phi_0 + \vec{\nabla} \varphi \right) \frac{\vec{\nabla} \Phi_0^2 + 2\vec{\nabla} \Phi_0 \cdot \vec{\nabla} \varphi}{b^2 (1+gx)^3} \right] = 0 \quad (2.103)$$

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \varphi}{(1+gx)} - \frac{1}{2} \frac{\vec{\nabla} \Phi_0^2 \vec{\nabla} \Phi_0 + 2 \left(\vec{\nabla} \Phi_0 \cdot \vec{\nabla} \varphi \right) \vec{\nabla} \Phi_0 + \nabla \Phi_0^2 \vec{\nabla} \varphi}{b^2 (1+gx)^3} \right] = 0 \quad (2.104)$$

where terms of second order in $\vec{\nabla} \varphi$ have been ignored.

In virtue of Eq. 2.91, 2.92, the leading order in the second term in Eq. 2.104 is the one where only Φ_0 appears. Therefore, we can approximate our equation to

$$\vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \varphi}{(1+gx)} - \frac{1}{2} \frac{\vec{\nabla} \Phi_0^2 \vec{\nabla} \Phi_0}{b^2 (1+gx)^3} \right] = 0 \quad (2.105)$$

from which

$$\vec{\nabla} \cdot \left[\frac{\vec{\nabla} \varphi}{(1+gx)} \right] = \vec{\nabla} \cdot \left[-\frac{1}{2} \frac{\vec{\nabla} \Phi_0^2 \vec{\nabla} \Phi_0}{b^2 (1+gx)^3} \right] \quad (2.106)$$

$$\frac{g}{(1+gx)} \nabla^2 \varphi - \frac{g^2}{(1+gx)^2} \partial_x \varphi = g \cdot \vec{\nabla} \cdot \left[-\frac{1}{2} \frac{\vec{\nabla} \Phi_0^2 \vec{\nabla} \Phi_0}{b^2 (1+gx)^3} \right] \quad (2.107)$$

Now, we can perform a change of variables

$$\chi = \frac{1+gx}{g} \quad (2.108)$$

Noticing that

$$dx = d\chi \quad (2.109)$$

we reach the following intermediate expression

$$\frac{\nabla^2 \varphi}{\chi} - \frac{\partial_x \varphi}{\chi^2} = \frac{1}{g^2} \cdot \vec{\nabla} \cdot \left[-\frac{1}{2} \frac{\vec{\nabla} \Phi_0^2 \cdot \vec{\nabla} \Phi_0}{b^2 \chi^3} \right] \quad (2.110)$$

where the first spatial derivative inside $\vec{\nabla}$, $\vec{\nabla} \cdot$ and ∇^2 is taken with respect to χ .

We could solve Eq. 2.110 if we knew the Green function G associated to it, i.e. the solution to

$$\frac{\nabla^2 G}{\chi} - \frac{\partial_x G}{\chi^2} = \delta(\chi - \chi_0) \delta(y - y_0) \delta(z - z_0) \quad (2.111)$$

In such a case, the potential φ would be simply given by the convolution $G \circ S$, where we have called S the source

$$\frac{1}{g^2} \cdot \vec{\nabla} \cdot \left[-\frac{1}{2} \frac{\nabla \Phi_0^2 \cdot \vec{\nabla} \Phi_0}{b^2 \chi^3} \right] \quad (2.112)$$

It turns out that we know the function G . In fact, Eq. 2.111 is the wave equation derived for the electric potential when solving Faraday's law in the Rindler spacetime using the (τ, X, Y, Z) sets of coordinates, for a pointlike charge [70]. The solution reads

$$G(\vec{X}, \vec{X}_0) = \frac{\chi^2 + (y - y_0)^2 + (z - z_0)^2 + \chi_0^2}{4\pi \sqrt{[\chi^2 + (y - y_0)^2 + (z - z_0)^2 + \chi_0^2]^2 - 4\xi^2 \xi_0^2}} \quad (2.113)$$

where we have shortened (χ, y, z) into $\vec{X}$ and (χ_0, y_0, z_0) in $\vec{X}_0$, χ_0 being $\frac{1}{g}$. Since this expression is precisely the electric potential for a static charge in the Rindler spacetime it represents not only the Green function, but also Φ_0 , hence

$$\Phi_0 = \frac{q [\chi^2 + (y - y_0)^2 + (z - z_0)^2 + \chi_0^2]}{\sqrt{[\chi^2 + (y - y_0)^2 + (z - z_0)^2 + \chi_0^2]^2 - 4\chi^2 \chi_0^2}} \quad (2.114)$$

Inserting Eq. 2.114 into Eq. 2.112, the expression for the source is

$$S(\vec{X}, \vec{X}_0) = -\frac{256q^3}{b^2 g^2} \cdot \frac{\chi \chi_0^6}{\left\{ [\chi^2 + (y - y_0)^2 + (z - z_0)^2 + \chi_0^2]^2 - 4\chi^2 \chi_0^2 \right\}^{\frac{7}{2}}} \quad (2.115)$$

As explained, our aim is the convolution between Eq. 2.113 and Eq. 2.115.

Therefore we want to compute the integral

$$I(\vec{X}) = -\frac{256q^3}{4\pi b^2 g^2} \int \left[\frac{\chi^2 + (y-b)^2 + (z-c)^2 + a^2}{\sqrt{[\chi^2 + (y-b)^2 + (z-c)^2 + a^2]^2 - 4\chi^2 a^2}} \right. \\ \left. \frac{a \chi_0^6}{\left\{ [a^2 + (b-y_0)^2 + (c-z_0)^2 + \chi_0^2]^2 - 4a^2 \chi_0^2 \right\}^{\frac{7}{2}}} \right] dadbdc \quad (2.116)$$

Computational difficulties have arisen to such a level that we had to drop pursuing a solution. The integral, Eq. 2.116, has two divergencies. Besides the classic divergence on the world line, an additional one arises where the integration variables match the point where we compute the field. The first divergence is expected and normally eliminated via the convolution integral. The second, however, has no classical counterpart. Its profound nature lies in the development of the potential in series and in the very high degree of non-linearity of the theory we are using. All this transposed into a major difference with respect to what happens when we compute the potential of a pointlike charge in a Maxwellian environment: usually, the source is a Dirac delta which helps us get rid of the singularity in the integrating function. Here, instead, the source, Eq. 2.115, is a field.

We have tried in various ways to tackle the computation of this integral. We first tried analytically, but there was very little hope. A second try was to use hypergeometric functions. Actually, such functions constitute the solution to integrals of several polynomial fractions containing at least one square root at the denominator. Unfortunately, the exponent of our result is so high that the results are not tabled. Also, we were not able to find the exact form we needed. Ultimately, we have resorted to a more direct approach, through a Monte Carlo integration. Of course this is a complete change of heart, because we give up the hope of finding an analytic result, but it was the best we could do under the circumstances. This essay is still ongoing and we cannot, at present day, put forward any result.

2.2 Electromagnetic shifts in the field of an Heisenberg-Euler dipole

2.2.1 Context

In this Section, we intend to study an application of another possible way in which non-linearities appear in an EM theory. As pointed out in the previous Chapter, this time we operate via an effective Lagrangian that accounts for the QED effects. We have chosen to work with the more established of all the possible theories that fall under this class, that is the Heisenberg-Euler theory.

There are several testable predictions that follow from the HE Lagrangian. Among them, we can mention the Schwinger effect [77] (see [78] for attempts to observe this effect in the laboratory), the birefringence of the vacuum under a strong magnetic field (see for example [79]), currently sought at the PVLAS or BMV experiment [71–75], and photon splitting [76] in a laser field [104]¹.

While all these efforts are being developed in terrestrial laboratories, some compact astrophysical objects may offer the possibility of testing the quantum vacuum. The magnetars [106] are neutron stars endowed with an overcritical magnetic field, see [107] for a review, with estimated values up to $10^2 B_c$ at their surface [108], where B_c is the critical field for the HE theory. Several consequences of such intense field have been studied in the literature, such as the lensing of the light emitted by background astronomical objects due to the optical properties of quantum vacuum in the presence of a magnetic field [109]², the polarization phase lags due to the index of refraction of the vacuum [111]³, and the influence of the quantum vacuum friction on the spindown of pulsars [113].

Many of these effects are related to photon propagation in a background field, which can be studied using the effective metric. As shown in [80, 81], the propagation of the high energy excitations of a non-linear EM theory on a fixed background is governed by an effective metric that depends on the background spacetime metric, on the background EM field configuration, and on the details of the non-linear dynamics obeyed by the EM field (see [82] for a review). There are numerous applications of the effective metric in pure gravity [81, 114], and cosmology [115–117]. In an astrophysical setting, it was shown in [85] for the case of the BI theory that high energy photons coming from a compact object endowed with an overcritical field would display an EM shift (EMS), in addition to the gravitational redshift. In [86] the authors derive Eq. 2.156 that we have emended of a computational for the low-field correction to Maxwell’s theory obtained from the HE Lagrangian. In the following we will give derive the EMS for the strong magnetic field case and discuss into greater detail the implications on the photon propagation.

The EMS can be toward the red or the blue and is rooted in the non-linearities of the Lagrangian, hence it is absent in Maxwell’s theory. The fact that perturbations (in this case, photons) can exchange energy with the background, and that the exchange is intimately connected to the effective metric has been explored many times in the literature; see Parker [118] for the case of wave propagation in stellar coronas, Moncrief [119] for the stationary and spherically symmetric accretion onto a Schwarzschild-Droste black hole, Mach and Malec [120] for the same type of accretion in Schwarzschild-(anti)de Sitter spacetimes, and Rivasplata Paz et al. [121] for the accretion of a ghost condensate onto the Schwarzschild-Droste black hole.

¹See [105] for a recent proposal to test non-linear vacuum electrodynamics with Michelson interferometry.

²See [110] for the polarization dependence of lensing, and its consequences on the thermal emission of the star.

³For a review of the exotic physics that takes place in the overcritical field regime, see [112].

Our intent is to investigate the EMS of photons in a dipole field which is a solution of the full QED one-loop corrected Lagrangian, in a flat spacetime background [122], through the effective metric. This setup can be considered as a first step in the process of the modeling of the EMS of a magnetar. We will analyze the angular dependence of the EMS for each of the two photon polarizations, and show that there may be a redshift or a blueshift, depending on the polarization of the photons.

2.2.2 The effective metric formalism

The effective metric formalism is the general framework that will allow us to treat a highly non-linear problem in a rather simplified way. The idea is rather simple: we have already examples in general relativity (GR) where particle propagation is studied. Being GR a geometric theory, trajectories are determined by geodesics in the spacetime of interest. What is the spacetime of interest? Is either the background metric or the full metric. In the former case, an extra force term will appear in the geodesic equation causing the particle to accelerate in the background metric. In the latter, the particle will not be subject to acceleration since it is moving in the full metric. In this manner, the particle still complies with the equivalence principle even when we consider its mass, or charge, and the radiation emitted. We choose the second approach. Thus, we include the non-linearities in the metric and proceed to study the motion of the photon in this new - effective - metric. This metric will be built such to include the EM interactions, too.

We start by taking an EM field that is continuous on its characteristic surface but with first derivative displaying a finite discontinuity along this very same plane [123, 124]. A characteristic surface is a surface, associated to a system of differential equations, which is the result of the envelope of so called exceptional planes. A plane is called exceptional in the point P , belonging to the boundary conditions to solve the Cauchy problem, if the plane is tangent to the boundary in P and if it exists a linear combination of the differential equations in the system such that this combination does not contain the derivatives normal to the plane. In mathematical terms, calling Σ the characteristic surface and using the notation

$$[J]_{\Sigma} = \lim_{\delta \rightarrow 0^+} (J|_{\Sigma+\delta} - J|_{\Sigma-\delta}), \quad (2.117)$$

we are requiring

$$[F_{\mu\nu}]_{\Sigma} = 0 \quad (2.118)$$

and

$$[\partial_{\lambda} F_{\mu\nu}]_{\Sigma} = D_{\mu\nu} k_{\lambda} \quad (2.119)$$

where we have called $D_{\mu\nu}$ the tensor that accounts for the discontinuities of the fields and

$$k_{\lambda} = \partial_{\lambda} \Sigma \quad (2.120)$$

is the propagation vector.

We must recall that the HE Lagrangian depends on the two invariants $F = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ and $G = \frac{1}{8}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$. Therefore, applying the Least Action Principle, we obtain [81]

$$\partial_\lambda (L_F F^{\mu\nu} + L_G f^{\mu\nu}) = 0 \quad (2.121)$$

having called $f^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ and used the notation according to which $L_X = \frac{\partial L}{\partial X}$. In [83] the propagation of the discontinuities is studied in by expliciting the equation of motion thanks to Eqs. 2.118 and 2.119. The result reads

$$[L_F D^{\mu\nu} + 2A F^{\mu\nu} + 2B f^{\mu\nu}] k_\nu = 0 \quad (2.122)$$

where

$$A = 2(\xi L_{FF} + \zeta L_{FG}) \quad (2.123)$$

$$B = 2(\xi L_{FG} + \zeta L_{GG}) \quad (2.124)$$

with

$$\xi = F^{\alpha\beta} D_{\alpha\beta} \quad (2.125)$$

$$\zeta = f^{\alpha\beta} D_{\alpha\beta} \quad (2.126)$$

We transform Eq. 2.122 into a scalar equation by contracting it either with $F_\mu^\alpha k_\alpha$ or with $f_\mu^\alpha k_\alpha$. The two results read

$$\left[\xi L_F + \frac{1}{2} B G \right] \eta^{\mu\nu} k_\mu k_\nu - 2A F_\alpha^\nu F^{\alpha\mu} k_\nu k_\mu = 0 \quad (2.127)$$

$$\left[\zeta L_F - B F + \frac{1}{2} A G \right] \eta^{\mu\nu} k_\mu k_\nu - 2B F_\alpha^\nu F^{\alpha\mu} k_\nu k_\mu = 0 \quad (2.128)$$

From these two last equations, a second order equation is built posing

$$\Omega = \frac{\zeta}{\xi} \quad (2.129)$$

that is

$$\Omega_1 \Omega^2 + \Omega_2 \Omega + \Omega_3 = 0 \quad (2.130)$$

with

$$\Omega_1 = -L_F L_{FG} + 2F L_{FG} L_{GG} + G (L_{GG}^2 - L_{FG}^2) \quad (2.131)$$

$$\Omega_2 = (L_F + 2G L_{FG}) (L_{GG} - L_{FF}) + 2F (L_{FF} L_{GG} + L_{FG}^2) \quad (2.132)$$

$$\Omega_3 = L_F L_{FG} + 2F L_{FF} L_{GG} + G (L_{FG}^2 - L_{FF}^2) \quad (2.133)$$

The solution of Eq. 2.130 is

$$\Omega_\pm = \frac{-\Omega_2 \pm \sqrt{\Delta}}{2\Omega_1} \quad (2.134)$$

with $\Delta = \Omega_2^2 - 4\Omega_1\Omega_3$. Therefore, collecting $k_\mu k_\nu$ in Eqs. 2.127, 2.128 the propagation equation looks as

$$^{(\pm)}g_{eff}^{\mu\nu}k_\mu k_\nu = 0 \quad (2.135)$$

and the effective metrics⁴

$$^{(\pm)}g_{eff}^{\mu\nu} = L_F \eta^{\mu\nu} - 4 \left[(L_{FF} + \Omega_\pm L_{FG}) F_\lambda^\mu F^{\lambda\nu} + (L_{FG} + \Omega_\pm L_{GG}) F_\lambda^\mu F^{\lambda\nu} \right] \quad (2.136)$$

The two signs correspond to the birefringence of the HE theory. Generally, non-linearity prompts birefringence. There are exceptions, though. Among those, remarkably, BI theory for which the Ω_i vanish identically⁵.

In our work, we deal with a static dipole, hence

$$G = 0 \quad (2.138)$$

Therefore, also [84]

$$L_G = 0 \quad (2.139)$$

$$L_{FG} = 0 \quad (2.140)$$

which can be used to simplify Eq. 2.136 into the more manageable forms

$$^{(1)}g_{eff}^{\mu\nu} = [L_F \eta^{\mu\nu} - 4L_{FF} F_\alpha^\mu F^{\alpha\nu}]_0 \quad (2.141)$$

$$^{(2)}g_{eff}^{\mu\nu} = [(L_F - 2FL_{GG}) \eta^{\mu\nu} - 4L_{GG} F_\alpha^\mu F^{\alpha\nu}]_0 \quad (2.142)$$

where the subscript zero indicates the quantity is evaluated at the background field.

2.2.3 Computation of the shift

The computation of the shift must be performed with some care. In fact, a photon propagating in a non-linear theory does exchange energy with the background and, hence, accelerates because of the non-linearities. In other words, it does not follow the geodesics of the background metric [80, 81, 83]. Instead, the photon will move on geodesics of the effective metric because only this metric incorporates the dynamics induced by the non-linearities in the theory. Therefore, an observer that is comoving with respect to the background field will see the motion of the light particle governed by

$$p^\mu \tilde{\nabla}_\mu p^\nu = p^\mu \partial_\mu p^\nu + \tilde{\Gamma}_{\alpha\beta}^\nu p^\alpha p^\beta = 0 \quad (2.143)$$

⁴The same result can be obtain perturbing Eq. 2.121 with respect to a fixed background keeping only terms linear in the perturbations and applying the eikonal approximation [125]

⁵Indeed, if the Hadamard procedure is applied correctly, one obtains the effective metric has an unique (meaning it does not depend on the polarization) form that excludes birefringence [126]

$$g^{\mu\nu} = \left(b^2 + \frac{1}{2}F \right) \eta^{\mu\nu} + F_\lambda^\mu F^{\lambda\nu} \quad (2.137)$$

where, for each polarization, $\tilde{\nabla}_\mu$ is the covariant derivative built using the effective metric and $\tilde{\Gamma}_{\alpha\beta}^\nu$ the corresponding Christoffel symbols⁶. All contractions must be done using the background metric.

For a time independent metric, it exists a constant of motion, which is p_0 , i.e. the time component of the 4-momentum. Conversely we will show that p_0 is no longer conserved along the geodesic in our case. We will have a new conserved quantity, arising from the Killing vector⁷ of the effective metric that governs the dynamics.

Sufficiently away from the dipole, we consider a Minkowskian background. Lowering the indexes of the 4-momentum in Eq. 2.143

$$(p^\mu \partial_\mu \eta^{\nu\sigma}) p_\sigma + \eta^{\nu\sigma} p^\mu \partial_\mu (p_\sigma) = -\tilde{\Gamma}_{\alpha\beta}^\nu p^\alpha p^\beta \quad (2.146)$$

and multiplying by $\eta_{\nu\sigma}$ we get

$$p^\mu \partial_\mu (p_\sigma) = -\eta_{\nu\sigma} \tilde{\Gamma}_{\alpha\beta}^\nu p^\alpha p^\beta - \eta_{\nu\sigma} p_\sigma (p^\mu \partial_\mu \eta^{\nu\sigma}) = -\eta_{\nu\sigma} \tilde{\Gamma}_{\alpha\beta}^\nu p^\alpha p^\beta \quad (2.147)$$

the second term in the RHS vanishes. However, nothing assures that the first does it too. More specifically, for $\sigma = 0$, it results from Eq. 2.147

$$p^\mu \partial_\mu (p_0) = -\eta_{00} \tilde{\Gamma}_{\alpha\beta}^0 p^\alpha p^\beta \quad (2.148)$$

From Eq. 2.148 we see immediately that p_0 would be a constant of motion only if the RHS vanished. However, as we have just pointed out, nothing assures, in general, that such a term vanishes. We thus define a new invariant of motion that is conserved along the geodesics. Being the effective metric time independent, we know that it must have an associated Killing vector. More specifically, this vector is $K = \frac{\partial}{\partial t}$ that, in components, is

$$K^\mu = \delta_0^\mu \quad (2.149)$$

We now define

$$\tilde{p}_K = \tilde{g}^{\mu\nu} p_\mu K_\nu \quad (2.150)$$

and exploit 1) Eq. 2.143, 2) the definition of a Killing vector and 3) the vanishing of the covariant derivative of the effective metric [83]. Under these assumptions,

⁶Given a metric, the Christoffel symbols are defined as follows

$$\Gamma_{\alpha\beta}^\nu = \frac{1}{2} g^{\nu\mu} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right) \quad (2.144)$$

⁷Given a Riemannian manifold, a Killing vector field is a vector field that preserves the metric, being the generator of isometries. In the direction of the Killing vector, distances on the manifold are not distorted. From a more practical point of view, the Killing vector field K_μ must satisfy

$$\nabla_\nu K_\mu + \nabla_\mu K_\nu = 0 \quad (2.145)$$

$\tilde{p}_K$ is indeed a conserved quantity

$$\begin{aligned} p^\alpha \tilde{\nabla}_\alpha \tilde{p}_K &= \left(p^\alpha \tilde{\nabla}_\alpha \tilde{g}^{\mu\nu} \right) p_\mu K_\nu + \tilde{g}^{\mu\nu} \left(p^\alpha \tilde{\nabla}_\alpha p_\mu \right) K_\nu + \tilde{g}^{\mu\nu} p^\alpha p_\mu \tilde{\nabla}_\alpha K_\nu \\ &= \tilde{g}^{\mu\nu} p^\alpha p_\mu \tilde{\nabla}_\alpha K_\nu = \frac{P^\nu p^\alpha + P^\alpha p^\mu}{2} \frac{\tilde{\nabla}_\alpha K_\nu - \tilde{\nabla}_\nu K_\alpha}{2} = 0 \end{aligned} \quad (2.151)$$

where $P^\nu = \tilde{g}^{\mu\nu} p_\mu$.

From Eq. 2.150, the conserved 4-momentum in the effective metric is

$$\tilde{p}_K = \tilde{g}^{\mu\nu} p_\mu K_\nu = \tilde{g}^{\mu\nu} p_\mu g_{\alpha\nu} \delta_0^\mu = \tilde{g}^{00} p_0 g_{00} \Rightarrow p_0 = g^{00} (\tilde{g}^{00})^{-1} \tilde{p}_K \quad (2.152)$$

Incidentally, we see from this result that at least some of the Christoffel symbols in Eq. 2.148 are necessarily different from zero.

From the definition of a frequency shift, we write

$$1 + z = \frac{g^{00}(E) u_0(E) p_0(E)}{g^{00}(R) u_0(R) p_0(R)} \quad (2.153)$$

We have labeled $R(E)$ the reception (emission) point. Furthermore, $u_0 = \frac{1}{\sqrt{g^{00}}}$ since $u^\mu u^\nu g_{\mu\nu} = 1$ and the photon doesn't have spatial momentum; combining this expression for u_0 , Eq. 2.153 and Eq. 2.152, we get

$$1 + z = \left[\frac{g^{00}(E)}{g^{00}(R)} \right]^{\frac{3}{2}} \frac{\tilde{g}^{00}(R)}{\tilde{g}^{00}(E)} \quad (2.154)$$

We observe that in Maxwell's theory and a photon propagating in Minkowski spacetime

$$1 + z = \left[\frac{g^{00}(E)}{g^{00}(R)} \right]^{\frac{1}{2}} = 1 \quad (2.155)$$

since $\tilde{g}^{\mu\nu} = g^{\mu\nu}$. This result is precisely what it is expected because Maxwell's theory does not give rise to a shift in frequency if the space is flat.

Having placed the dipole in a Minkowski background, Eq. 2.154 reduces to

$$1 + z = \frac{\tilde{g}^{00}(R)}{\tilde{g}^{00}(E)} \quad (2.156)$$

Once again, we remark that this expression correctly vanishes if the theory is linear as $\tilde{g}^{\mu\nu} = g^{\mu\nu}$. Furthermore, we point out that Eq. 2.156 corrects results present in the literature [85, 86].

For computing the electromagnetic shift in the HE dipole spacetime, we fix the reception point at spatial infinity, where the metric reduces to the flat background. Therefore, $\tilde{g}^{00}(R) = g^{00}(R) = 1$, and we get

$$^{(1)}z = L_F(E) - 1 \quad (2.157)$$

$$^{(2)}z = [L_F(E) - 2F(E)L_{GG}(E)] - 1 \quad (2.158)$$

In the next Section, we will specify these expressions, that hold for every non-linear theory, to the case of HE theory.

2.2.4 EMS for the HE dipole

The first element needed for the computation of the EMS is the derivative of the HE Lagrangian Eq. 1.157. To this end, we rewrite Eq. 1.157 in a more convenient form [15, 77]:

$$L_{EH} = -F - \frac{1}{8\pi^2} \int_0^{i\infty} \frac{ds}{s^3} e^{-m_e^2 s} \left[(es)^2 ab \coth(eas) \cot(ebs) - \frac{(es)^2}{3} (a^2 - b^2) - 1 \right] \quad (2.159)$$

where

$$a = \left[\sqrt{F^2 + G^2} + F \right]^{\frac{1}{2}} \quad (2.160)$$

$$b = \left[\sqrt{F^2 + G^2} - F \right]^{\frac{1}{2}} \quad (2.161)$$

m_e is the electron mass and e is the electron charge. We have assumed the so called soft photon approximation, which implies that the photon energy is much smaller than its rest mass, i.e. $\omega \ll m_e$. In the case of a static magnetic field, where no electric field is present, the derivatives of L_{EH} can be analytically computed exploiting the following relations

$$\partial_F = (a^2 + b^2)^{-1} (a\partial_a - b\partial_b) \quad (2.162)$$

$$\partial_G = (a^2 + b^2)^{-1} (a\partial_a + b\partial_b) \quad (2.163)$$

and performing the derivation inside the brackets in Eq. 2.159. Then, the limit $b \rightarrow 0$ is taken. The remaining integrals in Eq. 2.159 can be performed via regularization techniques presented in [127]. The convergence of the integrals is implicitly assumed through the Feynman prescription $m_e^2 \rightarrow m_e^2 - i\epsilon$. The result of this long and involute computation yields [84]

$$L_F = -1 - \frac{\alpha}{2\pi} \left[\frac{1}{3} + \frac{1}{2\kappa^2} - 8\zeta' \left(-1, \frac{1}{2\kappa} \right) + \frac{1}{2\kappa} \ln \Gamma \left(\frac{1}{2\kappa} \right) - \frac{1}{\kappa} \ln \left(\frac{1}{2\kappa} \right) + \frac{2}{3} \ln \left(\frac{1}{2\kappa} \right) - \frac{1}{\kappa} \ln 2\pi \right] \quad (2.164)$$

$$\mathcal{L}_{GG} = \frac{\alpha}{2\pi B_c^2 \kappa^2} \left[-\frac{1}{3} - \frac{2}{3} \psi \left(1 + \frac{1}{2\kappa} \right) - \frac{1}{2\kappa^2} + \frac{2}{3} \kappa + 8\zeta' \left(-1, \frac{1}{2\kappa} \right) - \frac{1}{2\kappa} \ln \Gamma \left(\frac{1}{2\kappa} \right) + \frac{1}{\kappa} \ln 2\pi - \frac{1}{\kappa} \ln \left(\frac{1}{2\kappa} \right) \right] \quad (2.165)$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine structure constant $\sim \frac{1}{137}$, $\Gamma(h)$ is the gamma special function, $\psi(h)$ is the derivative with respect to h of $\ln \Gamma(h)$ and $\zeta'(-1, h)$ is the derivative with respect to the first argument of the Riemann-Hurwitz zeta function [128, 129]. κ is instead given by the following expression

$$\kappa = \frac{B}{B_c} \quad (2.166)$$

2.2. ELECTROMAGNETIC SHIFTS IN THE FIELD OF AN HEISENBERG-EULER DIPOLE63

B being the modulus of the magnetic field.

We have stated that Eqs. 2.141-2.142 are evaluated on the background. Hence, we need to find the background magnetic field associated to a dipole. Without loss of generality, we assume the dipole to be oriented along the z axis. In terms of spherical harmonics, the background field can be given by

$$B_0 = -\nabla\Phi_0 \quad (2.167)$$

where [122]

$$\Phi_0(r, \theta, \phi) = \sqrt{\frac{4\pi}{3}} \frac{m}{r^2} Y_{10}(\theta, \phi) \quad (2.168)$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta = \sqrt{\frac{3}{4\pi}} u \quad (2.169)$$

and m is the dipolar moment. Since the relevant expressions in both effective metrics are already $O(\alpha)$, only the zero-order solution for the field is needed to calculate the EMS⁸⁹.

It follows

$$B_0 = -\vec{\nabla}\Phi_0 = \left(\frac{2 \cdot m \cdot u}{r^3}, \frac{m \cdot \sqrt{1-u^2}}{r^3}, 0 \right) \quad (2.170)$$

and using the expression for the gradient in spherical coordinates

$$\vec{\nabla} = \left(\frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \right) \quad (2.171)$$

Therefore

$$B_0^2 = \frac{4m^2 u^2}{r^6} + \frac{m^2 (1-u^2)}{r^6} = \frac{m^2}{r^6} (1+3u^2) \quad (2.172)$$

Hence

$$\begin{aligned} B_0 &= \frac{m}{r^3} \sqrt{1+3u^2} \\ &= B_c \cdot \frac{m}{B_c \cdot r^3} \sqrt{1+3u^2} \end{aligned} \quad (2.173)$$

For simplicity, we introduce the parameter λ which is the ratio between the mean dipole background at $r = r_E$ and the critical field

$$\lambda = \frac{m}{r_E^3 B_c} \quad (2.174)$$

⁸The $O(\alpha)$ correction for the dipole has been calculated in [122].

⁹The eikonal approximation is valid when the modulus of the wave vector k_μ is much larger than the (dimensionless) gradient of the field [125]. In the case of a magnetic dipole, this implies that $k \gg 1/r$. For $r = 10$ km, which is the radius of a typical neutron star, we get that $\lambda \ll 10$ km. Hence, only very low energy photons are excluded by the eikonal approximation.

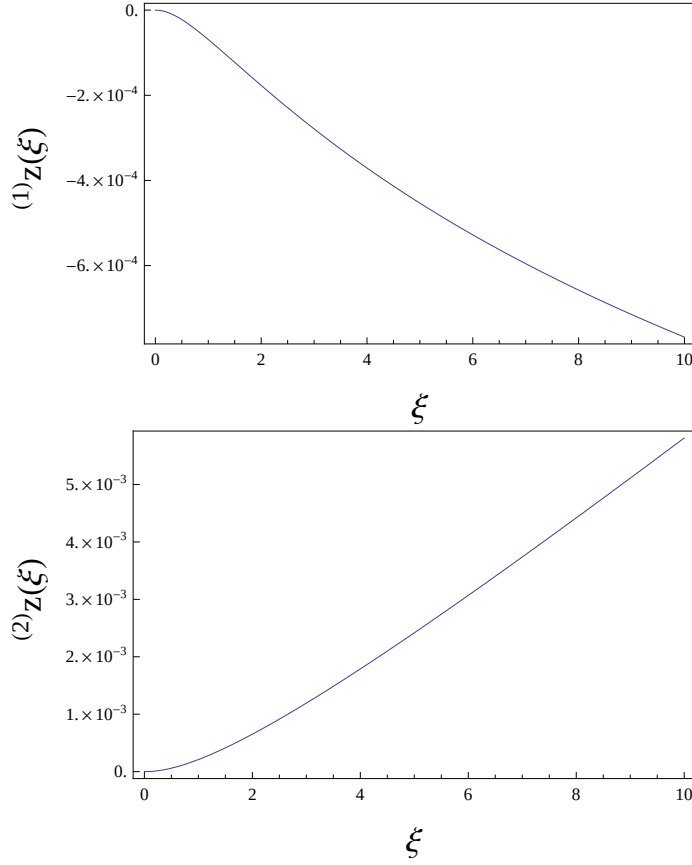


Figure 2.2: The upper and bottom panels display the dependence of ${}^{(1)}z$ and of ${}^{(2)}z$ with κ , respectively. [130]

Then it follows

$$\kappa = \frac{B_0}{B_c} = \lambda \sqrt{1 + 3u^2} \quad (2.175)$$

Hence, the EMS given by Eqs. 2.157 and 2.158 is a function of θ for each value of λ .

Figure 2.2 presents the dependence of the EMS with the variable κ for both polarizations. The figures show that the EMS is small in both cases as expected from an effect arising from a quantum correction. It is important to remark that the EMS is negative for the first polarization (hence a redshift) and positive (so it is actually a blueshift) and larger for the second.

Figure 2.3 shows the dependence with $\cos \theta$ of the EMS for three values of λ , which are relevant for magnetars [107]. The plots show that in both cases the EMS increases (in absolute value) with the field, but while ${}^{(1)}z$ is maximum at the equator and minimum at the poles, ${}^{(2)}z$ behaves in the opposite way. It

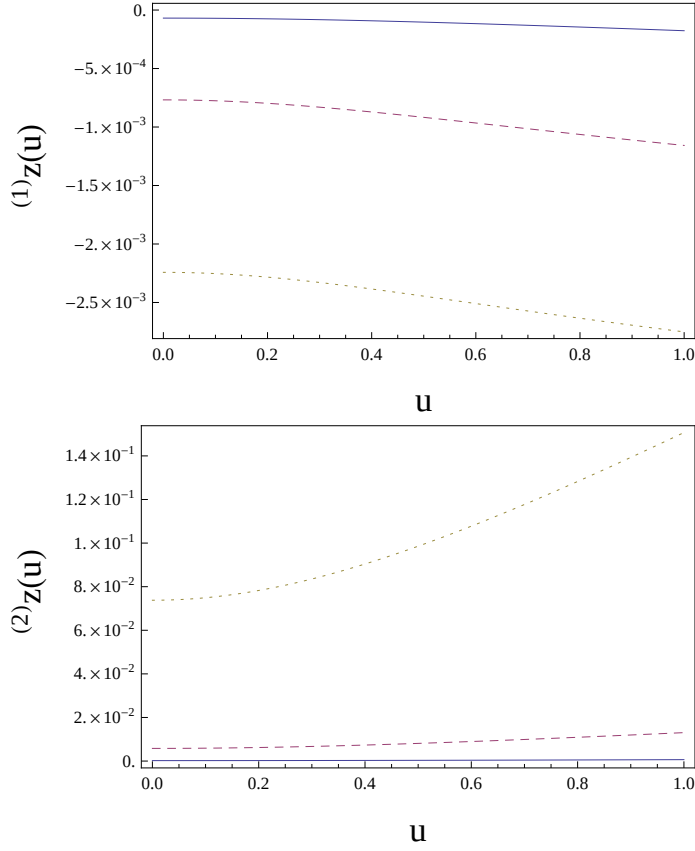


Figure 2.3: Plot of $(1)z$ (left) and $(2)z$ in terms of $u = \cos \theta$, for $\lambda = 1, 10, 100$ (continuous, dashed, and dotted line respectively). In the left plot, the EMS is negative and decreases with the field. In the right plot the opposite occurs. [130]

also follows from the plots that the EMS is typically one order of magnitude larger for the second polarization.

2.3 Conclusions

What conclusions can we draw from the exposition in this Chapter?

For the BI radiating charge, we have tried to apply a technique used in Maxwellian physics encountering insurmountable difficulties for our time constraints. The benefits of BI theory would be evident if the field of a radiating charge could be computed: since the field of this theory is finite everywhere the self-force problem is expected to be much simpler and richer of insights. We have tried to simplify the computation of the field for a relativistic, uniformly accelerating charge using the Rindler coordinates. The advantage of this proce-

ture is that the motion of the particle is extremely simple, being it at rest. The disadvantage is the use of a curved, complicated set of coordinates. This adds to the already non-trivial structure of the differential equations that BI theory confronts us with. As shown in [100], in the linear case such complications do not represent an insurmountable obstacle since, thanks to an analogy with the geodesic equation of a particle moving next to a Schwarzschild-Droste black hole, the solution for the 4-potential is known. At present, we were not able to find such a correspondence for the differential equation we obtain in the BI case. Such equation is derived via an expansion on the field around its Maxwellian value, which is possible because BI theory generates only tiny variations with respect to the Maxwell field.

We have tried to solve such equation through convolution, since the Green function of the problem was known. Unfortunately, the complexity of the ensuing integrals is such that we were unable to solve them, nor analytically nor computationally. We have focused our attempts mainly on the analytical part, also looking for possible solutions coming from the use of hypergeometric functions. Only after realizing that such attempts lead nowhere, we tried to approach the problem from the computational point of view. Because of the lack of expertise and time no substantial progresses were made in that regard. The author and his collaborators intend to continue working on these issues in the future. We want to stress, however, that such mathematical problems might be the result of the fact that we have expanded the BI solution: the entire series must be convergent, but - to the best of our knowledge - there is no literature assuring us that each single term is finite.

Concerning the overcritical magnetic dipole in the HE theory, we have derived and computed the EMS that photons experience when emitted at a given point of a dipole field with QED one-loop corrections and travel to infinity. The EMS evidences a polarization dependent effect, since a photon may gain or lose energy depending on its polarization (see [119,120] for a gravitation analogue). This “photon acceleration” effect has been discussed before in a different setting in [131], although the polarization plays no role there. The energy exchange between photons and the background field is due to their interaction through the non-linearities of the theory (since the EMS is null when $\alpha = 0$).

This new effect may be important at least in two different areas. The first one is the astrophysics of magnetars. To give a full description of such objects gravitation and rotation should not be neglected, but the significance of the EMS can be seen in a simpler setting, taking into account that the EMS adds to the gravitational redshift (see [85] for the proof of this assertion in the case of the EM redshift). For a star of two solar masses and a radius of 10 km (typical values for a magnetar), the gravitational redshift (calculated using the Schwarzschild-Droste metric [87]) would be approximately 0.2. Hence, as shown in Figure 2.3, for strong enough background fields, the EMS can reach values that are of the order of the gravitational redshift (for one of the polarizations).

This effect would lead to a variation in the shift of photons coming from the surface of the star and hence in the ratio of the mass and the radius.

There are other concurring effects leading to a detected frequency being

different from the one expected. Among the effects to be compared to the EMS, there is the Doppler shift due to the peculiar velocity of the magnetar, and the influence of the magnetic field on the spectral lines. For the latter, we remark that modeling the influence of an overcritical magnetic field on the spectral lines is highly non-trivial. In such a regime, the usual treatment of the magnetic effects does not apply. Indeed, the Coulombian interaction must be treated as a perturbation to the magnetic interaction, conversely to the usual Zeeman effect. For an explicit computation of the frequency shift, the expressions for the energy levels of an atom in an overcritical magnetic field would be needed, and these, to our knowledge, are not available in the literature [112, 132, 133]. Furthermore, the shift in the spectral lines due to a magnetic field and the Heisenberg-Euler effect are of a different nature. While the former depends only on the change of energy levels due to the magnetic field, the latter is a consequence of both the magnetic field and the propagation of the photon.

An interesting recent potential source of frequency shifts comes from the non-minimal coupling between gravity and the Higgs field [134–136]. In the notation of the second of these references, the non-minimal coupling between the Higgs and the Kretschmann scalar would lead to a shift given by

$$\frac{\delta\nu}{\nu} = \sqrt{3}\xi' R_S \frac{\lambda_\mu^2}{r^3} \quad (2.176)$$

where ξ' is the coupling constant, R_S is the Schwarzschild-Droste radius, and λ_μ the Compton wavelength for the Higgs. Taking typical values ($M = 3$ km, $r = 20$ km, and $\lambda_\mu = 2 \times 10^{-21}$ km), it follows that

$$\frac{\delta\nu}{\nu} = 6\sqrt{3}\xi' \times 10^{-40} \quad (2.177)$$

Hence, for this effect to be comparable to the Heisenberg-Euler shift quite large values of the Higgs-curvature coupling parameter are needed, $\xi' \approx 10^{37}$.

Our results may be relevant also in experimental settings to detect the signature of the QED vacuum, in particular those that use an overcritical dipole magnetic field, such as PVLAS [71, 74, 75] and BMV [72, 73].

Chapter 3

EFFECTIVE PHOTON MASS FROM SUSY AND LORENTZ BREAKING

Résumé

Dans ce Chapitre, nous avons essayé de placer un photon massif dans le Modèle Standard. Ce dernier est le lagrangien de la physique des particules et résume en lui-même toutes les interactions parmi les particules fondamentales et explique les forces électromagnétique, nucléaire faible et forte. Initialement, le Modèle Standard prédisait des particules sans masse qui ne sont pas conformes avec ce que l'on voit dans la nature. Le problème a été résolu [137–139] en introduisant un nouveau champ qui, grâce au mécanisme de rupture de la symétrie, engendre des masses correctes. Ce champ est connu comme le champ de Higgs et a été finalement mesuré en 2015 [140]. est plus faible qu'attendue. Une tentative pour expliquer une telle différence est d'aller au-delà du Modèle Standard grâce à la théorie de la supersymétrie (SUSY). Dans cette théorie, chaque particule bosonique possède un équivalent fermionique et vice versa. Les interactions parmi les nouvelles particules qui apparaissent et le boson de Higgs conduiraient à la bonne valeur expérimentale.

Dans le contexte de SUSY, nous considérons les extensions du Modèle Standard où la symétrie de Lorentz est cassée à des énergies proches de l'échelle de Plank. Notre but est d'étudier la propagation de la lumière et voir si nous pouvons trouver des comportements massifs. Nous considérons quatre théories différentes. Les deux premières seront CPT impaires, les deux suivantes CPT-paires. À l'intérieur des deux groupes, la deuxième classe sera obtenue à partir de la première via l'intégration de tous les champs supersymétriques – plus spécifiquement du photino, qui est le partenaire supersymétrique du photon. Pour la classe CPT impaire, nous adoptons le modèle de Carroll-Field-Jackiw,

qui est une version modifiée du lagrangien de Chern-Simons. Pour implémenter le terme non-maxwellien dans un espace-temps de dimensionnalité paire, un vecteur externe de fond est nécessaire. Le prix à payer pour cela est la rupture de l'invariance de Lorentz. Dans SUSY, ce fond est donné par un condensat généré à haute énergie par la perte de la symétrie de Lorentz. Nous avons écrit les relations de dispersion à la fois pour le cas où le photino est laissé comme un degré de liberté et pour le cas où il est intégré. Elles sont [141, 142]

$$(k^\mu k_\mu)^2 + (v^\mu v_\mu) (k^\nu k_\nu) - (v^\mu k_\mu)^2 = 0 \quad (3.1)$$

$$(k^\mu k_\mu)^2 + \frac{(v^\mu v_\mu) (k^\nu k_\nu)}{(1 - H - M)^2} - \frac{(v^\mu k_\mu)^2}{(1 - H - M)^2} = 0 \quad (3.2)$$

où H et M sont des scalaires donnés par des combinaisons appropriées des condensats de, $k^\mu = (\omega, \vec{k})$ et v^μ est le fond qui provoque la rupture de Lorentz. Les deux sont ensuite les mêmes sauf pour un changement dans la valeur de v^μ , qui est très petit. Nous soulignons que ces relations de dispersion sont du quatrième ordre en ω et que seulement des puissances paires de la fréquence angulaire apparaissent. Par conséquent, nous avons deux solutions (doubles) à cette équation. Nous devons étudier la propagation pour ces deux modes. Pour le cas CPT pair, nous utilisons le modèle k_F -term. Il s'agit d'un modèle purement supersymétrique et qui a pour but d'étudier comment les condensats fermioniques affectent la physique du photon et du photino. Pour cette théorie, les relations de dispersion sont connues seulement en laissant le photino comme un degré de liberté. Elles sont

$$\omega = (1 + \rho \pm \sigma) |\vec{k}| \quad (3.3)$$

où

$$\rho = \frac{1}{2} \tilde{K}_\alpha^\alpha \quad (3.4)$$

$$\sigma^2 = \frac{1}{2} \left(\tilde{K}_{\alpha\beta} \right)^2 - \rho^2 \quad (3.5)$$

$$\tilde{K}_{\alpha\beta} = K_{\mu\nu\alpha\beta} \hat{k}_\mu \hat{k}_\nu \quad (3.6)$$

$$\hat{k}^\mu = \frac{k^\mu}{|\vec{k}|} \quad (3.7)$$

sont toutes des quantités dépendant du terme de rupture de symétrie. Nous avons dérivé les relations de dispersion pour le cas où la physique du photino a été intégrée [143], obtenant

$$b\chi k^4 - k^2 + (3a + bk^2) \chi^{\alpha\beta} k_\alpha k_\beta = 0 \quad (3.8)$$

où a et b sont des constantes (dimensionnelles), χ est la trace de $\chi^{\alpha\beta}$ et cette dernière est le tenseur qui provoque la rupture de Lorentz dans la théorie. Il

est important de remarquer que dans les deux cas impairs, le fond qui cause la violation de la symétrie est un tenseur, à la différence de ce qui se passe pour le modèle de Carroll-Field-Jackiw.

Ayant donné les relations de dispersion pour toutes les théories que nous avons l'intention d'étudier, la discussion est centrée sur une analyse approfondie de ces dernières. Pour la relation de dispersion de Carroll-Field-Jackiw, nous traitons seulement le cas non-intégré, puisque celui où le photino n'est plus un degré de liberté est simplement une réduction de la valeur du vecteur de fond du premier. Bien que l'on obtienne la vitesse de groupe comme une fonction du nombre d'onde, nous focalisons notre attention principalement sur deux façons légèrement inhabituelles d'approcher la dispersion. Nous donnons le retard entre deux photons émis à des fréquences différentes depuis la même source. L'effet dépend de l'angle θ entre la direction de propagation et le vecteur de fond. Pour $\cos\theta \neq 0$, les vitesses en terme de fréquence angulaire sont les mêmes pour les deux modes émergeant de l'équation de dispersion et sont

$$v_{g\pm}^{\cos\theta \neq 0} = 1 - \frac{v^2}{8\omega^2} (2 + \cos^2\theta) \quad (3.9)$$

Pour $\cos\theta = 0$, un des modes a un comportement maxwellien tandis que l'autre est dispersé

$$v_{g+}^{\cos\theta=0} = 1 \quad (3.10)$$

$$v_{g-}^{\cos\theta=0} = 1 - \frac{v^2}{2\omega^2} \quad (3.11)$$

Dans ces expressions, v est le module du vecteur v^μ . De plus, nous avons fixé la composante du temps de ce 4-vecteur à zéro du fait que sa présence permettrait l'apparition de modes superluminaux. Synthétiquement, nous pouvons conclure que la vitesse de groupe a la forme

$$v_g = 1 - av^2 \frac{1}{\omega^2} \quad (3.12)$$

où a est un facteur numérique. Par conséquent, le retard pour deux photons à différentes fréquences est

$$\Delta t = l \cdot a \cdot v^2 \cdot \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \quad (3.13)$$

En comparant cela à l'expression correspondante pour un photon de de Broglie-Proca, nous remarquons un comportement massif. La masse du photon est donnée par

$$m_\gamma = 2 a v \quad (3.14)$$

Une estimation de l'ordre de grandeur de v a été récemment donnée dans [144]. Avec leur résultat nous obtenons une masse de l'ordre de $10^{-55} kg$,

Nous avons étudié la relation de dispersion de Carroll-Field-Jackiw aussi en termes des forces de champ. L'avantage de cette approche est que nous pouvons

étudier comment la dispersion dépend de la polarisation. Pour des polarisations circulaires, nous trouvons des résultats qui sont compatibles avec les vitesses de groupe obtenues par la procédure normale. Pour une polarisation générale, on voit plutôt qu'il y a possibilité que des modes avec des fréquences imaginaires apparaissent.

Pour le secteur CPT-pair, les difficultés sont dues à la nature tensorielle du fond. Par conséquent, pour calculer la vitesse de groupe nous devons prendre la dérivée de la fréquence angulaire par rapport à chaque composante du vecteur nombre d'onde et ensuite recomposer la vitesse totale via le théorème de Pythagore. Pour les deux classes nous obtenons un résultat similaire

$$v_g = 1 - C (t_1 \sin^2 \theta \cos^2 \varphi + t_2 \sin^2 \theta \sin^2 \varphi + t_3 \cos^2 \theta) \quad (3.15)$$

où C est une constante dépendant des composantes du tenseur de fond uniquement. Nous constatons que nous avons un comportement non-maxwellien, puisque la vitesse de groupe est différente de la valeur obtenue dans la théorie classique. Cependant, nous n'avons pas de dispersion puisque la correction est une constante, une fois que nous avons choisi la direction de propagation, étant le tenseur de fond ne dépendant ni du temps ni de la position. De plus, une telle correction est très petite puisque l'ordre de grandeur typique des composantes du tenseur est $10^{-19} eV$.

La dernière partie de ce Chapitre est consacrée à l'explication de comment la masse du photon devrait se produire dans le secteur CPT impair. Pour montrer cela nous écrivons le lagrangien de Carroll-Field-Jackiw en termes de composantes du 4-potentiel. Nous perdons la forme explicite covariante de l'expression, mais nous sommes capable de voir que le potentiel scalaire électrique ϕ apparaît seulement à travers son gradient et que ce n'est pas un vrai degré de liberté, puisque il peut être éliminé du lagrangien via une redéfinition du champ

$$\chi = \vec{\nabla} \phi + \dot{\vec{A}} - 2\vec{v} \times \vec{A} \quad (3.16)$$

$\vec{A}$ étant le potentiel vecteur magnétique et le point au-dessus d'une lettre indique la dérivée temporelle. L'équation du mouvement pour le champ χ est $\chi = 0$, signifiant qu'il n'y a pas de dynamique pour ce champ. Par conséquent, la partie dynamique du lagrangien est

$$L = \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - 2 (\vec{v} \times \vec{A})^2 - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) \quad (3.17)$$

Nous nous focalisons sur le second terme qui peut être écrit comme

$$2 |\vec{v}|^2 \tilde{A}_2^2 + 2 |\vec{v}|^2 \tilde{A}_3^2 \quad (3.18)$$

après qu'une rotation appropriée ait été utilisée, transformant $\vec{A}$ en $\tilde{\vec{A}}$. On voit que, dans cette expression, nous avons un terme massif *à la* de Broglie-Proca, de Broglie-Proca, où le rôle de la masse est joué par le module au carré du vecteur de fond.

Pour résumer, nous avons prouvé que le retard a une dépendance en $\frac{1}{\omega^2}$ de la même façon qu'elle apparaît dans le cas de de Broglie-Proca. À la différence de ce qui se passe dans cette théorie, la masse n'est cependant pas une constante. Elle dépend du vecteur de fond qui provoque la violation de la symétrie de Lorentz et de la direction. En d'autres mots, il s'agit d'une masse effective. À cause de cela, pour obtenir une estimation de cette valeur, nous avons besoin d'estimer l'ordre de grandeur du vecteur de fond. Un travail récent [144] le fixe à $10^{-19}eV$. Cela résulte en une masse d'ordre de $10^{-55}kg$, seulement un ordre de grandeur plus faible que la limite acceptée par le Particle Data Group [47]. De plus, une étude sur la dispersion en termes d'ondes polarisées a montré qu'il est possible d'avoir des régions où la vitesse de la lumière est complexe. Dans tous les cas CPT-impairs, nous avons obtenu un comportement massif. Nous avons essayé de donner une explication à cela par l'analyse plus minutieuse du lagrangien. Nous l'avons écrit dans une forme covariante non-explicite, introduisant les composantes du champ du photon. Le potentiel électrique n'est pas une variable dynamique et on peut l'éliminer du lagrangien. Un terme avec la structure classique d'un terme massif apparaît, mais c'est le fond au carré qui joue le rôle de la masse. Ceci est cohérent avec ce que nous avons précédemment vu dans les cas particuliers que nous avons étudiés et cela nous donne une raison plus fondamentale pour laquelle la masse du photon devrait être liée au vecteur de fond.

3.1 Context

A legitimate question is how a massive photon relates to the Standard Model (SM hereafter). Even more importantly is to search for a mechanism providing mass to the photon. We will provide an initial single reply to both questions.

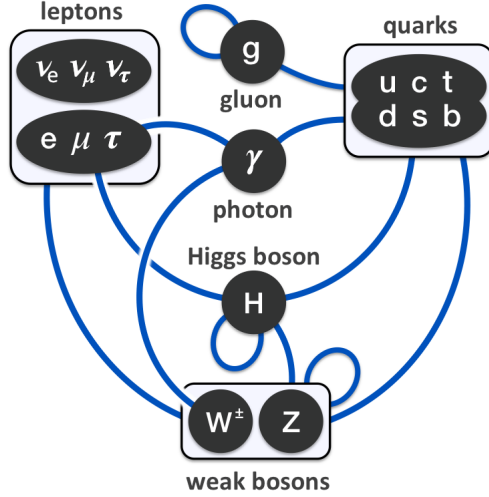
For the most part, we base our understanding of particle physics on the SM. The SM is the Lagrangian of particle physics and summarizes in itself all the interactions among fundamental particles, and accounts for electromagnetic, weak, and strong nuclear forces. The model has been finalized theoretically in the mid seventies, and has found several experimental confirmations ever since. In 1995, the top quark was found [145]. In 2000, the tau neutrino was measured [146]. Last, but not least, in 2015 the most elusive particle, the Higgs boson, was found [140]; its relevance is huge because it is the field that allows the spontaneous symmetry breaking mechanism through which other elementary particles that interact with the Higgs gain their expected mass.

The SM is divided in two sectors, fermionic and bosonic. The fermions are particle of semi-integer spin. To each fermion, corresponds an anti-particle. Inside the sector, the fermions are divided into two groups, according to the force through which they interact. Quarks interact through the strong nuclear force. Leptons through the weak force. Quarks are not visible individually. They combine to form composed particles called hadrons, when they get together as a quark-antiquark duo, or mesons, when three quarks reunite. Being charged, quarks also interact electromagnetically with other fermions.

Of the fermionic family, the remaining particles are called leptons and their direct dynamics is influenced by the weak nuclear interaction. Leptons are further divided in neutrinos, that don't carry electric charge, while the remaining particles do.

All ordinary (baryonic) matter is made of fermions. The integer spin particles are instead called bosons and they are the interaction carriers in the SM. In fact, in the context we are working in, forces are seen as baryonic matter exchanging other particles. The force carrier for the electromagnetism is the photon. Strong nuclear interactions is mediated by eight gluons whose interactions give rise to the so called Quantum ChromoDynamics (QCD). The W^+ , W^- and Z bosons, instead, are the mediators of the weak interaction. They are massive and electrically charged. Being charged, they allow to describe the electric and weak nuclear forces in a single, reunited context, which is called electroweak theory. Fig. 3.1 shows, schematically, the basic particles and interactions of the SM.

In the SM, a special place, is held by the Higgs boson. SM would imply that most particles are massless, unless this special field is introduced. Of course, particles all massless would be in contrast with every day experience. Hence, in 1964, Higgs and others [137–139] came up with a mechanism that, thanks to the introduction of a new particle, the Higgs boson, is able to explain why we observe massive particles. There is however a problem. The detected mass of Higgs boson is too light: in 2015 the ATLAS and CMS experiments showed that the Higgs boson mass is $125.09 \pm 0.32 \text{ GeV}$ [140]. For such a mass value for the

Figure 3.1: Spin $\frac{1}{2}$ fermions and gauge bosons in the SM

Higgs, the SM would be valid up to the Plank energy scales [147]. Instead, it is widely believed that new physics should appear at the TeV scales, which are now the experimental limit up to which the SM was tested [148]. Consequently, we need a theory that can explain the light mass of the Higgs particle and still assures that new physics arises beyond the current technological limits. An interesting attempt to go beyond the SM is for sure Supersymmetry (SUSY) - see [149] for a review. This theory predicts the existence of new particles that are not included in the SM.

The interaction between the Higgs and these new SUSY particles would cancel out some contribution the Higgs boson mass, making it as light as detected. The SM is assumed to be Lorentz invariant at low energy scales, like the ones we can probe with current technology. Anyway, it is reasonable to expect that this prediction is valid only up to certain energy scales, as proven by the work of Kostelecký and Samuel [150–156], where they show that in the context of bosonic strings - i.e. for very high energies - the condensation¹ of tensor fields is possible.

The aforementioned facts show that there are valid reasons to undertake an investigation of physics beyond the SM and also consider Lorentz symmetry violation (LSV). However, it might be argued that we could see those effects at hardly attainable high energies. While this is true, there is a general framework where we can test the low-energy manifestations of LSV. This framework is the so called Standard Model Extension (SME) [157, 158], whose effective Lagrangian is given by the usual SM Lagrangian corrected by SM operators of any

¹In QCD, a field that is representation of a certain symmetry is called a condensate if it spontaneously breaks the symmetry and acquires a vacuum expectation value.

CPT-even	CPT-odd
PLANK SCALE	10^{19} GeV
LOS Y VIOLATION	$10^{17} - 10^{18}$ GeV
GUT	10^{16} GeV
SUSY BREAKING	$10^{11} - 10^{13}$ GeV
k_F - model	Carroll-Field-Jackiw model

Figure 3.2: We show the order in which the symmetries are supposed to break: the order is not certain and in our work we adopted that of [143], although others are possible. At Planck scale they are all exact symmetries. The breaking of Lorentz symmetry (LOS Y) is the first and happens above the energy scale of the Great Unified Theory (GUT). At $10^{11} - 10^{13}$ GeV we have the breaking of SUSY. At the observable level we find the models presented in this work and that carry reminiscence of such symmetries being broken.

dimensionality suitably contracted with Lorentz breaking tensorial background coefficients to get a scalar Lagrangian.

In this context, LSV has been thoroughly investigated phenomenologically. Studies include electrons, photons, muons, mesons, baryons, neutrinos and Higgs sectors [159]. Limits on the parameters associated to the breaking of relativistic covariance are set by quite a few experiments [159–161]. LSV has also been tested in the context of electromagnetic cavities and optical systems [162–168]. Also fermionic models in presence of LSV have been proposed: spinless and/or neutral particles with a non-minimal coupling to a LSV background magnetic properties have been put in relation to fermionic matter or gauge bosons [169–180].

In this article we study the effects visible in the low energy regime to the photon propagation physics in the context where we have LSV. Following [181–191], we see that LSV corresponds to a more fundamental physics of which we have little experience because it happens at a scale of energy greater than that obtainable in modern particle accelerators. At that energy, SUSY is still an exact symmetry, even if we assume that it might break at scales close to the primordial ones. However, LSV may induce SUSY breaking because the background vector (or tensor) that implies the LSV is part of a SUSY multiplet² (see for instance [143]). In Fig. 3.2 we give a graphic representation of the energy scale hierarchy.

²A multiplet is a group of particles that belong to a fundamental representation of a group.

The interest of studying the implication of these models on light propagation lies in the fact that, until we will not be able to exploit gravitational wave information through ground/space based interferometers or pulsar timing arrays, our understanding of the universe will largely depend on our electromagnetic observations - but for what concerns neutrinos and cosmic rays. Fundamental physics is therefore burdened with the task of testing the foundation of electromagnetism, of which photons are the carriers. Astrophysics has just to interpret the universe accordingly.

We have previously shown that non-Maxwellian theories have been proposed over the course of the last century and that they are indeed worth taking into account. Considering the direction of the work we will present here, the massive theories are particularly relevant. If the photon is massive, propagation is affected by this massiveness and the effect is important if we want to perform a precision measurement of astrophysical and cosmological phenomena.

This Chapter will be structured as follows. We will recall some extensions to the standard models that imply LSV. For this work, we will consider four classes, in couples of two. All of them are embedded in the supersymmetric scenario. We will proceed here to a brief summary of the features that are relevant to our discussion, referring the reader to the literature for a more in depth analysis. The first two will be CPT³ odd, the second CPT-even. Inside each class, the second and fourth class will be obtained from the first and third via integration of all the supersymmetric fields. After that, for all these models we will write down and examine the dispersion relations (DR). We will see that the group velocity for the photon takes up a behavior similar to that of a massive particle. This will suggest to try and treat these models as if the QFT effects that go beyond the SM could be summarized into a massive term. Then, we will show how the Lagrangians for these models can be recast in a non-explicitly covariant form so that they reproduce the structure of a de Broglie-Proca Lagrangian, where now the massive term depends on the symmetry breaking parameters. At the end, we will provide some reflections and conclusions.

3.1.1 Chapter outline

In order to help the reader in following the logical flow of this Chapter, we propose here an outline of the contents.

- For the presentation of the CPT-odd Lagrangians, we consider
 - CPT-odd case (Carroll-Field-Jackiw model), where the Lorentz violating perturbation is a background vector v_μ

³First, a gauge theory is Abelian if its own symmetry group is commutative. CPT stands for Charge-Parity-Time Reversal and represent three operator that, applied to a physical state of an Abelian gauge theory return either the physical state itself or minus the state. In the first case the theory is said to be CPT-even, in the second CPT-odd. The SM is CPT-even. The implication of the CPT symmetry is that the “mirror image” of the universe would undergo the same physics. C implies passing from matter to anti-matter, or vice versa; P implies parity inversion as the mirrored positions of objects; T implies time reversal, since time has the opposite sign.

- * Lagrangian for the case where the photino⁴ is left as a separate degree of freedom (Class 1)
- * Lagrangian for the case where the photino has been integrated out (Class 2)
- CPT-even case (k_F -model), where the Lorentz violating perturbation is a tensor
 - * Lagrangian for the case where the photino is left as a separate degree of freedom (Class 3)
 - * Lagrangian for the case where the photino has been integrated out (Class 4)

For Classes 1 and 3 the photon propagation is not influenced by the photino; conversely, for Classes 2 and 4.

- The study of the CPT-odd case is done for both Lagrangians at the same time, because the ensuing DR differ by a simple rescaling of the background Lorentz symmetry breaking vector v_μ
 - A) We obtain the group velocity in function of the wave number, in the hypothesis that $v_0 = 0$.
 - B) For $v_0 = 0$, we analyze the time delay for two photons emitted at different frequencies, finding a massive behavior where the mass role is played by the background vector absolute value.
 - C) We study the DR for polarized waves.
 - For circular polarizations and $v_0 = 0$, we obtain solutions coherent with the case A) where the spatial part of the background vector $\vec{v}$ has been taken aligned with the propagation direction.
 - For circular polarization and $v_0 \neq 0$, the appearance of complex solutions is possible.
 - We analyze a wave randomly polarized wave where the direction of propagation lies along the $\hat{z}$ axis.
 - * If $v_0 = 0$ and $v_3 = 0$, the result is identical to A), where the propagation in the latter occurs along the $\hat{z}$ axis. Again we evidence a massive behavior like in B).
 - * If $v_0 \neq 0$ and $v_3 = 0$, the behavior resembles the previous point but there is the possibility of complex frequencies.
 - * If $v_0 = 0$ and $v_3 \neq 0$, the solutions are the same as in the above case, but with the roles of v_0 and v_3 inverted.
 - * If $v_0 \neq 0$ and $v_3 \neq 0$, there is no straightforward way to solve the equation. We have analyzed a parametric solution and shown that complex frequencies are once more possible.

⁴In SUSY each boson has a fermion as supersymmetric counterpart and vice versa. The photino is the fermionic partner associated to the photon. It is a particle that it is not included in the SM.

- The study of the CPT-even case is done for the two Lagrangian separately.
 - For the Lagrangian where the photino has not been integrated out, we consider three possible forms for the symmetry breaking background tensor in ascending order of complexity.
 - * If the tensor is proportional to the Kroenecker's delta, the vacuum behaves like an isotropic medium.
 - * For the remaining two cases (diagonal with or without vanishing trace) we find that the group velocity is reduced by a factor depending on the direction of propagation and on the tensor components. This corresponds to the vacuum behaving like a non-isotropic medium
 - For the Lagrangian where the photino has been integrated out, we find that the group velocity is reduced by a factor depending on the direction of propagation and on the tensor components.
- Finally, we explain why we expect a massive behavior for the CPT-odd case and why the role of the mass is played by the background vector.

The following section is very technical. We do give the useful definitions where required; however, a in-depth discussion of QCD and SME is not our purpose. We want to emphasize our fundamental physics setting, for which we privilege the discussion on the phenomenological consequences, namely light propagation, of Lorentz violating SMEs. We limit ourselves to supplying the reader with some insight on how these models are devised in order to explain the physical principles that drove the derivation of the DR. Ultimately the DR are all we need for the suite. The reader only interested in the LSV consequences on the photon propagation can jump to the Sec. 3.3

3.2 Models for SUSY and Lorentz symmetry breaking

3.2.1 Classes 1 and 2: Carroll-Field-Jackiw Model

Class 1 and 2 are strictly related. The original idea behind them has nothing to do with SUSY [141]. Carroll, Field and Jackiw (CFJ) introduce a modified Chern-Simons term in the EM Lagrangian. The Chern-Simons Lagrangian is only valid in odd spaces; this is why there must be some slight adjustment to pass to a 3+1 dimensional case. The price for this operation is the loss of Lorentz invariance. In the original paper, this was exploited to set experimental limits on the Lorentz violating parameter, but, with hindsight, we can embed this theory in a ampler scenario where LSV is assumed.

The Lagrangian reads

$$L_1 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}v_\mu A_\nu F_{\sigma\rho} \quad (3.19)$$

We see that a coupling between the EM field and a background vector v_μ arises. This vector is the culprit for breaking Lorentz invariance. First, we want to point out what the physical relation between this addition and Chern-Simons EM is. There are many examples (quantum Hall effect, high- T_c superconductivity, etc) where a phenomenon is taken bound on a plane in first approximation, excluding any interesting dynamics from happening on the perpendicular plane. Thus, the vector v_μ can be chosen to lie on the plane as well, reducing the problem to a 2+1 dimensional spacetime. Such a theory would be a modified Maxwell's EM, but it would be Lorentz and gauge invariant and very close to Chern and Simon's result of a topological massive photon.

If we perform a gauge transformation on the EM field of the form $\Delta A_\mu = \partial_\mu \chi$, then

$$\Delta L_1 = \frac{1}{4} \chi \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho} (\partial_\mu v_\nu - \partial_\nu v_\mu) \quad (3.20)$$

to within a divergence, because the Maxwellian part is gauge invariant, as we know. To restore such invariance, the vanishing of Eq. 3.20 must be imposed. This is obviously the case if v_μ is a constant vector, because then, in flat spaces, $\partial_\mu v_\nu = 0$. However, from a classical standpoint, this is rather unsettling because such a vector would correspond to determining a preferential direction in space. Namely, non-vanishing spatial components would break the rotational invariance and non-vanishing time components would break the invariance under boosts. From these considerations we clearly see that such a term in the Lagrangian necessarily break Lorentz invariance.

We proceed to write the modified Maxwell equations. Via the variation method, we obtain the dynamical pair of Maxwell's equations

$$\partial_\mu F^{\mu\nu} + \frac{1}{2} \epsilon^{\nu\kappa\lambda\rho} v_\kappa F_{\lambda\rho} = 0 \quad (3.21)$$

Explicitly, for $\nu = 0$,

$$\begin{aligned} \partial_\mu F^{\mu 0} + \frac{1}{2} \epsilon^{0ijk} v_i F_{ij} &= 0 \\ \Rightarrow -\partial_i (-E_i) + \frac{1}{2} \epsilon_{ijk} (-v_i) (-\epsilon_{jkl} B_l) &= 0 \\ \Rightarrow \vec{\nabla} \cdot \vec{E} + \vec{v} \cdot \vec{B} &= 0 \end{aligned} \quad (3.22)$$

and, for $\nu = i$,

$$\begin{aligned} \partial_\mu F^{\mu i} + \frac{1}{2} \epsilon^{i0jk} v_0 F_{jk} + \frac{1}{2} \epsilon^{ij0k} v_j F_{0k} + \frac{1}{2} \epsilon^{ijk0} v_j F_{k0} &= 0 \\ \Rightarrow -\partial_t \vec{E} + \partial_j F^{ji} - \frac{1}{2} \epsilon_{ijk} v_0 (-\epsilon_{jkl} B_l) + \epsilon_{ijk} (-v_j) E_k &= 0 \\ \Rightarrow \vec{\nabla} \times \vec{B} + v_0 \vec{B} - \vec{v} \times \vec{E} = \partial_t \vec{E} \end{aligned} \quad (3.23)$$

where the changes of signs in the second lines of each equation are due to the passage to the Euclidean vectors.

We can complete the set of Maxwell equations with the ones coming from Ricci Curbastro-Bianchi identities. These are left untouched by the CFJ term. So they read

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \quad (3.24)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (3.25)$$

We write these equations in the momentum space, performing a Fourier transform

$$i\vec{k} \cdot \vec{E} + \vec{v} \cdot \vec{B} = 0 \quad (3.26)$$

$$\vec{k} \times \vec{E} = \omega \vec{B} \quad (3.27)$$

$$\vec{k} \cdot \vec{B} = 0 \quad (3.28)$$

$$i\vec{k} \times \vec{B} + v_0 \vec{B} - \vec{v} \times \vec{B} = -i\omega \vec{E} \quad (3.29)$$

Thanks to Eq. 3.27 we obtain $\vec{B}$ in terms of $\vec{E}$:

$$\vec{B} = \frac{\vec{k}}{\omega} \times \vec{E} \quad (3.30)$$

Inserting this into Eq. 3.29, we get

$$\begin{aligned} i\vec{k} \times \left(\frac{\vec{k}}{\omega} \times \vec{E} \right) + v_0 \frac{\vec{k}}{\omega} \times \vec{E} - \vec{v} \times \vec{E} &= -i\omega \vec{E} \\ \Rightarrow \frac{i}{\omega} (\vec{k} \cdot \vec{E}) \vec{k} - \frac{i}{\omega} \vec{k}^2 \vec{E} + \frac{v_0}{\omega} (\vec{k} \times \vec{E}) - \vec{v} \times \vec{E} &= -i\omega \vec{E} \\ \Rightarrow (\omega^2 - \vec{k}^2) \vec{E} + (\vec{k} \cdot \vec{E}) \vec{k} &= i (v_0 \vec{k} \times \vec{E} - \omega \vec{v} \times \vec{E}) \end{aligned} \quad (3.31)$$

Eq. 3.31 can be easily arranged in the form

$$R_{ij} E_j = 0 \quad (3.32)$$

where R_{ij} is the matrix

$$R_{ij} = ik^2 \delta_{ij} + ik_i k_j - v_0 \epsilon_{ijk} k_k + \epsilon_{ijk} \omega v_k \quad (3.33)$$

As a first consistency check, we must compute the determinant of this matrix. $\det R = 0$ gives us the DR, which should be the same as those derived with the potentials. Indeed, writing each term in the matrix and taking the determinant, we obtain

$$(k^\mu k_\mu)^2 + (v^\mu v_\mu) (k^\nu k_\nu) - (v^\mu k_\mu)^2 = 0 \quad (3.34)$$

where

$$k^\mu = (\omega, \vec{k}) \quad (3.35)$$

This is the DR we are going to use. Where is SUSY hidden in all this? In the original work [141], there was no link to a supersymmetric theory. Neither the origin of the the background vector is given, though. We claim that

v_μ has a microscopic justification which originates from fundamental fermionic condensates present in SUSY.

A supersymmetric extension of the CFJ model is done in [192]. The Lagrangian presented in this work includes, of course, also various other supersymmetric partners. As shown in [142] the component field Lagrangian⁵ can be written in terms for the physical fields as

$$\begin{aligned} L_{1SUSY} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\epsilon^{\mu\alpha\beta\nu}v_\mu F_{\alpha\beta}A_\nu - \frac{i}{2}\bar{\Lambda}\bar{\gamma}^\mu\partial_\mu\Lambda + \\ & + \left(\text{Re}(F) + \frac{1}{4}\bar{\Psi}\Psi\right)\bar{\Lambda}\Lambda - i\left(\text{Im}(F) + \frac{1}{4}i\bar{\Psi}\gamma_5\Psi\right)\bar{\Lambda}\gamma_5\Lambda + \\ & -\frac{1}{4}\left(v_\mu + \bar{\Psi}\gamma_\mu\gamma_5\Psi\right)\left(\bar{\Lambda}\gamma^\mu\gamma_5\Lambda\right) + \sqrt{2}\bar{\Lambda}\Sigma^{\mu\nu}\gamma_5\Psi F_{\mu\nu} \end{aligned} \quad (3.36)$$

where $F = F_{\mu\nu}F^{\mu\nu}$, γ^μ are the Dirac's matrices⁶ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ and the Majorana fermionic fields⁷ Λ and Ψ indicate the photino and the background fermion, which is supposed to be responsible for v_μ . Always in [142], it is shown that the DR is the same as the one in Eq. 3.34. However, integrating Eq. 3.36 to eliminate the contribution from the SUSY photino field Λ , we obtain that the effective action for the photon reads

$$L_2 = -\frac{1}{4}(1 - H - M)F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda} \quad (3.38)$$

where $M_{\mu\nu} = -C_\mu S_\nu$, being H , C_μ and S_μ structures depending on the SUSY background fermion condensates. The DR is obtained via Least Action Principle as before and reads

$$(k^\mu k_\mu)^2 + \frac{(v^\mu v_\mu)(k^\nu k_\nu)}{(1 - H - M)^2} - \frac{(v^\mu k_\mu)^2}{(1 - H - M)^2} = 0 \quad (3.39)$$

⁵In the supersymmetric context, fields are described by multiplets of fields that include the physical fields themselves and auxiliary fields. A field is considered physical if it is a mass eigenstate, that is if it has a kinetic term in the canonical sense. Auxiliary fields are those that appear in a purely algebraic way inside the Lagrangian and can therefore be eliminated through the corresponding field equations (see Sec. 3.4 for an example of this procedure). For obtaining Eq. 3.36 we have spelled out the Lagrangian in terms of complete multiplets, *i.e.* containing the auxiliary fields too. To obtain the expression we present, though, these auxiliary fields have been removed and we are left only with the gauge fields (containing photon and photino) and the matter fields (containing the SUSY background that causes the symmetry breaking).

⁶Dirac's matrices are a set of four 4×4 matrices that are defined by the relation

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}I_4 \quad (3.37)$$

where, as usual, $\eta^{\mu\nu}$ is the Minkowski metric and I_4 is the identity in 4 dimension. As shown in [53], these matrices are important in the construction of the Dirac equation, that is the equation that regulates the dynamics of spin $\frac{1}{2}$ fermions.

⁷Fermions are introduced in two different ways. Either a fermion and its antiparticle are two different particles or the two are the same. In the first case, we talk of Dirac fermions. In the second, about Majorana fermions. Up to now, all fermions in the SM have been seen to be Dirac fermions. The only open question is about the nature of neutrinos. The problem of their massiveness has not been satisfyingly settled yet. Neutrinos could be Majorana fermions.

3.2.2 Classes 3 and 4: k_F -term model

Using the supersymmetrized CFJ model, we have seen that it is indeed possible to have a spontaneous LSV induction starting from fermionic condensate background. In other words, the fermionic fields of the theory spontaneously assume a vacuum expectation value and break the Lorentz symmetry creating a condensate. It is normal to try and see if what holds for the CPT-odd sector holds also for the CPT-even one. More specifically, in [143], the authors investigate the so called k_F -term SME, with a particular attention to how the fermionic condensates affect the physics of the photon and photino.

The starting point is a Lagrangian in the form

$$L_{CPT-even} = (k_F)_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} \quad (3.40)$$

where the tensor $(k_F)_{\mu\nu\alpha\beta}$ is double traceless and

$$(k_F)_{\mu\nu\alpha\beta} = -(k_F)_{\nu\mu\alpha\beta} = -(k_F)_{\mu\nu\beta\alpha} = (k_F)_{\alpha\beta\mu\nu} \quad (3.41)$$

The idea is to write the tensor in terms of a single bosonic vector ξ_μ which signals LSV. This field must be supposed to be part of a chiral field⁸ whose fermionic condensates generates the LSV. To achieve this, we start by not considering the fully anti-symmetric part in $(k_F)_{\mu\nu\alpha\beta}$ because it would only account to a total derivative in the action. Then we exclude the component yielding birefringence. Exploiting the Ansatz in [193, 194], we write

$$(k_F)_{\mu\nu\alpha\beta} = \frac{1}{2} (\eta_{\mu\alpha} \tilde{\kappa}_{\nu\beta} - \eta_{\mu\beta} \tilde{\kappa}_{\nu\alpha} + \eta_{\nu\beta} \tilde{\kappa}_{\mu\alpha} - \eta_{\nu\alpha} \tilde{\kappa}_{\mu\beta}) := K_{\mu\nu\alpha\beta} \quad (3.42)$$

where

$$\tilde{\kappa}_{\alpha\beta} = \xi_\alpha \xi_\beta - \eta_{\alpha\beta} \frac{\xi_\rho \xi^\rho}{4} \quad (3.43)$$

This, in turn, implies a Lagrangian in the form

$$L_3 = \frac{1}{4} \left(\frac{1}{2} \xi_\mu \xi_\nu F_\alpha^\mu F^{\alpha\nu} + \frac{1}{8} \xi_\rho \xi^\rho F_{\mu\nu} F^{\mu\nu} \right) \quad (3.44)$$

All these simplifications are actually physically well sound. In fact, had we taken into account the full complexity of the k_F term, then we would have had to deal with a higher spin superfield, whose appearance is instead avoided thanks to transferring the effects to the ξ^μ vector. The Lagrangian Eq. 3.50 will be obtained carrying out the supersymmetrization of Eq. 3.44 taking into account that ξ^μ defines the SUSY background field.

In [143, 195] the DR for the photon is given and reads

$$\omega = (1 + \rho \pm \sigma) |\vec{k}| \quad (3.45)$$

⁸The chirality is the handedness of the spin along of the direction of motion. If chiral symmetry is respected, the fermions are paired in such a way that inverting the chirality one particle is changed in its partner. From the algebraic point of view, this means that the field describing the fermions is a representation of the chiral symmetry group.

where

$$\rho = \frac{1}{2} \tilde{K}_\alpha^\alpha \quad (3.46)$$

$$\sigma^2 = \frac{1}{2} \left(\tilde{K}_{\alpha\beta} \right)^2 - \rho^2 \quad (3.47)$$

$$\tilde{K}_{\alpha\beta} = K_{\mu\nu\alpha\beta} \hat{k}_\mu \hat{k}_\nu \quad (3.48)$$

$$\hat{k}^\mu = \frac{k^\mu}{|\vec{k}|} \quad (3.49)$$

As we did for Class 2, we can obtain an effective photonic Lagrangian by integrating out the photino sector (and all others supersymmetric sectors as well). The resulting Lagrangian reads [143]

$$L_4 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a}{2} \chi_{\mu\nu} F_\kappa^\mu F^{\nu\kappa} + \frac{b}{2} \chi_{\mu\nu} \partial_\alpha F^{\alpha\mu} \partial_\beta F^{\beta\nu} \quad (3.50)$$

To the best of our knowledge, the DR for this theory is not present in literature. We proceed to derive it here. The steps are the following:

- write L in terms of the fields
- get the Euler-Lagrange equations
- perform the Fourier transform

The Lagrangian in terms of the potential is

$$\begin{aligned} L = & \frac{1}{2} A^\mu (\eta_{\mu\nu} \square - \partial_\mu \partial_\nu) A^\nu + \frac{a}{2} A^\mu (-\chi^{\alpha\beta} \partial_\alpha \partial_\beta \eta_{\mu\nu} + 2\chi_{\alpha\nu} \partial_\mu \partial^\alpha - \chi_{\mu\nu} \square) A^\nu \\ & + \frac{b}{2} A^\mu (\chi_{\mu\nu} \square^2 - \chi_{\mu\alpha} \square \partial^\alpha \partial_\nu - \chi_{\nu\alpha} \square \partial^\alpha \partial_\mu + \partial_\mu \partial_\nu \chi^{\alpha\beta} \partial_\alpha \partial_\beta) A^\nu \end{aligned} \quad (3.51)$$

Varying with respect to A^μ we obtain

$$\begin{aligned} & [\eta_{\mu\nu} \square - \partial_\mu \partial_\nu + a (-\chi^{\alpha\beta} \partial_\alpha \partial_\beta \eta_{\mu\nu} + \chi_{\alpha\nu} \partial_\mu \partial^\alpha + \chi_{\nu\alpha} \partial^\alpha \partial_\mu) \\ & + b (\chi_{\mu\nu} \square^2 + \chi_{\mu\alpha} \square \partial^\alpha \partial_\nu - \chi_{\nu\alpha} \square \partial^\alpha \partial_\mu + \partial_\mu \partial_\nu \chi^{\alpha\beta} \partial_\alpha \partial_\beta)] A^\nu = 0 \end{aligned} \quad (3.52)$$

Now we perform the Fourier transform and obtain

$$\begin{aligned} & [-k^2 \eta_{\mu\nu} + k_\mu k_\nu + a (\chi^{\alpha\beta} k_\alpha k_\beta \eta_{\mu\nu} - \chi_{\mu\alpha} k^\alpha k_\nu - \chi_{\nu\alpha} k^\alpha k_\mu) \\ & + b (\chi_{\mu\nu} k^4 + \chi_{\nu\alpha} k^2 k^\alpha k_\mu - \chi_{\mu\alpha} k^2 k^\alpha k_\nu + \chi^{\alpha\beta} k_\alpha k_\beta k_\mu k_\nu)] \tilde{A}^\nu = 0 \end{aligned} \quad (3.53)$$

We choose the Lorentz gauge

$$k_\nu \tilde{A}^\nu = 0 \quad (3.54)$$

so we can simplify the previous expression to

$$\begin{aligned} & [-k^2 \eta_{\mu\nu} + a (\chi^{\alpha\beta} k_\alpha k_\beta \eta_{\mu\nu} - \chi_{\nu\alpha} k^\alpha k_\mu) \\ & + b (\chi_{\mu\nu} k^4 + \chi_{\nu\alpha} k^2 k^\alpha k_\mu)] \tilde{A}^\nu = 0 \end{aligned} \quad (3.55)$$

This shows that we have a matricial equation in the form

$$M_{\mu\nu}\tilde{A}^\nu = 0 \quad (3.56)$$

which has non-trivial solutions only if

$$\det M_{\mu\nu} = 0 \quad (3.57)$$

The terms are rearranged as follows

$$\left[(-k^2 + a\chi^{\alpha\beta}k_\alpha k_\beta)\eta_{\mu\nu} - (a + bk^2)\chi_{\nu\alpha}k^\alpha k_\mu + bk^4\chi_{\mu\nu}\right]\tilde{A}^\nu = 0 \quad (3.58)$$

so that we can clearly see that

$$M_\mu^\nu = -k^2 \left[\delta_\mu^\nu + a \frac{\chi^{\alpha\beta}k_\alpha k_\beta}{k^2} \delta_\mu^\nu + \frac{(a + bk^2)}{k^2} \chi_\alpha^\nu k^\alpha k_\mu - bk^2 \chi_\mu^\nu \right] \quad (3.59)$$

has the structure of the identity plus something small, because the parameters a and b are dependent on the symmetries violating terms which are extremely small. Therefore

$$\det M_\mu^\nu = \det(\mathbb{I} + X) = e^{\text{tr}[\ln(\mathbb{I}+X)]} \quad (3.60)$$

with X small. Expanding the logarithm,

$$\begin{aligned} \det(\mathbb{I} + X) &\sim e^{\text{tr}[X - \frac{X^2}{2}]} \\ &= e^{\text{tr}X - \frac{1}{2}\text{tr}X^2} \\ &\sim 1 + \text{tr}X - \frac{1}{2}\text{tr}X^2 + \frac{1}{2}(\text{tr}X)^2 + O(X^3) \end{aligned} \quad (3.61)$$

Using Eq. 3.59 we finally obtain, at first order

$$\begin{aligned} \det(\mathbb{I} + X) &= -k^2 \left[1 - 4a \frac{\chi^{\alpha\beta}k_\alpha k_\beta}{k^2} + -\frac{(a + bk^2)}{k^2} \chi_\alpha^\mu k^\alpha k_\mu - bk^2 \chi_\mu^\mu \right] = 0 \\ &\Rightarrow -k^2 + 4a\chi^{\alpha\beta}k_\alpha k_\beta - (a + bk^2)\chi_\alpha^\mu k^\alpha k_\mu + bk^4\chi_\mu^\mu = 0 \\ &\Rightarrow b\chi k^4 - k^2 + (3a + bk^2)\chi^{\alpha\beta}k_\alpha k_\beta = 0 \end{aligned} \quad (3.62)$$

where $\chi = \chi_\mu^\mu$.

3.3 Dispersion Relations

3.3.1 Class 1 and 2

3.3.1.1 Time delay

We have seen that the DR is

$$\begin{aligned} \omega^4 - 2\omega^2 \left| \vec{k} \right|^2 + \left| \vec{k} \right|^4 - |\vec{v}|^2 \omega^2 + \left| \vec{k} \right|^2 |\vec{v}|^2 - \left(\vec{v} \cdot \vec{k} \right)^2 &= \\ \omega^4 - \left(2 \left| \vec{k} \right|^2 + |\vec{v}|^2 \right) \omega^2 + \left| \vec{k} \right|^4 + \left| \vec{k} \right|^2 |\vec{v}|^2 - \left(\vec{v} \cdot \vec{k} \right)^2 &= 0 \end{aligned} \quad (3.63)$$

Calling

$$A = 2 \left| \vec{k} \right|^2 + |\vec{v}|^2 \quad (3.64)$$

$$B = \left| \vec{k} \right|^4 + \left| \vec{k} \right|^2 |\vec{v}|^2 - \left(\vec{v} \cdot \vec{k} \right)^2 \quad (3.65)$$

we have

$$\omega^4 - A\omega^2 + B = 0 \quad (3.66)$$

We perform the change of variable

$$\Omega = \omega^2 \quad (3.67)$$

and obtain

$$\Omega^2 - A\Omega + B = 0 \quad (3.68)$$

The solutions are

$$\begin{aligned} \Omega_{\pm} &= \frac{A}{2} \pm \frac{(A^2 - 4B)^{\frac{1}{2}}}{2} \\ &= \frac{2 \left| \vec{k} \right|^2 + |\vec{v}|^2}{2} \pm \frac{\left\{ \left(2 \left| \vec{k} \right|^2 + |\vec{v}|^2 \right)^2 - 4 \left[\left| \vec{k} \right|^4 + \left| \vec{k} \right|^2 |\vec{v}|^2 - \left(\vec{v} \cdot \vec{k} \right)^2 \right] \right\}^{\frac{1}{2}}}{2} \end{aligned} \quad (3.69)$$

We can expand the term inside the curl brackets as follows

$$\begin{aligned} &\left(2 \left| \vec{k} \right|^2 + |\vec{v}|^2 \right)^2 - 4 \left[\left| \vec{k} \right|^4 + \left| \vec{k} \right|^2 |\vec{v}|^2 - \left(\vec{v} \cdot \vec{k} \right)^2 \right]^{\frac{1}{2}} = \\ &4 \left| \vec{k} \right|^4 + 4 \left| \vec{k} \right|^2 |\vec{v}|^2 + |\vec{v}|^4 - 4 \left| \vec{k} \right|^4 - 4 \left| \vec{k} \right|^2 |\vec{v}|^2 + 4 \left(\vec{v} \cdot \vec{k} \right)^2 = \\ &|\vec{v}|^4 + 4 \left(\vec{v} \cdot \vec{k} \right)^2 = |\vec{v}|^4 + 4 \left| \vec{k} \right|^2 |\vec{v}|^2 \cos^2 \theta \end{aligned} \quad (3.70)$$

So

$$\Omega_{\pm} = \omega_{\pm}^2 = \frac{2 \left| \vec{k} \right|^2 + |\vec{v}|^2}{2} \pm \frac{\sqrt{|\vec{v}|^4 + 4 \left| \vec{k} \right|^2 |\vec{v}|^2 \cos^2 \theta}}{2} \quad (3.71)$$

The frequency and spatial momentum can be rendered dimensionless dividing them by the background

$$\tilde{\omega} = \frac{\omega}{|\vec{v}|} \quad (3.72)$$

$$\left| \vec{\tilde{k}} \right| = \frac{\left| \vec{k} \right|}{|\vec{v}|} \quad (3.73)$$

Doing so we obtain

$$\tilde{\Omega}_{\pm} = \tilde{\omega}_{\pm}^2 = \frac{2|\vec{k}|^2 + 1}{2} \pm \frac{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}}{2} \quad (3.74)$$

$$\tilde{\omega}_{\pm} = \sqrt{\frac{2|\vec{k}|^2 + 1}{2} \pm \frac{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}}{2}} \quad (3.75)$$

We are interested in the group velocity

$$\frac{d\tilde{\Omega}_{\pm}}{d|\vec{k}|} = \frac{d\tilde{\omega}_{\pm}^2}{d|\vec{k}|} = 2\tilde{\omega}_{\pm} \frac{d\tilde{\omega}_{\pm}}{d|\vec{k}|} \Rightarrow v_g = \frac{1}{2\tilde{\omega}_{\pm}} \cdot \frac{d\tilde{\Omega}_{\pm}}{d|\vec{k}|} \quad (3.76)$$

The derivative of $\tilde{\Omega}_{\pm}$ with respect to the modulus of the momentum is

$$\begin{aligned} \frac{d\tilde{\Omega}_{\pm}}{d|\vec{k}|} &= \frac{4|\vec{k}|}{2} \pm \frac{1}{4} \frac{8|\vec{k}| \cos^2 \theta}{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}} \\ &= 2|\vec{k}| \pm \frac{2|\vec{k}| \cos^2 \theta}{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}} \end{aligned} \quad (3.77)$$

Therefore

$$\begin{aligned} v_g &= \frac{1}{\tilde{\omega}_{\pm}} \cdot \left[|\vec{k}| \pm \frac{|\vec{k}| \cos^2 \theta}{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}} \right] \\ &= \frac{1}{\sqrt{\frac{2|\vec{k}|^2 + 1}{2} \pm \frac{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}}{2}}} \cdot \left[|\vec{k}| \pm \frac{|\vec{k}| \cos^2 \theta}{\sqrt{1 + 4|\vec{k}|^2 \cos^2 \theta}} \right] \end{aligned} \quad (3.78)$$

It is however more telling to write the group velocity in terms of the frequency. We recast Eq. 3.63 as

$$\omega^4 - \left(2|\vec{k}|^2 + |\vec{\alpha}|^2 \right) \omega^2 + |\vec{k}|^4 + |\vec{\alpha}|^2 |\vec{k}|^2 \sin^2 \theta = 0 \quad (3.79)$$

from which

$$|\vec{k}|^4 + \left(-2\omega^2 + |\vec{v}|^2 \sin^2 \theta \right) |\vec{k}|^2 + \omega^4 - |\vec{v}|^2 \omega^2 = 0 \quad (3.80)$$

For simplicity, we call $|\vec{v}| = v$ and $|\vec{k}| = k$ for simplicity. Solving for k we get

$$k^2 = \frac{(2\omega^2 - v^2 \sin^2 \theta) \pm \sqrt{(2\omega^2 - v^2 \sin^2 \theta)^2 - 4(\omega^4 - v^2 \omega^2)}}{2} \quad (3.81)$$

The term in the square root can undergo some algebra, yielding

$$\begin{aligned} k^2 &= \frac{(2\omega^2 - v^2 \sin^2 \theta) \pm v \sqrt{4\omega^2 \cos^2 \theta + v^2 \sin^4 \theta}}{2} \\ &= \omega^2 - \frac{1}{2} v^2 \sin^2 \theta \pm v \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{\frac{1}{2}} \end{aligned} \quad (3.82)$$

We observe that even for the minus sign the right-hand term is positive, so we don't incur in imaginary solutions for this specific case. We will later show that it is not a general features. The absence of imaginary solutions is due to the condition $v_0 = 0$. Otherwise, the appearance of this parameter gives rise to the tachions.

We now differentiate Eq. 3.82

$$2kdk = 2\omega d\omega \pm \frac{v}{2} \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} \cdot 2\omega \cos^2 \theta d\omega \quad (3.83)$$

Therefore

$$\frac{k}{\omega} = v_g \pm \frac{v}{2} \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} \cos^2 \theta v_g \quad (3.84)$$

So,

$$v_{g\pm} = \frac{k}{\omega} \left[1 \pm \frac{v}{2} \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} \cdot 2\omega \cos^2 \theta \right]^{-1} \quad (3.85)$$

where the $\pm$ has been inserted to keep track of the two possible solutions. To make this an expression in terms of ω only, we need to write down k in term of the frequency. From Eq. 3.82 two cases arise. If $\cos \theta \neq 0$

$$k^2 = \omega^2 - \frac{1}{2} v^2 \sin^2 \theta \pm v \omega \cos \theta \left(1 + \frac{v^2}{4\omega^2} \frac{\sin^4 \theta}{\cos^2 \theta} \right)^{\frac{1}{2}} \quad (3.86)$$

If $\cos \theta = 0$

$$k^2 = \omega^2 - \frac{1}{2} v^2 \pm \frac{1}{2} v^2 \quad (3.87)$$

First we focus on the case $\cos \theta \neq 0$ first and expand the solution to the order $\mathcal{O}(v^2)$

$$\begin{aligned} k^2 &\sim \omega^2 - \frac{1}{2} v^2 \sin^2 \theta \pm v \omega \cos \theta \left(1 + \frac{v^2}{8\omega^2} \frac{\sin^4 \theta}{\cos^2 \theta} \right) + \mathcal{O}(v^3) \\ &\sim \omega^2 - \frac{1}{2} v^2 \sin^2 \theta \pm v \omega \cos \theta + \mathcal{O}(v^3) \end{aligned} \quad (3.88)$$

Collecting ω^2 and taking the square root,

$$\begin{aligned}
k &= \omega \left(1 - \frac{v^2 \sin^2 \theta^2}{2\omega^2} \pm \frac{v \cos \theta}{\omega} \right)^{\frac{1}{2}} + \mathcal{O}(v^3) \\
&\sim \omega \left(1 - \frac{v^2 \sin^2 \theta^2}{24\omega} \pm \frac{v \cos \theta}{2\omega} - \frac{v^2 \cos^2 \theta}{8\omega^2} \right) + \mathcal{O}(v^3) \\
&\sim \omega \pm \frac{v \cos \theta}{2} - \frac{v^2}{4\omega} + \frac{v^2 \cos^2 \theta^2}{8\omega} + \mathcal{O}(v^3)
\end{aligned} \tag{3.89}$$

Therefore

$$\frac{k}{\omega} \sim 1 \pm \frac{v \cos \theta}{2\omega} - \frac{v^2}{4\omega^2} + \frac{v^2 \cos^2 \theta^2}{8\omega^2} \tag{3.90}$$

Inserting this into the expression for the group velocity, the result is

$$\begin{aligned}
v_{g\pm}^{\cos \theta \neq 0} &= \left(1 \pm \frac{v \cos \theta}{2\omega} - \frac{v^2}{4\omega^2} + \frac{v^2 \cos^2 \theta^2}{8\omega^2} \right) \cdot \\
&\quad \cdot \left[1 \pm \frac{v}{2} \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} \cdot 2\omega \cos^2 \theta \right]^{-1}
\end{aligned} \tag{3.91}$$

The the order $\mathcal{O}(\alpha^2)$ it reads

$$\begin{aligned}
v_{g\pm}^{\cos \theta \neq 0} &\sim \left(1 \pm \frac{v \cos \theta}{2\omega} - \frac{v^2}{4\omega^2} + \frac{v^2 \cos^2 \theta^2}{8\omega^2} \right) \cdot \\
&\quad \cdot \left[1 \mp \frac{v}{2} \cos^2 \theta \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} \right]
\end{aligned} \tag{3.92}$$

It is important to notice that we must take only the sign combinations $+-$ and $-+$, not the cross products.

Developing further the second term, we obtain

$$\begin{aligned}
1 \mp \frac{v}{2} \cos^2 \theta \left(\omega^2 \cos^2 \theta + \frac{v^2}{4} \sin^4 \theta \right)^{-\frac{1}{2}} &\sim \\
&\sim 1 \mp \frac{v}{2} \cos^2 \theta \frac{1}{\omega \cos \theta} \left(1 - \frac{v^2 \sin^4 \theta}{8\omega^2 \cos^2 \theta} \right) \\
&\sim 1 \mp \frac{v}{2} \cos^2 \theta \frac{1}{\omega \cos \theta} + \mathcal{O}(v^3)
\end{aligned} \tag{3.93}$$

Therefore

$$v_{g\pm}^{\cos \theta \neq 0} \sim \left(1 \pm \frac{v \cos \theta}{2\omega} - \frac{v^2}{4\omega^2} + \frac{v^2 \cos^2 \theta^2}{8\omega^2} \right) \cdot \left(1 \mp \frac{v}{2} \cos^2 \theta \frac{1}{\omega \cos \theta} \right) \tag{3.94}$$

yielding

$$v_{g+}^{\cos \theta \neq 0} = 1 - \frac{v^2}{8\omega^2} (2 + \cos^2 \theta) \tag{3.95}$$

$$v_{g-}^{\cos \theta \neq 0} = 1 - \frac{v^2}{8\omega^2} (2 + \cos^2 \theta) \tag{3.96}$$

We see that there is no difference in the group velocity for the two solutions. Also, we notice that the discrepancy of our light speed with respect to unity is extremely small because the parameter v is arising from SUSY and hence is bound to be a tiny correction to the SM.

In the case $\cos\theta = 0$, using Eq. 3.85 it is immediate to see that

$$\begin{aligned} v_{g\pm}^{\cos\theta=0} &= \frac{k}{\omega} = \frac{1}{\omega} \left(\omega^2 - \frac{1}{2}v^2 \pm \frac{1}{2}v^2 \right)^{\frac{1}{2}} \\ &\sim 1 - \frac{v^2}{4\omega^2} \pm \frac{v^2}{4\omega^2} \end{aligned} \quad (3.97)$$

So

$$v_{g+}^{\cos\theta=0} = 1 \quad (3.98)$$

$$v_{g-}^{\cos\theta=0} = 1 - \frac{v^2}{2\omega^2} \quad (3.99)$$

In this case we have birefringence. More specifically, for a solution light doesn't experience dispersion, while the other does with a ω^{-2} dependence. It also worth noticing that all dispersive solutions have the same functional dependence. What changes is only the factor in front of it.

To better display the physical consequences of these results, we will compute the time delay that two waves with different frequency would show. The expression for the time delay can be found in [11,196]. So we have

$$\Delta t = l \left(\frac{1}{v_{g1}} - \frac{1}{v_{g2}} \right) = l \left(\frac{v_{g2} - v_{g1}}{v_{g1}v_{g2}} \right) \quad (3.100)$$

We write the dispersive group velocities we have found in the form

$$v_g = 1 - av^2 \frac{1}{\omega^2} \quad (3.101)$$

where a is of the order of the unity whereas v^2 is extremely small. Therefore

$$\begin{aligned} v_{g1}v_{g2} &= \left(1 - av^2 \frac{1}{\omega_1^2} \right) \left(1 - av^2 \frac{1}{\omega_2^2} \right) \\ &\sim 1 - av^2 \frac{1}{\omega_1^2} - av^2 \frac{1}{\omega_2^2} \end{aligned} \quad (3.102)$$

to first order in v^2 . And

$$\begin{aligned} \frac{v_{g2} - v_{g1}}{v_{g1}v_{g2}} &\sim av^2 \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \left(1 - av^2 \frac{1}{\omega_1^2} - av^2 \frac{1}{\omega_2^2} \right) \\ &\sim av^2 \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \end{aligned} \quad (3.103)$$

Hence,

$$\Delta t = l a v^2 \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \quad (3.104)$$

In Fig. 3.3 we plot this time delay for values of θ going from 0 to $\pi/2$. We see that the dependance on the angle is very feeble. To draw this plot, we have supposed a source at 4 *kpc* and set the value of v at 10^{-19} *eV* [197]. This is a crucial choice. Thanks to the time delays we can prospect to test the existence of a massive photon and also the models we have used. As we will show later, it is a general feature of CFJ theories to present a massive, gauge invariant term. This term is proportional to the breaking parameter v , so a choice for this latter corresponds to setting a value to the photon mass. As shown in [197], the value for v can be tested via ground based experiments. More stringent limits than the ones we have chosen do exist. In [141] the authors point out that the parameter v must be smaller of 10^{-34} *eV*. To give a better understanding of what this would mean, in term of photon mass, we make a comparison between Eq. 3.104 and the equivalent expression for a de Broglie-Proca photon [198]. This latter reads

$$\Delta t_{DBP} = \frac{l m_\gamma^2}{2} \cdot \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \quad (3.105)$$

Therefore

$$m_\gamma = 2 a v \quad (3.106)$$

We suggest the reader to retain this result and confront it with the results of Sec. 3.4. The mass is proportional to the parameter v . For the two values we have singled out in the literature, we obtain masses that are very consistent with the known experimental limits. The most stringent value $v = 10^{-34}$ *eV* corresponds to 10^{-69} *kg* which is the lowest possible measurable mass, because of Heisenberg's uncertainty principle, if we set the age of the universe to $13 \cdot 10^9$ years. Even more interesting is the other value: 10^{-19} *eV* turns into 10^{-55} *kg*, which is only an order of magnitude lower than the accepted PDG [47] value.

Before proceeding, we note that that Class 2 is just a rescaling of Class 1. The correcting factor is $\frac{1}{(1 - H - M)^2}$. The scalars H and M are of the same order of magnitude of v_μ ; therefore, this factor is extremely close to unity. So much that, from a numerical point of view, there is no significant change in the display of the results.

3.3.1.2 Dispersion relations for the field strengths

For the CFJ model, we might be interested to obtain the DR in terms of the fields to link the solutions to the physical polarizations of the fields themselves. This must reproduces compatible results with those obtained with the potentials. However, the physical interpretation of said results should be clearer in this alternative approach.

The ensuing advantage is to relate the behavior of the group velocity with the polarization of the fields. A reasonable starting point is a wave circularly

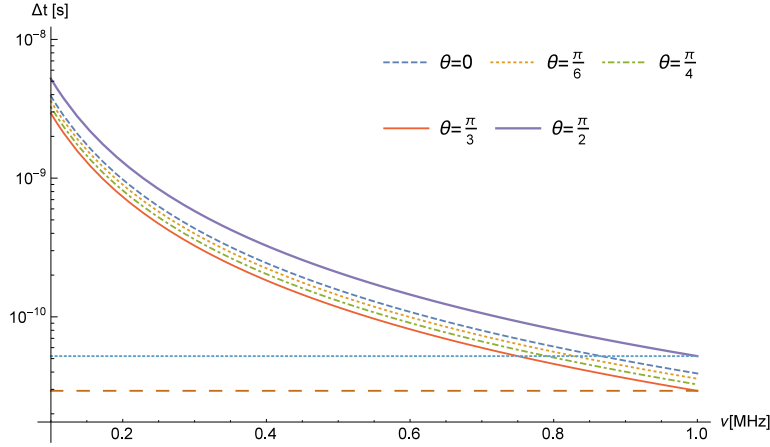


Figure 3.3: Time delay for Class 1 as a function of θ , for a frequency range between 0.1 and 1 MHz at the distance of 4 kpc . We have chosen a low frequency range in order to stay in the regime at which current experiments like OLFAR and LOFAR work. The distance has been chosen at 4 kpc because it is the average distance at which a pulsar is located and a lot of time delay measurements are performed on pulsars. [199]

polarized. We can have a right-handed (+) or left-handed (−) polarization

$$\vec{E}_+ = E_0 e^{i(\vec{k} \cdot \vec{z} - \omega t)} (\hat{x} + i\hat{y}) \quad (3.107)$$

$$\vec{E}_- = E_0 e^{i(\vec{k} \cdot \vec{z} - \omega t)} (\hat{x} - i\hat{y}) \quad (3.108)$$

Since the orientation of our reference system is free, we can choose the wave to propagate along the $\hat{z}$ axis. Under these assumptions and performing the Fourier transform of the fields, the latter read

$$\vec{E}_+ = E_0 (\hat{x} + i\hat{y}) \quad (3.109)$$

$$\vec{E}_- = E_0 (\hat{x} - i\hat{y}) \quad (3.110)$$

Therefore, the DR for such a wave is

$$\begin{aligned} E_0 (\omega^2 - k_3^2) (\hat{x} - i\hat{y}) &= E_0 [iv_0 k_3 \hat{z} \times (\hat{x} - i\hat{y}) - i\omega (v_1 \hat{x} + v_2 \hat{y} + v_3 \hat{z}) \times (\hat{x} - i\hat{y})] \\ \Rightarrow (\omega^2 - k_3^2) (\hat{x} - i\hat{y}) &= iv_0 k_3 (\hat{y} - i\hat{x}) - i\omega (iv_1 \hat{z} - v_2 \hat{z} + v_3 \hat{y} - iv_3 \hat{x}) \end{aligned} \quad (3.111)$$

So, we have these three equations

$$\omega^2 - k_3^2 = v_0 k_3 - \omega v_3 \quad (3.112)$$

$$i(\omega^2 - k_3^2) = iv_0 k_3 - i\omega v_3 \quad (3.113)$$

$$v_1 \omega + iv_2 \omega = 0 \quad (3.114)$$

From Eq. 3.114 we obtain the constraint

$$v_1 = v_2 = 0 \quad (3.115)$$

So the remaining two yield

$$\omega^2 - k_3^2 = v_0 k_3 - \omega v_3 \quad (3.116)$$

The solution to this equation is

$$\omega_{\pm} = \frac{-v_3 \pm \sqrt{v_3^2 - 4(-k_3^2 + v_0 k_3)}}{2} \quad (3.117)$$

As a check, we put $v_0 = 0$ to compare with Eq. 3.71. Indeed, we see that this last equation, after renormalizing the quantities as in Eq. 3.72-3.73, yields

$$\tilde{\omega}_{\pm} = \frac{-1 \pm \sqrt{1 + 4\tilde{k}_3^2}}{2} \quad (3.118)$$

We can only accept the solution with the plus sign, because $\tilde{k}_3^2$ is always positive, so the minus sign would give rise to negative frequencies. If we take the square of the only physical solution, we obtain

$$\tilde{\omega}_+^2 = \frac{1 + \tilde{k}_3^2}{2} + \frac{\sqrt{1 + 4\tilde{k}_3^2}}{2} \quad (3.119)$$

which is exactly $\tilde{\omega}_+^2$ in Eq. 3.74, if we set $\cos\theta = 1$. This means that the vector $\vec{v}$ is parallel to the propagation as in this case. Therefore, we can obtain the group velocity by simply taking the derivative of Eq. 3.118 with the plus sign:

$$v_{g+} = \frac{2\tilde{k}_3}{\sqrt{1 + 4\tilde{k}_3^2}} \quad (3.120)$$

The group velocity behavior is shown in the Fig. 3.4

We can proceed likewise for the left-handed polarization. We obtain, for the three orthonormal directions

$$\omega^2 - k_3^2 = -v_0 k_3 + \omega v_3 \quad (3.121)$$

$$-i(\omega^2 - k_3^2) = iv_0 k_3 - i\omega v_3 \quad (3.122)$$

$$-v_1 \omega + iv_2 \omega = 0 \quad (3.123)$$

Once again we have the condition $v_1 = v_2 = 0$ and we obtain

$$\omega_{\pm} = \frac{v_3 \pm \sqrt{v_3^2 - 4(-k_3^2 + v_0 k_3)}}{2} \quad (3.124)$$

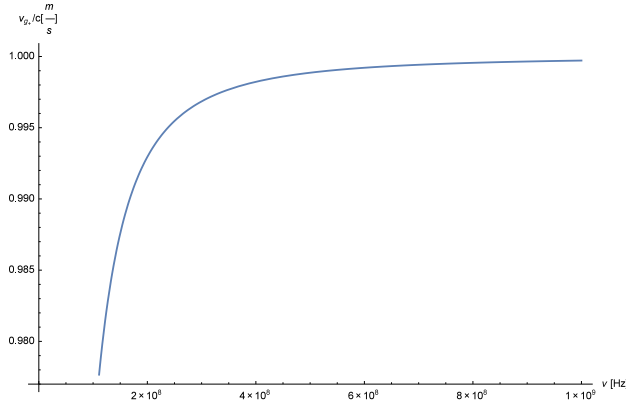


Figure 3.4: Group velocity for a wave circularly polarized and right-handed, plotted in function of the frequency in the range $1\text{ MHz} - 1\text{ GHz}$ that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10.

We can check once more that this is a square root of the more general solutions Eq. 3.78 once we have properly oriented our reference system and set $v_0 = 0$. Under such assumptions, we obtain the group velocity

$$v_{g-} = \frac{2\tilde{k}_3}{\sqrt{1 + 4\tilde{k}_3^2}} \quad (3.125)$$

However this formalism allows us to try and study also the case where $v_0 \neq 0$, which was not feasible with the other approach, because the solutions were too complicated to draw reasonable conclusions. To achieve this, we start with a wave which has a general polarization. Its Fourier transform reads

$$\vec{E} = E_0 (\alpha\hat{x} + \beta\hat{y} + \gamma\hat{z}) \quad (3.126)$$

Plugging this into Eq. 3.31 we get

$$\begin{aligned} -\frac{i}{\omega}k_3^2\alpha\hat{x} - \frac{i}{\omega}k_3^2\beta\hat{y} + \frac{v_0}{\omega}\alpha\hat{y} - \frac{v_0}{\omega}\beta\hat{x} - (\beta v_1\hat{z} - v_1\gamma\hat{y}) - (-v_2\alpha\hat{z} - v_2\gamma\hat{x}) + \\ - (v_3\alpha\hat{y} - v_3\beta\hat{x}) = -i\omega\alpha\hat{x} - i\omega\beta\hat{y} - i\omega\gamma\hat{z} \end{aligned} \quad (3.127)$$

So we have the system of algebraic equations

$$-\frac{i}{\omega}k_3^2\alpha - \frac{v_0}{\omega}\beta + v_2\gamma + v_3\beta = -i\omega\alpha \quad (3.128)$$

$$-\frac{i}{\omega}k_3^2\beta + \frac{v_0}{\omega}\alpha + v_1\gamma - v_3\alpha = -i\omega\beta \quad (3.129)$$

$$-\beta v_1 + v_2\alpha = -i\omega\gamma \quad (3.130)$$

In the case where $\beta = 0$

$$v_0 = v_1 = v_3 = 0 \quad (3.131)$$

$$\alpha \in \mathbb{R} \quad (3.132)$$

$$\gamma = i \frac{v_2}{\omega \alpha} \quad (3.133)$$

and, more importantly,

$$\omega^2 = k_3^2 + \frac{v_2^2}{\alpha^2} \quad (3.134)$$

We see that the DR resembles that of a massive photon, where the background tensor plays the role of the mass. We will see that this is a general feature of this approach in Sec. 3.4.

Of course, this is only a particular solution. However, now that we have caught a glimpse of what happens, we can look at the DR for $v_0 \neq 0$ and $\vec{k}$ along the z axis with new eyes. This relation reads

$$\omega^4 - (2k_3^2 + v_1^2 + v_2^2 + v_3^2) \omega^2 + 2v_0 v_3 k_3 \omega + k_3^4 + (v_1^2 + v_2^2 - v_0^2) k_3^2 = 0 \quad (3.135)$$

There are some interesting combination of the parameters to consider. The term that causes us some trouble is the linear one. Hence the parameters which we will inspect closely are v_0 and v_3 .

Firstly we take $v_0 \neq 0$ and $v_3 = 0$. In this case, we have

$$\begin{aligned} \omega^2 &= \frac{(2k_3^2 + |v|^2) \pm \sqrt{(2k_3^2 + |v|^2)^2 - 4(k_3^4 + |v|^2 k_3^2 - v_0^2 k_3^2)}}{2} \\ \Rightarrow \tilde{\omega}^2 &= \frac{(2\tilde{k}_3^2 + 1) \pm \sqrt{(2\tilde{k}_3^2 + 1)^2 - 4(\tilde{k}_3^4 + \tilde{k}_3^2 - \tilde{v}_0^2 \tilde{k}_3^2)}}{2} \\ \Rightarrow \tilde{\omega}^2 &= \frac{2\tilde{k}_3^2 + 1 \pm \sqrt{1 + 4\tilde{v}_0^2 \tilde{k}_3^2}}{2} \end{aligned} \quad (3.136)$$

where we have rescaled the quantities as usual. For the plus sign, we see that the quantity is always positive and therefore we can take the square root of this expression, derive and obtain the group velocity

$$v_{g+} = \frac{\sqrt{2}\tilde{k}_3 \left(1 + \frac{v_0}{\sqrt{1 + 4\tilde{v}_0^2 \tilde{k}_3^2}} \right)}{\sqrt{\frac{2\tilde{k}_3^2 + 1 + \sqrt{1 + 4\tilde{v}_0^2 \tilde{k}_3^2}}{2}}} \quad (3.137)$$

which can be plotted, obtaining the following behavior:

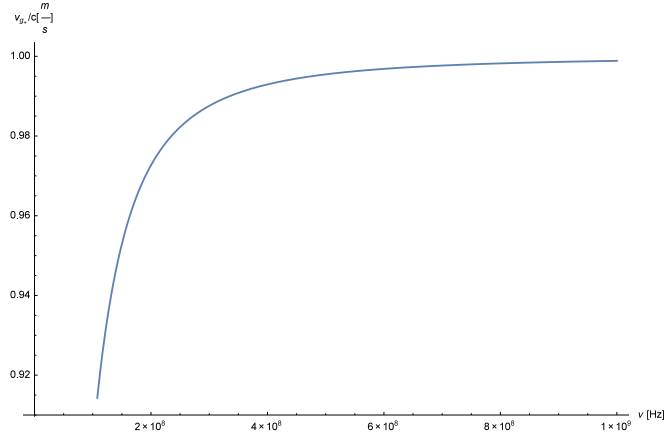


Figure 3.5: The group velocity corresponding to $\tilde{\omega}_+$ plotted as a function of the frequency in the range $1\text{ MHz} - 1\text{ GHz}$, that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10.

For the minus sign, instead, we must be careful with the existence conditions. Namely, we can see that, for a timelike vector,

$$v_0^2 > |v|^2 \Rightarrow \frac{v_0^2}{|v|^2} > 1 \quad (3.138)$$

Hence, the point where the numerator becomes vanishing is

$$\tilde{k}_3^2 = \tilde{v}_0^2 - 1 \quad (3.139)$$

This shows that there is the possibility of purely imaginary frequencies. On the other hand for spacelike or null-like vectors, this is not the case, because the condition can never be satisfied.

For the minus sign the group velocity is

$$v_{g-} = \frac{\sqrt{2}\tilde{k}_3 \left(1 - \frac{v_0}{\sqrt{1 + 4\tilde{v}_0^2 \tilde{k}_3^2}} \right)}{\sqrt{\frac{2\tilde{k}_3^2 + 1 + -\sqrt{1 + 4\tilde{v}_0^2 \tilde{k}_3^2}}{2}}} \quad (3.140)$$

If we take $v_0 = 0$ in these formulas, the solution reads

$$\omega^2 = \frac{2\tilde{k}_3^2 + |v|^2 \pm |v|^2}{2} \quad (3.141)$$

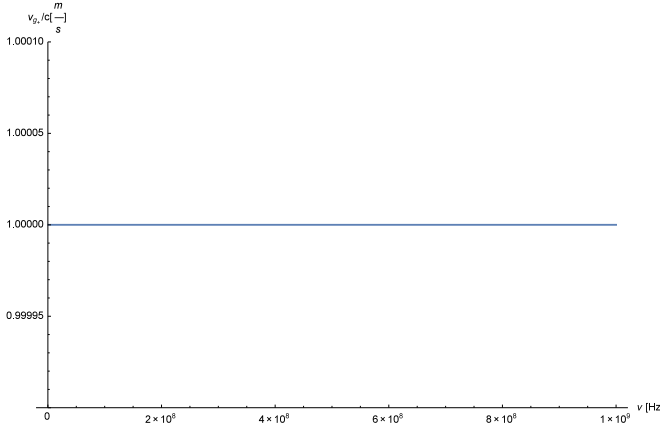


Figure 3.6: The group velocity corresponding to $\tilde{\omega}_-$ plotted as a function of the frequency in the range $1\text{ MHz} - 1\text{ GHz}$, that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10.

Thus, we either have no dispersion at all, or a massive photon behavior

$$\omega^2 = k_3^2 + |v|^2 \quad (3.142)$$

Again we remark that the background vector plays the role of the mass and we suggest the reader to head to Sec. 3.4 for a more detailed explanation of why this occurs.

Like we did in the previous Section, also here we work out the group velocity in term of ω . Using Eq. 3.151, we write

$$2\omega_{1,2}^2 = 2k_3^2 + |v|^2 \pm |v|^2 \sqrt{1 + 4 \frac{v_0^2}{|v|^4} k_3^2} \quad (3.143)$$

However, k_3 can be taken small if we are interested in the low frequency regime and v_μ is spacelike and can be assumed that $\frac{v_0^2}{|v|^4} \ll 1$. Thus,

$$2\omega_{1,2}^2 \sim 2k_3^2 + |v|^2 \pm |v|^2 \left(1 + 2 \frac{v_0^2}{|v|^4} k_3^2 \right) \quad (3.144)$$

So

$$\begin{aligned} \omega_{1,2} &= \left[k_3^2 + \frac{|v|^2}{2} \pm \left(\frac{|v|^2}{2} + \frac{v_0^2}{|v|^2} k_3^2 \right) \right]^{\frac{1}{2}} \\ &= \left[\left(1 \pm \frac{v_0^2}{|v|^2} \right) k_3^2 + \frac{|v|^2}{2} \pm \frac{|v|^2}{2} \right]^{\frac{1}{2}} \end{aligned} \quad (3.145)$$

Therefore we have two possibilities. Either

$$\omega_1 = \left(\beta_1^2 k_3^2 + |v|^2 \right)^{\frac{1}{2}} \quad \left(\beta_1 = 1 + \frac{v_0^2}{|v|^2} \right) \quad (3.146)$$

where we have a dispersive behavior with the parameter v acting, once more as the mass of the photon, or

$$\omega_2 = \beta_2 k_3 \quad \left(\beta_2 = 1 - \frac{v_0^2}{|v|^2} \right) \quad (3.147)$$

with a Maxwellian photon. This nature is even more evident if we set $v_0 = 0$ so that the parameters β reduce to the unit.

To obtain the group velocities we take the derivative of these ω 's. From Eq. 3.144 k_3 can be explicitly written as

$$k_3 = \frac{\omega}{\beta_1} \left(1 - \frac{|v|^2}{\omega^2} \right)^{\frac{1}{2}} \quad (3.148)$$

thus,

$$v_{g1} = \frac{d\omega_1}{dk_3} = \frac{\beta_1^2 k_3}{\omega} = \beta_1 \left(1 - \frac{|v|^2}{\omega^2} \right)^{\frac{1}{2}} \sim \beta_1 \left(1 - \frac{|v|^2}{2\omega^2} \right) \quad (3.149)$$

Obviously, the other solution yields

$$v_{g2} = 1 - \frac{v_0^2}{|v|^2} \quad (3.150)$$

We must underline that this result is consistent, if we set $v_0 = 0$ to Eq. 3.98, 3.99. The choice of $\theta = \pi/2$, in that context is equivalent as setting the propagation along the z axis. Here we can obtain the same solutions but multiplied by a factor in front, depending on the time component of the CFJ background vector. This coefficient is not, however, totally trivial and gives rise to some very interesting features. Eq. 3.149 is never superluminal because v_μ is spacelike.

However, since $\beta_1 = 1 + \frac{v_0^2}{|v|^2}$, there is the chance that Eq. 3.150 is. That

actually happens for $\sqrt{2}\omega > |v|^2 \left(1 + \frac{v_0^2}{|v|^2} \right)^{\frac{1}{2}}$. This is not surprising since we

have already shown that v_0 is associated to superluminal modes in literature. Only setting it to zero, we can get rid of these modes.

If we take $v_0 = 0$ and $v_3 \neq 0$, the situation is very similar. The DR reads, calling $v_1^2 + v_2^2 = |\alpha|^2$

$$\omega^4 - \left(2k_3^2 + |v|^2 \right) \omega^2 + k_3^4 + |\alpha|^2 k_3^2 = 0 \quad (3.151)$$

So

$$\begin{aligned}\omega_{\pm}^2 &= \frac{\left(2k_3^2 + |v|^2\right) \pm \sqrt{\left(2k_3^2 + |v|^2\right)^2 - 4\left(k_3^4 + |\alpha|^2 k_3^2\right)}}{2} \\ \Rightarrow \tilde{\omega}^2 &= \frac{2\tilde{k}_3^2 + 1 \pm \sqrt{1 + 4\tilde{v}_3^2 \tilde{k}_3^2}}{2}\end{aligned}\quad (3.152)$$

where the same considerations we made for the case $v_0 \neq 0$ and $v_3 = 0$ hold.

The case in which both parameters are different from zero is obviously the most complex. It is not trivial to solve the equation but we can obtain some information on the kind of solutions it yields. We assume that the vector v^μ is lightlike. In this case, if $k_3 = v_3$ we have that the term independent of ω in the DR is vanishing. Therefore one of the solutions is zero and we can reduce the order of the equation. We have then three zeros which we will call α , β , γ and given by the following system:

$$\alpha + \beta + \gamma = 0 \quad (3.153)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -2k_3^2 - |v|^2 \quad (3.154)$$

$$\alpha\beta\gamma = -2v_0v_3^2 \quad (3.155)$$

Furthermore, we assume that $\alpha = a \in \mathbb{R}$ and $\gamma = \beta^*$. Therefore

$$\beta = b + ic \Rightarrow \gamma = b - ic \quad (3.156)$$

So the system turns into

$$a + 2b = 0 \quad (3.157)$$

$$-4b + b^2 + c^2 = -2k_3^2 - |v|^2 \quad (3.158)$$

$$a(b^2 + c^2) = -2v_0v_3^2 \quad (3.159)$$

From the Eq 3.158 we get, assuming that $b = c$

$$b = \sqrt{2k_3^2 + |v|^2} \quad (3.160)$$

It goes without saying that this is a very specific case, with a lot of assumptions. However, the point that it shows is that, for suitable combination of the parameters, we can have complex frequencies.

3.3.2 Class 3

For the second class of DR, we have that

$$\omega^2 - (1 + \rho + \sigma)^2 k^2 = 0 \quad (3.161)$$

where

$$\rho = \frac{1}{2} \tilde{K}_\alpha^\alpha \quad (3.162)$$

$$\sigma^2 = \frac{1}{2} \tilde{K}_{\alpha\beta} \tilde{K}^{\alpha\beta} - \rho^2 \quad (3.163)$$

$$\tilde{K}^{\alpha\beta} = t^{\alpha\beta} t^{\mu\nu} \frac{p_\mu p_\nu}{|k|^2} \quad (3.164)$$

$t^{\mu\nu}$ being the SUSY background tensor. It is worthwhile recalling that for such a tensor, only the spatial components are present, because temporal ones are related to unphysical fields like tachions and ghosts. Therefore in Eq. 3.164 we can only take the ij components

$$\tilde{K}^{ij} = t^{ij} t^{mn} \frac{p_m p_n}{|k|^2} = t^{ij} t^{mn} \frac{k_m k_n}{|k|^2} \quad (3.165)$$

because $p^\mu = (\omega, \vec{k})$.

Moreover, the tensor t is always symmetric, hence we can always diagonalize it. Therefore, most simple case to consider is when the background tensor is a multiple of the identity. In this case, Eq. 3.165 becomes

$$\tilde{K}^{ij} = t^2 \delta^{ij} \delta^{mn} \frac{k_m k_n}{|k|^2} = t^2 \delta^{ij} \quad (3.166)$$

This means that both ρ and σ don't depend on $\vec{k}$ or ω . Thus the factor in front of k^2 in Eq. 3.161 carries no functional dependence. Therefore, we have a situation where the vacuum acts like a medium, whose refraction index is given by

$$n = (1 + \rho + \sigma)^{-1} \quad (3.167)$$

A more interesting case is the when we choose t^{ij} to be diagonal and traceless, but not a multiple of the identity. In such a case we have

$$t^{ij} = t_i \delta^{ij} \quad (3.168)$$

where, for once, we don't use Einstein's indexes summation rule. Eq. 3.165 can be rewritten as

$$\tilde{K}^{ij} = t_i \delta^{ij} \left(t_m \delta^{mn} \frac{k_m k_n}{|k|^2} \right) \quad (3.169)$$

Explicitly, the part within brackets is

$$\frac{1}{|k|^2} \begin{pmatrix} t_1 & 0 & 0 \\ 0 & t_2 & 0 \\ 0 & 0 & t_3 \end{pmatrix} \begin{pmatrix} p_1^2 & p_1 p_2 & p_1 p_3 \\ p_2 p_1 & p_2^2 & p_2 p_3 \\ p_3 p_1 & p_3 p_2 & p_3^2 \end{pmatrix} = \frac{t_1 k_1^2}{|k|^2} + \frac{t_2 k_2^2}{|k|^2} + \frac{t_3 k_3^2}{|k|^2} := P \quad (3.170)$$

Now, using Eq. 3.162, Eq. 3.163 can be transformed into

$$\sigma^2 = \frac{1}{2} \text{tr}(\tilde{K}^2) - \left(\frac{1}{2} \text{tr} \tilde{K} \right)^2 \quad (3.171)$$

Being $\tilde{K}$ traceless, the second term identically vanishes and we are left with the first one.

$$\text{tr}(\tilde{K}^2) = P^2 \text{tr}(t_{ij} t^{jk}) = P^2 (t_1^2 + t_2^2 + t_3^2) \quad (3.172)$$

So

$$\sigma = P \sqrt{t_1^2 + t_2^2 + t_3^2} := P F \quad (3.173)$$

and plugging this result in the DR, we get

$$\begin{aligned} & \omega^2 - [1 + F \cdot P]^2 |k|^2 \\ &= \omega^2 - F^2 \left[\frac{1}{F} + \left(\frac{t_1 k_1^2}{|k|^2} + \frac{t_2 k_2^2}{|k|^2} + \frac{t_3 k_3^2}{|k|^2} \right) \right]^2 |k|^2 \\ &= \omega^2 - \frac{F^2}{|k|^4} \left[\frac{|k|^2}{F} + (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right]^2 |k|^2 = 0 \\ &\Rightarrow \omega^2 = \frac{F^2}{|k|^2} \left[\frac{|k|^2}{F} + (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right] \end{aligned} \quad (3.174)$$

Finally,

$$\omega = |k| + \frac{F}{|k|} (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \quad (3.175)$$

We can now derive the group velocity for such a DR. Of course, the appearance of the momenta along the three spatial directions break the isotropy of the problem, hence we must derive such a group velocity for each direction. For the k_1 direction, the group velocity component reads

$$\begin{aligned} v_{g1} &= \frac{d\omega}{dk_1} = \frac{d\omega}{d|k|} \frac{d|k|}{dk_1} \\ &= \left[1 + \frac{d}{d|k|} \frac{F}{|k|} (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right] \cdot \frac{d\sqrt{k_1^2 + k_2^2 + k_3^2}}{dk_1} \\ &= \left[1 - \frac{F}{|k|^2} (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right] \frac{k_1}{|k|} \end{aligned} \quad (3.176)$$

We observe that, if we put $k_2 = k_3 = 0$, hence falling back into the case where t is proportional to the identity, we find again a linear behavior:

$$v_{g1} = 1 - F t_1 \quad (3.177)$$

The most general case is, of course, if t^{ij} is diagonal but not traceless. In that case, the term in brackets inside Eq. 3.171 does not vanish. But

$$\frac{1}{2} \text{tr} \tilde{K} = \frac{1}{2} P (t_1 + t_2 + t_3) \quad (3.178)$$

so Eq. 3.171 becomes

$$\sigma^2 = \frac{F^2 P^2}{2} - \frac{P^2}{4} (t_1 + t_2 + t_3)^2 \quad (3.179)$$

$$= \left[\frac{F^2}{2} - \frac{(t_1 + t_2 + t_3)}{4} \right] P^2 \quad (3.180)$$

The DR is

$$\omega = (1 + \rho + \sigma) |k| \quad (3.181)$$

which becomes, explicitly,

$$\begin{aligned} \omega &= \left\{ 1 + \frac{P}{2} (t_1 + t_2 + t_3) + \left[\frac{F^2}{2} - \frac{(t_1 + t_2 + t_3)}{4} \right]^{\frac{1}{2}} P \right\} |k| \\ &= \left\{ 1 + \left[\frac{t_1 + t_2 + t_3}{2} + \left(\frac{F^2}{2} - \frac{t_1 + t_2 + t_3}{4} \right)^{\frac{1}{2}} \right] P \right\} |k| \\ &:= (1 + C \cdot P) |k| \end{aligned} \quad (3.182)$$

In conclusion, then, we have the same behavior as for the case where t is traceless, just with a different constant in front of the isotropy breaking factor P . In this case, the group velocity along the first direction is

$$\begin{aligned} v_{g1} &= \frac{d \{ (1 + C \cdot P) |k| \}}{d |k|} \frac{d \sqrt{k_1^2 + k_2^2 + k_3^2}}{d k_1} \\ &= \left\{ 1 + \frac{d}{d |k|} \left[\frac{C}{|k|} (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right] \right\} \frac{k_1}{|k|} \\ &= \left[1 - \frac{C}{|k|^2} (t_1 k_1^2 + t_2 k_2^2 + t_3 k_3^2) \right] \frac{k_1}{|k|} \end{aligned} \quad (3.183)$$

There is a problem with the approach we have taken though. We don't usually measure one component of the group velocity. So, to obtain a result that we could, eventually, compare with data, we need to reconstruct the full velocity. Using spherical coordinates we have

$$k_1 = |k| \sin \theta \cos \varphi \quad (3.184)$$

$$k_2 = |k| \sin \theta \sin \varphi \quad (3.185)$$

$$k_3 = |k| \cos \theta \quad (3.186)$$

So

$$v_{g1} = [1 - C (t_1 \sin^2 \theta \cos^2 \varphi + t_2 \sin^2 \theta \sin^2 \varphi + t_3 \cos^2 \theta)] \sin \theta \cos \varphi := A \cdot \sin \theta \cos \varphi \quad (3.187)$$

Likewise

$$v_{g2} = A \cdot \sin \theta \sin \varphi \quad (3.188)$$

$$v_{g3} = A \cdot \cos \theta \quad (3.189)$$

Hence

$$v_g = [1 - C (t_1 \sin^2 \theta \cos^2 \varphi + t_2 \sin^2 \theta \sin^2 \varphi + t_3 \cos^2 \theta)] \quad (3.190)$$

We see therefore that we have a non-Maxwellian behavior, but no dispersion. We obtain a tiny correction to the usual light speed. The parameters t_i are suppressed by powers of the exponential of the Plank energy, so they are very small. The specific value depends on the estimates for such parameters.

3.3.3 Class 4

For Class 4 we have the following DR.

$$b\chi k^4 - (b\chi^{\alpha\beta} k_\alpha k_\beta + 1) k^2 + 3a\chi^{\alpha\beta} k_\alpha k_\beta = 0 \quad (3.191)$$

Similarly to what happened for Class 2, we have that the tensor $\chi_{\alpha\beta}$ is symmetric. So it is always possible to diagonalize it. Also, the temporal components can be set to zero because they are linked to unphysical particles. For these reasons

$$\chi = \chi_1 + \chi_2 + \chi_3 \quad (3.192)$$

$$\chi_{ij} = \chi_i \delta_{ij} \quad (3.193)$$

where, in this last expression, the i index is not summed. We have then that

$$\chi^{\alpha\beta} k_\alpha k_\beta = \chi_1 k_1^2 + \chi_2 k_2^2 + \chi_3 k_3^2 := D \quad (3.194)$$

We get

$$\begin{aligned} b\chi k^4 - (bD + 1) k^2 + 3aD &= 0 \\ \Rightarrow b\chi (\omega^2 - |k|^2)^2 - (bD + 1) (\omega^2 - |k|^2) + aD &= 0 \end{aligned} \quad (3.195)$$

Expanding for ω ,

$$\begin{aligned} b\chi (\omega^4 - 2\omega^2 |k|^2 + |k|^4) - (bD + 1) \omega^2 + (bD + 1) |k|^2 + 3aD &= 0 \\ b\chi \omega^4 - (bD + 1 + 2b\chi |k|^2) \omega^2 + [b\chi |k|^4 + (bD + 1) |k|^2 + 3aD] &= 0 \end{aligned} \quad (3.196)$$

Calling $\Omega = \omega^2$, we have the two solutions

$$\Omega_+ = \frac{(bD + 1 + 2b\chi |k|^2)}{2b\chi} \quad (3.197)$$

$$+ \frac{\sqrt{(-bD - 1 - 2b\chi |k|^2)^2 - 4b\chi [b\chi |k|^4 + (bD + 1) |k|^2 + 3aD]}}{2b\chi}$$

$$\Omega_- = \frac{(bD + 1 + 2|k|^2)}{2b\chi} \quad (3.198)$$

$$- \frac{\sqrt{(-bD - 1 - 2b\chi |k|^2)^2 - 4b\chi [b\chi |k|^4 + (bD + 1) |k|^2 + 3aD]}}{2b\chi}$$

So

$$\omega_+ = \sqrt{\frac{(bD + 1 + 2|k|^2)}{2b\chi}} + \sqrt{\frac{(-bD - 1 - 2b\chi|k|^2)^2 - 4b\chi[b\chi|k|^4 + (bD + 1)|k|^2 + 3aD]}{2b\chi}} \quad (3.199)$$

$$\omega_- = \sqrt{\frac{(bD + 1 + 2|k|^2)}{2b\chi}} - \sqrt{\frac{(-bD - 1 - 2b\chi|k|^2)^2 - 4b\chi[b\chi|k|^4 + (bD + 1)|k|^2 + 3aD]}{2b\chi}} \quad (3.200)$$

Once again, the parameter D breaks the isotropy. Hence, we must look for a group velocity for each direction. For instance, using Eq. 3.76

$$v_{g1} = \frac{1}{2\omega_{\pm}} \cdot \frac{d\Omega_{\pm}}{dk_1} = \frac{1}{2\omega_{\pm}} \cdot \frac{d\Omega_{\pm}}{d|k|} \cdot \frac{d|k|}{dk_1} = \frac{k_1}{2\omega_{\pm}|k|} \cdot \frac{d\Omega_{\pm}}{d|k|} \quad (3.201)$$

So we have the four group velocities

$$v_{g1} = \frac{k_1}{2\omega_{\pm}|k|} \cdot \frac{2|k| \cdot \left(1 \pm \frac{(bD+1+2|k|^2) \cdot (b\chi-1)}{\sqrt{(bD+1+2|k|^2)^2 - 4b\chi[|k|^4 + (bD+1)|k|^2 + 3aD]}}\right)}{b\chi} \quad (3.202)$$

This result, however, it must be interpreted order to order. Indeed, we must keep in mind that we have performed a perturbative expansion. The order zero in Eq. 3.195 is obviously the Maxwell relation between angular frequency and wave number, which yields $\omega^{(0)} = |k|$. The order one, in the i direction, reads instead

$$b\omega^4 - b(k_i^2 + 2|k|^2)\omega^2 + b|k|^4 + bk_i^2|k|^2 + 3ak_i^2 = 0 \quad (3.203)$$

This expression must be evaluated at $\omega^{(0)} = |k|$ so it simplifies to $3ak_i^2 = 0$. Exploiting this result for each direction we obtain

$$v_{gx} = \frac{k_1}{|k|} - a \frac{\chi_1 k_1^2 + \chi_2 k_2^2 + \chi_3 k_3^2}{k^3} \quad (3.204)$$

As we did for Class 2, we must reconstruct the full group velocity to obtain physically meaningful results. Following the steps we have already outlined, we obtain

$$v_g = 1 - a(\chi_1 \sin^2 \theta \cos^2 \varphi + \chi_2 \sin^2 \theta \sin^2 \varphi + \chi_3 \cos^2 \theta) \quad (3.205)$$

3.4 A de Broglie-Proca-like approach

We show in this Section that it is possible to derive a de Broglie-Proca term from the Carroll-Field-Jackiw Lagrangian. The purpose is to show that the interaction of the photon with the background gives rise to a mass for the photon. Of course, this mass will depend on the parameters of our model, more specifically the background vector v^μ . As we will show, this can be put in correlation with the results we obtained from the DR applied to polarized fields.

Now we manipulate the CFJ Lagrangian Eq. 3.19 trying to write it in terms of the potentials

$$L = \frac{1}{2} (E^2 - B^2) + v_0 \vec{A} \cdot \vec{B} - (\vec{v} \cdot \vec{B}) \phi - \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - \vec{v} \cdot (\vec{A} \times \vec{\nabla} \phi) \quad (3.206)$$

where we have used

$$\epsilon^{0ijk} v_0 A_i \partial_j A_k = \epsilon_{ijk} v_0 \vec{A} \cdot \vec{B} \quad (3.207)$$

$$\epsilon^{i0jk} v_i A_0 \partial_j A_k = -\phi \epsilon_{ijk} v_i \partial_j A_k \quad (3.208)$$

$$\epsilon^{ij0k} v_i A_j \partial_0 A_k = -\epsilon_{ijk} v_i A_j \partial_0 A_k \quad (3.209)$$

$$\epsilon^{ijk0} v_i A_j \partial_k A_0 = -\epsilon_{ijk} v_i A_j \partial_k \phi \quad (3.210)$$

We can go further and write

$$\begin{aligned} L = & \frac{1}{2} (\vec{\nabla} \phi + \dot{\vec{A}})^2 - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) \\ & - \phi \vec{v} \cdot (\vec{\nabla} \times \vec{A}) - \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - (\vec{v} \times \vec{A}) \cdot \vec{\nabla} \phi \end{aligned} \quad (3.211)$$

Furthermore

$$\begin{aligned} \phi \vec{v} \cdot (\vec{\nabla} \times \vec{A}) &= \phi v_i \epsilon_{ijk} \partial_j A_k \\ &= \partial_j (v_i \epsilon_{ijk} \phi A_k) - v_i \epsilon_{ijk} (\partial_j \phi) A_k \\ &= \vec{\nabla} \phi \cdot (\vec{v} \times \vec{A}) \end{aligned} \quad (3.212)$$

where we have eliminated the first term in the second line because it is a boundary term that vanishes upon integration.

So we finally get

$$L = \frac{1}{2} (\vec{\nabla} \phi + \dot{\vec{A}})^2 - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) - 2 \vec{\nabla} \phi \cdot (\vec{v} \times \vec{A}) - \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) \quad (3.213)$$

The important remark is that ϕ always appears through its gradient. This means that $\vec{\nabla} \phi$ is the true degree of freedom. We also see that there are no time derivatives of this field, hence it has no dynamics. In other words, it plays the role of an auxiliary field. That is why it can be eliminated from the Lagrangian.

We call

$$\vec{\nabla} \phi = \vec{S} \quad (3.214)$$

then we can rewrite the CFJ Lagrangian as

$$L = \frac{1}{2} \left(\vec{S} + \dot{\vec{A}} - 2\vec{v} \times \vec{A} \right)^2 - 2 \left(\vec{v} \times \vec{A} \right)^2 + 2\dot{\vec{A}} \cdot (\vec{v} \cdot \vec{A}) - \frac{1}{2} \left(\vec{\nabla} \times \vec{A} \right)^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) - \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) \quad (3.215)$$

Defining χ as

$$\chi = \vec{S} + \dot{\vec{A}} - 2\vec{v} \times \vec{A} \quad (3.216)$$

we get

$$L = \frac{1}{2} \chi^2 - 2 \left(\vec{v} \times \vec{A} \right)^2 + \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - \frac{1}{2} \left(\vec{\nabla} \times \vec{A} \right)^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) \quad (3.217)$$

Passing through the Euler-Lagrange equations, we obtain that

$$\chi = 0 \quad (3.218)$$

Therefore χ is eliminated and we are left with the CFJ Lagrangian

$$L = \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - 2 \left(\vec{v} \times \vec{A} \right)^2 - \frac{1}{2} \left(\vec{\nabla} \times \vec{A} \right)^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) \quad (3.219)$$

We notice now that in the second term the potential $\vec{A}$ does not appear with derivatives. We write

$$\begin{aligned} \left(\vec{v} \times \vec{A} \right)^2 &= \epsilon_{ijk} \epsilon_{imn} v_j A_k v_m A_n \\ &= (\delta_{jm} \delta_{kn} - \delta_{jn} \delta_{km}) v_j A_k v_m A_n \\ &= (|\vec{v}|^2 \delta_{kn} - v_k v_n) A_k A_n \end{aligned} \quad (3.220)$$

So

$$L = \vec{v} \cdot (\vec{A} \times \dot{\vec{A}}) - 2M_{kn}(\vec{v}) A_k A_n - \frac{1}{2} \left(\vec{\nabla} \times \vec{A} \right)^2 + v_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) \quad (3.221)$$

where

$$M_{kn}(\vec{v}) = |\vec{v}|^2 \delta_{kn} - v_k v_n \quad (3.222)$$

This is a symmetrical matrix. Every symmetrical matrix can be diagonalized:

$$\begin{aligned} -2M_{kn}(\vec{v}) A_k A_n &= -2A^T M A \\ &= -2A^T R^T R M R^T R A \end{aligned} \quad (3.223)$$

with $R \in SO(3)$ such that it diagonalize M and A^T being the transposed potential vector. We label

$$\tilde{M} = R M R^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (3.224)$$

We have that

$$\det \tilde{M} = 0 \Rightarrow m_1 = 0 \quad (3.225)$$

$$\text{Tr} \tilde{M} = m_1 + m_2 + m_3 \Rightarrow m_1 = m_2 = |\vec{v}|^2 \quad (3.226)$$

Therefore the term

$$\tilde{A}_i \tilde{M}_{ij} \tilde{A}_j = |\vec{v}|^2 \tilde{A}_2^2 + |\vec{v}|^2 \tilde{A}_3^2 \quad (3.227)$$

which is exactly a de Broglie-Proca term as we wanted. The role of the mass is played by the modulus of the vector $\vec{v}$ just as it happened in the case of the wave with propagation along k_3 and general polarization in the Section 3.3.

3.5 Conclusions

In this Chapter, we have approached the problem of non-Maxwellian photons from a more fundamental perspective, linking their existence to a breaking of the Lorentz symmetries. Despite massive photons having been proposed in several works, to the best of our knowledge in the available literature there is no comprehensive discussion concerning what generates the mass of the photon or an eventual non-Maxwellian behavior (see [2] for some ideas). It is our belief that answering this question is a crucial task that fundamental physics must charge itself of in order to truly understand the nature of the EM interaction carrier. The work in this Chapter has been driven by this purpose. Given the complexity of the subject, we have only scratched the surface of the problem but intend to carry on our research in future works.

The approach we have chosen is to investigate well established supersymmetric theories that go beyond the SM. It was already known [141, 143, 192, 195] that some of these theories yield dispersion relations, but a careful study of them had never been performed. What we did was to derive the DR for the cases where that had not been done yet and charge ourselves with the task of studying the consequences in some detail. We have tried to be as general as possible, but there are of course many theories and we do not have the pretense of having covered them all. Nevertheless, we have tried to cover both the CPT-even and CPT-odd sector. It is our belief that a theory that accurately represents nature should have both an odd and an even part. The interaction between the two sectors is not clear at all at the moment. In particular, one major question remains open. If we are in presence of a non-Maxwellian behavior for both sectors, how would the photon of the ensuing joint theory propagates? Would the two contribution be simply additive or would there be more complicated interactions? We do not know how to answer to this question, but it is indeed a very stimulating avenue of research for the future.

Our starting point are the Lagrangians for these models. Both in the CPT-even and CPT-odd case, we present two of them, one for the photon when the photino sector is left untouched and one when the effects of this supersymmetric partner are integrated out. We must keep in mind that we are working in a SUSY environment, in which for every boson there is a fermionic partner.

These partners have not been experimentally found yet. Thus, it is possible to integrate over them to obtain an effective theory where their role is taken into account but not explicitly visible.

For the CPT-even sector, we adopt the k_F -term model [143]. From the dispersion relation we see that the propagation of the photon is affected by the action of the background tensor related to the Lorentz violating term in the Lagrangian. The consequence is that the propagation is affected differently in any direction. We have proceeded step by step, trying to start from a very special choice for the tensor to the most general one. If the background tensor is proportional to the Kroeneker's delta, we have shown that the dispersion relation is that of a light ray propagating through a medium. In this case, it is the vacuum that has an effective refraction index due to the interaction of the photon with the background. This generates a modification of the speed of light, but not a functional dependance with the position or the time. We have then shown that we do obtain a direction dependent effect, for more complicated choices of the background tensor. However, the combined effect of these variations along each direction results in a modification of the overall speed of light. This is consistent with what happens in the most simple case. Further, the variation to the light velocity is proportional to a factor that goes like the background components squared. These components are extremely small because they represent the Lorentz violating corrections that we should see at our energy scale due to the LSV occurring near the Planck scale. It is known that Lorentz symmetry holds very well, if we don't investigate extremely high energy phenomena. Therefore we are obliged to consider these variation extremely small. Because of this, the correction to the light speed is extremely small, too. However, we do see that such variation depends on the propagation angle.

For the CPT-odd sector, we study the supersymmetrized Carroll-Field-Jackiw model [141,143]. For the CFJ model the LSV is given by a 4-vector and we have chosen to set the time component to zero to avoid the appearance of superluminal modes. The dispersion relation is of the fourth order. Solving the equation, we see that we have two physical solutions whose behavior is highly complex. That's why we preferred showing as a result not the dispersion of the speed of light, but the time delay of two photons emitted at different frequencies and coming to us from a distant source. We have proven that the time delay has a $\frac{1}{\omega^2}$ dependance, just as it happens in the de Broglie-Proca case. Unlike what happens for this theory, the mass is not a constant though. It depends on the Lorentz violating background vector and on the direction. In other words, it is an effective mass. Because of this, to obtain an estimate of this mass value we need to assess the order of magnitude of the background vector. A recent work [197] sets it to 10^{-19} eV. That results in a mass of the order of 10^{-55} kg, only an order of magnitude lower than the accepted Particle Data Group limit [47].

Having obtained two velocities for the CFJ-model, we had the idea to interpret them as speeds related to the polarization of the fields. For this reason, we have proceeded to a rather unusual analysis of the dispersion relation in terms

of the fields. We have chosen these latter in a well defined polarization, performed a Fourier transform and studied the resulting wave equation. For sake of completeness, we have kept our equations as general as possible. This means that we have considered also cases in which (unlike what we did for the time delay) the time component of the background vector is not zero. As a first step we have shown that the solutions obtained with this procedure are consistent with the ones obtained from the usual treatment of the DR. Secondly, we have analyzed different cases, drawing solutions even for a wave that is parametrically polarized. A study on these polarization parameters showed that it is possible to have regions where the speed of light is complex.

In all the CPT-odd cases, we have obtained a massive-like behavior. We tried to give an explanation to this by analyzing the Lagrangian more carefully. We have written it in a non explicitly covariant form, introducing the photon field components. The electric potential is not a dynamic variable and we can eliminate it from the Lagrangian. In this latter a term that has the classic structure of a de Broglie-Proca mass arises, only with the background playing the role of the mass. This is consistent with what we had previously seen in the peculiar cases we had studied and gives us a more fundamental reason of why the mass of the photon would be linked to the background vector.

Chapter 4

OBSERVATIONAL LIMITS ON THE PHOTON MASS

In Chapter 1, we have mentioned several experimental limits that have been set on the photon mass throughout the years. We have also pointed out that not all these measurements are equally reliable. Some, e. g. [45, 46, 49–51], rely heavily on the idealized models. In a 2010 review, the authors affirm [2] :

“Quoted photon mass limits have at times been overly optimistic in the strengths of their characterizations. This is perhaps due to the temptation to assert too strongly something that one ‘knows’ to be true.”

Recently, there have been attempts to put forward more sensible and solid values using ESA Cluster satellite data [48]. Indeed, the quest to find reliable means to measure the photon mass upper limit is ongoing. In the following, we propose a method based on the study of time delays in the so called Fast Radio Bursts (FRBs).

Résumé

Dans ce Chapitre, nous présentons des résultats concrets qui utilisent de véritables données de mesure. Malgré l’existence de nombreuses expériences pour fixer une limite supérieure à la masse du photon, nous trouvons la méthode présentée ici particulièrement intéressante puisqu’elle se base sur des mesures de très hautes précisions faites par des astronomes. Dans la seconde partie de ce Chapitre, nous avons porté notre attention sur la mission OLFAR et sa possible contribution scientifique.

Nos résultats se basent sur les données obtenues par FRB 150418, un Fast Radio Burst (FRB) pour lequel le décalage vers le rouge de la galaxie l’hébergeant a été déterminé et qui a la valeur de $z = 0.492 \pm 0.008$ [200]. Les FRBs sont des phénomènes astrophysiques sur lesquels nous connaissons peu de choses. Leurs noms proviennent des très courtes pulsations mesurées aux fréquences radio. Seulement quelques-unes ont été mesurées du fait que leur courte durée

les rend difficiles à résoudre en temps pour les instruments. Le premier FRB a été détecté en 2007 et seulement dix-sept d'entre eux ont été enregistrés [201]. Nous ignorons quelle est l'origine de ces FRB, mais nous avons pu constater que certains d'entre eux se répétaient ; par conséquent, il est raisonnable de supposer que les sources ne se détruisent pas durant le processus qui génère la pulsation. Il est intéressant de noter pour notre discussion que nous ignorons où se situent les FRB. C'est un problème courant en astronomie quand les objets sont trop loin ou pas assez stables pour utiliser la trigonométrie afin de trouver leur distance. Un paramètre qui traduit l'information concernant les distances est la Mesure de Dispersion (DM), qui n'est rien que l'intégrale sur la densité de l'électron du milieu au travers duquel il se propage, le long du parcours suivi par le photon depuis la source jusqu'à l'observateur

$$DM = \int_S^O n_e dl \quad (4.1)$$

où les index S et O font référence à la source et à l'observateur respectivement, n_e est la densité d'électron libre du milieu et dl est l'élément de ligne.

La DM représente le délai temporel avec lequel les photons à basses fréquences arrivent à nous. Si l'on prend seulement en compte la dispersion due au plasma d'électrons galactique, les valeurs mesurées sont inexplicables [202]. Par conséquent, l'excès de DM est normalement attribué soit aux effets proches de la source soit au Milieu Intergalactique (IGM). Dans notre travail, nous négligeons les effets proches de la source. Le centre de notre idée est qu'une partie de cet excès dans la dispersion peut être attribuée à la massiveté du photon. Comme mentionné précédemment, la difficulté de la méthode que nous allons présenter repose sur comment dissocier la contribution du photon massif de celle de l'IGM.

Pour FRB 150418, le retard pour deux photons émis avec des fréquences $\nu_1 = 1.2 \text{ GHz}$ et $\nu_2 = 1.5 \text{ GHz}$ respectivement est $\sim 0.8 \text{ s}$. Le retard est relié au DM selon la relation suivante

$$\Delta t_{12}^{DM} = \frac{e^2}{2\pi m_e} \int \frac{n_e}{\nu^2} dl = \frac{e^2}{2\pi m_e} \frac{1}{\nu^2} DM \quad (4.2)$$

$$= 415 \left(\frac{\nu}{1 \text{ GHz}} \right)^{-2} \frac{DM}{10^5 \text{ pc cm}^{-3}} \text{ s} \quad (4.3)$$

où e est la charge élémentaire, m_e la masse de l'électron, les fréquences ont été écrites en GHz et le DM a les unités de pc cm^{-3} . Dans le cas de FRB 150418, le DM correspondant au retard mesuré est [200]

$$DM_{FRB150418} = 776.2(5) \text{ cm}^{-3} \text{ pc} \quad (4.4)$$

La contribution due à la galaxie hôte est estimée approximativement à $\sim 37 \text{ cm}^{-3} \text{ pc}$; celle due à la Voie Lactée est $\sim 219 \text{ cm}^{-3} \text{ pc}$. En soustrayant ces composantes, il nous reste

$$DM_{IGM} \sim 520(138) \text{ cm}^{-3} \text{ pc} \quad (4.5)$$

Les sources principales d'incertitude pour cette valeur viennent de l'utilisation du modèle NE2001 pour la Voie Lactée [203] (dont les limitations sont montrées en [204]) et, principalement, des inhomogénéités de l'IGM.

En étudiant la propagation de la lumière dans un modèle Λ CDM, l'expression générale pour une IGM induite par DM est

$$DM_{IGM} = \int_0^z dz' \frac{n_e(z')}{(1+z') \sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} \quad (4.6)$$

$$= \frac{3H_0 \Omega_{IGM}}{8\pi m_p} H_e(z) \quad (4.7)$$

où z est le décalage vers le rouge cosmique, Ω_m est la fraction de densité de matière présente aujourd'hui, Ω_Λ est la fraction de densité de la constante cosmologique, Ω_{IGM} est la fraction ionisée de la densité des particules baryoniques dans l'IGM, m_p est la masse du proton, H_0 est la constante de Hubble au temps présent et

$$H_e(z) = \int_0^z \frac{(1+z') dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} \quad (4.8)$$

In [200], Eqs. 4.5 et 4.7 ont été utilisées pour obtenir une valeur Ω_{IGM} indépendamment du fond diffus cosmologique qui donne $\Omega_{IGM} = 0.041 \pm 0.002$ [205]. Les auteurs obtiennent

$$\Omega_{IGM} = 0.049 \pm 0.013 \quad (4.9)$$

Ce qui est cohérent, mais légèrement différent de la valeur donnée en [205], permettant d'héberger un photon massif.

En fait, pour un photon de de Broglie-Proca, le retard est

$$\Delta t_{m_\gamma} = \frac{m_\gamma^2}{2H_0} F(E_1, E_2) H_\gamma(z) \quad (4.10)$$

$$H_\gamma(z) = \int_0^z \frac{dz'}{(1+z')^2 \sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} \quad (4.11)$$

$$F(E_1, E_2) = \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) \quad (4.12)$$

Prenant aussi en compte le délai dû à la massiveté du photon, nous pouvons écrire

$$\Delta t_{12}^{FRB} = \frac{m_\gamma^2}{2H_0} F(E_1, E_2) H_\gamma(z) + \Delta t_{12}^{DM} \quad (4.13)$$

à partir de laquelle

$$m_\gamma = \left(1.05 \cdot 10^{-14} \text{ eV } s^{-\frac{1}{2}} \right) \sqrt{\frac{h_0}{F_{GHz} H_\gamma} (\Delta t_{12}^{FRB} - \Delta t_{12}^{DM})} \quad (4.14)$$

où h_0 est la constante de Hubble réduite et $F_{GHz} = F \left(\frac{E_1}{1GHz}, \frac{E_2}{1GHz} \right)$. Eq. 4.14 peut être utilisée pour fixer une limite expérimentale supérieure à la masse du photon. La limite la plus rigoureuse se produit lors de l'attribution de tout le retard à la massiveté du photon, c'est-à-dire en fixant $\Delta t_{12}^{DM} = 0$. Pour le FRB 150418 nous obtenons

$$m_\gamma < 4.6 \cdot 10^{-50} \text{ kg} \quad (4.15)$$

Cependant, il s'agit d'une estimation grossière. En effet, il est peu judicieux de négliger complètement la dispersion due à l'IGM. Une idée plus raisonnable est d'attribuer la différence dans la valeur estimée de Ω_{IGM} entre [200] et [205] à l'effet de la masse du photon, ce qui donne

$$m_\gamma < 3.2 \cdot 10^{-50} \text{ kg} \quad (4.16)$$

Ces deux résultats sont à 95% CL, ce qui signifie un intervalle de confiance de 2σ .

Nous sommes convaincus que ces limites peuvent être rendues encore plus robustes dans le futur. Nous pensons qu'il est difficile de faire mieux concernant la mesure d'un événement unique du fait que l'impossibilité d'estimer correctement les anisotropies de l'IGM nous empêche d'avoir une amélioration significative sur les barres d'erreur de DM_{IGM} . Au lieu de cela, nous montrons qu'une approche statistique nous permet d'augmenter notre précision. L'idée est de calculer un estimateur sur un échantillon de données fourni. Comme estimateur, nous avons choisi la racine de la moyenne au carré de la possible contribution de DM venant des effets à proximité de la source. Il s'agit d'un bon choix puisque l'origine de tous les FRB devrait être similaire donc il est logique de supposer que les conditions sont approximativement les mêmes autour d'eux. L'estimateur dépendra de façon paramétrique de la masse du photon et, par minimisation du χ^2 on calcule la valeur la plus appropriée pour m_γ . Avec cette technique, la limite supérieure est réduite d'un ordre de magnitude.

Nous aimerions attirer l'attention du lecteur sur le fait que, dans la théorie de de Broglie-Proca, la différence de vitesse entre deux photons avec une fréquence différente est inversement proportionnelle à ν^2 et directement proportionnelle à la masse m_γ . À cause de cette dépendance fonctionnelle, les signaux à basse fréquence, c'est-à-dire de radiation de faible énergie, sont particulièrement aptes à tester la massiveté du photon. Inversement, les données de haute énergie venant des Gamma Ray Bursts (GRBs) ou Active Galactic Nuclei (AGNs) ne sont pas compétitives pour contraindre m_γ lorsque comparées aux expériences menées, *e.g.*, sur le vent solaire. Dans la dernière partie de ce Chapitre, nous discutons la possibilité d'obtenir scientifiquement des tests significatifs de radiation à basse fréquence via des nano-satellites à bas prix construits avec des composants commerciaux. Tous les milieux ont une fréquence (appelée fréquence plasma) en-dessous laquelle un signal est réfléchi et, donc, ne peut pas se propager. Par conséquent, il est évident que la meilleure configuration pour étudier les basses fréquences est l'espace, où il n'y a aucune atmosphère

causant ce problème. L'IGM possède aussi une fréquence plasma, mais sa valeur est bien plus petite.

Jusqu'à maintenant, les satellites restent des instruments couteux avec une technologie conçue spécifiquement pour sa mission. Cependant, il peut exister un marché de niche pour de l'instrumentation à bas prix. Bien sûr, celui-ci ne peut pas être compétitif avec le premier mentionné. Néanmoins, la technologie commerciale permet d'envoyer dans l'espace un grand nombre d'appareils, qui, ensemble, ont autant des capacités qu'un satellite usuel. C'est la raison pour laquelle il y a une nécessité d'aller au-delà de la formation standard des satellites. Plus spécifiquement, nous discutons du projet OLFAR, dont les nano-satellites sont organisés en essaim.

Nous résumons brièvement les principales techniques qui gouvernent cet essaim. Plus particulièrement :

1. l'organisation de l'essaim : parmi les nano-satellites, il n'y a pas d'"unité mère" qui organise et contrôle l'acquisition de donnée ou le traitement ; de plus, les satellites sont tous capables d'effectuer les mêmes tâches (dans notre cas : détection et échantillonnage des signaux cosmiques dans la fenêtre de fréquence désignée, traitement de l'information via corrélation distribuée et envoi des données traitées vers la Terre) de sorte que la panne d'un ou de plusieurs appareils ne compromettrait pas le résultat final;
2. orbite et lancement : OLFAR se placera derrière la Lune pour éviter au mieux les interférences possibles et le lancement sera effectué en tirant parti d'autres appareils qui vont être envoyés sur l'orbite terrestre, après, la destination finale serait atteinte grâce à des systèmes de micro-propulsions;
3. puissance : les panneaux solaires ne peuvent pas être suffisamment gros afin de maintenir une activité permanente de l'instrumentation. Par conséquent, leur temps d'hibernation doit être planifié de façon à ce que les satellites puissent se recharger.

La contribution scientifique d'OLFAR peut être résumée comme suit. L'essaim couvre une large surface. Cela implique que la résolution spatiale est excellente puisque cette grande surface est couverte par un grand nombre de détecteurs. L'appareil d'enregistrement le plus simple est, bien sûr, une antenne, et par conséquent les fréquences radio sont les candidats naturels pour la première utilisation d'une telle technologie. OLFAR se concentrera sur la création d'une carte du ciel pour des fréquences en-dessous de 30 MHz , à savoir dans la fenêtre $0.1 - 10\text{ MHz}$. L'amplitude terrestre correspondante nécessaire serait de plus de 100 km , ce qui est irréalisable du fait des problèmes cités précédemment quant à l'opacité de l'atmosphère. Atteindre la Lune avec un essaim de nano-satellites remplirait ce manque que nous avons dans l'exploration du ciel. OLFAR est aussi supposé être utile pour la navigation. OLFAR éliminera complètement l'effet perturbant de l'atmosphère. Ainsi, des données affectées seulement par les deux problèmes restants seront fournies aux physiciens. La mesure de DM aux fréquences OLFAR aidera à améliorer la modélisation des formes des pulsations et mènera à un timing plus précis. Aussi, et c'est la principale raison

pour laquelle nous nous intéressons aux perspectives offertes par OLFAR, nous pouvons tester l'ISM et, éventuellement, dissocier la dispersion due au plasma de celle due à un photon massif

4.1 Fast Radio Bursts

FRBs are transient radio pulses that last only a few milliseconds (hence the name) generated by astrophysical or cosmological phenomena at high energy. The first FRB was discovered in 2007 by Lorimer’s team with the Parkes radio dish in Australia [206]. This first measurement was welcomed by the wariness of the scientific community, who attributed this reading to some kind of interference. However, after the discovery of other similar phenomena [207], what is now known as the “Lorimer’s burst” has become the first FRB of which we have knowledge. At present, we are aware of only seventeen such bursts [201].

We know very little about the FRB nature, as it is explained in a very recent review [208]. Many of such phenomena are so short that receivers are not quick enough to resolve them in time. This means in some cases, that the duration we attribute to FRBs is more an upper limit of their real length. We do know, however, that some of them have repeated themselves [201, 209]. This suggests that, whatever phenomenon causes them, it cannot destroy the source. So stellar collapses, merging of binaries and catastrophic collisions are excluded. The most plausible explanation is that of the young remnants of stellar collapses, like neutron stars or black holes, are - at least temporarily - endowed with a gravitational potential such that allows a sudden and short emission of energy. In this class of phenomena there are, e.g., Soft Gamma Repeaters (SGRs) [210–217] or giant pulses from young pulsars [217–221]. Among exotic ideas that have been proposed are interactions between pulsars and planets [222] or asteroids and comets [223, 224]. All these explanation have in common the feature of possibly taking place at cosmological distances. Alternatively, it has been proposed that stellar flares might give a coherent explanation of these phenomena [225].

Given the short duration of the FRBs, whatever the origin, we can be sure that the emitting region has to be small. If the emitting region were larger, it would entail a longer pulse. Since some FRBs have even a pulse duration $\lesssim 1$ ms, this implies not only a small emitting region but also a great intensity at the source. In the absence of relativistic bulk motion, the observed duration Δt is larger than $\Delta r/c$, Δr being the dimension of the emitting region along the direction of the radiation. Because of the aforementioned durations under the millisecond ($c\Delta t < 3 \cdot 10^7$ cm), regions of emission bigger than 10^{5-6} cm are excluded. This means that such regions can’t be larger than the radius of a neutron star or of a stellar mass black hole.

The distance of FRBs is debated. This is hardly uncommon in astronomy. Most objects we observe in the sky are so far away from us that it is impossible to measure their distance by trigonometric parallax. This is due to the imprecise angular resolution of our instruments but mainly to the short duration of the pulse. Using the parallax would require the object to be visible for months, so that Earth has the time to move along its orbit. Nevertheless, we have the means to estimate their position. The pulse energy doesn’t arrive all at once. It has been seen that light at smaller frequencies arrives later than that at higher frequencies. From the observations the dependance of the time delay upon the frequency follows a power law ν^α with α being fit by the value -2 to very high

accuracy. Generally, this time delay is attributed to the light traveling through a diluted plasma. Therefore, the intensity of this delay must be proportional to the so called Dispersion Measure (DM) which is nothing but the integral along the path followed by the photon from the source to the observer over the electron density of the medium through which it propagates

$$DM = \int_S^O n_e dl \quad (4.17)$$

where the indexes S and O refer to the source and observer respectively, n_e is the free electron density of the medium and dl is the line element.

Because of its nature, the DM conveys a lot of information about the location of the FRB. However, the interpretation is not straightforward. If we only take into consideration the interstellar plasma in our and, eventually, in the host galaxy where the FRB can take place, the DM would simply estimate the length the light has traveled. (Un)fortunately, observed DMs are typically at least an order of magnitude too large to be straightforwardly explained [202].

So, what could account for the excess of the DM? Among the possible causes, certainly there are

- near source plasma: the specific characteristics of the medium in the proximity of the source are such that they heavily affect the propagation, and thereby increase the DM
- intergalactic medium (IGM): to the very diluted plasma present among galaxies would correspond a cumulative effect for the DM along cosmological distances.

Other alternatives are less convincing. For instance, the DM could not be attributed solely to the host galaxy, unless the FRB is extremely near its center where dense plasma clouds can be found. Also intergalactic clouds of plasma or the gas present amongst a cluster of galaxies would not affect the DM significantly because they would not increase the mean value of the IGM density ($1.6 \cdot 10^{-7} (1+z)^3 \text{ cm}^{-3}$, z being the cosmic redshift).

In our work, we considered the excessive DM to be due to either the IGM or, possibly, from a massive photon. However, for completeness, we will say something on the near source plasma.

4.1.1 Near source dispersion

If we consider that the contribution of the environment next to the source is very relevant, we must assume the FRBs to be close to us on a cosmic scale, possibly even in our own galaxy. Otherwise, we would have an additional IGM contribution yielding an even larger DM.

Suppose that we have a plasma cloud responsible for the dispersion next to the source. What can we infer about it from the data we have at our disposal? First of all, the electron density can't exceed the following value [226, 227]:

$$n_e < \frac{2}{3} |-\alpha - 2| \frac{m_e \omega}{4\pi e^2} \sim 5 \cdot 10^7 \text{ cm}^{-3} \quad (4.18)$$

where m_e is the electron mass, e the elementary charge and α the aforementioned exponent according to which the time delay depends on the angular frequency of the radiation ω . We have already pointed out that measures concerning this number are pretty accurate. Specifically,

$$|-\alpha - 2| < 0.003 \quad (4.19)$$

and no inconsistency with $\alpha = 2$ has been ascertained so far. Nevertheless, a different frequency dependance has been discussed recently.

Knowing the value of the electron density, we can estimate the required vastness of the plasma cloud yielding the DM we have measured. The latter ranges from 350 to 1600 $pc\ cm^{-3}$ (where pc stands for parsec, $\sim 3,26$ light years). For taking into account the DMs, the clouds should be bigger than $3 \cdot 10^{13}\ cm$, *i.e.* hundreds of times a solar radius. Such values are mostly reasonable if we consider paths passing next to the galactic nucleus, where such clouds can form. Actually, they have been observed around the nucleus of our own galaxy [210]. This would leave an open question though: why would all known FRBs have been generated next to galactic nuclei?

Another plausible explanation would hint to a cloud generated by a supernova, more specifically by a young supernova remnants (SNR). The cloud of plasma generated by the supernova would be fast expanding, and would account for the dimensional problem. Furthermore, it would link the FRB to a well identified object.

4.2 Time delay for FRB 150418

From now on, we neglect near source dispersion. A massive photon would affect the DM measurement wherever the FRB is located. For the time being, though, we neglect such possible effect. Then, the alternative cause of a large DM is an extra-galactic location of the FRB. For the same reason it must be luminous and bright. Recently, FRB 150418 has been identified and it appears to have these qualities. It should be located in a galaxy at a redshift $z = 0.492 \pm 0.008$ [200]. But, in [228, 229] the authors raise serious questions on the trustworthiness of the attribution of FRB 150418, claiming that it might be a mere coincidence with an Active Galactic Nucleus (AGN) flare. However, the discussion is still open [230–232]. In the following, we assume the correctness of the extra-galactic location given in [200].

For FRB 150418, the time delay between the light at $\nu_1 = 1.2\ GHz$ and $\nu_2 = 1.5\ GHz$ is $\Delta t_{12}^{FRB} \sim 0.8\ s$. From this value, we have derived a photon mass upper limit [196].

Light traveling through a ionized medium has the velocity

$$v = \left(1 - \frac{\nu_p^2}{\nu^2}\right)^{\frac{1}{2}} \sim 1 - \frac{\nu_p^2}{2\nu^2} \quad (4.20)$$

where, as usual, we have set $c = 1$ and

$$\nu_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} \sim 8,98 \cdot 10^3 \cdot \sqrt{n_e} \text{ Hz} \quad (4.21)$$

in cgs units. The flying time delay is the integral of the speed along the trajectory. So

$$\Delta t_{12}^{DM} = \frac{e^2}{2\pi m_e} \int \frac{n_e}{\nu^2} dl = \frac{e^2}{2\pi m_e} \frac{1}{\nu^2} DM \quad (4.22)$$

$$= 415 \left(\frac{\nu}{1 \text{ GHz}} \right)^{-2} \frac{DM}{10^5 \text{ pc cm}^{-3} \text{ s}} \quad (4.23)$$

having renormalized the units in the last step.

The DM contains contributions from the interstellar plasma in the host galaxy and in our galaxy on top of the IGM effect. As discussed in [200], the DM for FRB 150418 is $776.2(5) \text{ cm}^{-3} \text{ pc}$. The contributions due to the host galaxy is estimated approximately $\sim 37 \text{ cm}^{-3} \text{ pc}$; the other due to the Milky way $\sim 219 \text{ cm}^{-3} \text{ pc}$. Subtracting these components we are left with

$$DM_{IGM} \sim 520(138) \text{ cm}^{-3} \text{ pc} \quad (4.24)$$

The main sources of uncertainty of this value originate from applying the NE2001 model to the Milky Way [203] (limitations are shown in [204]) and mainly from inhomogeneities of the IGM.

For cosmological distances, the comoving line element can be expressed as

$$dl = \left| \frac{dt}{dz} \right| dz \quad (4.25)$$

where

$$\left| \frac{dt}{dz} \right| = (1+z)^{-1} H(z)^{-1} \quad (4.26)$$

$$H(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \quad (4.27)$$

In this expression, Ω_m is the present day matter density fraction and Ω_Λ is the cosmological constant density fraction. We are working in a Λ CDM cosmology¹, so the expansion of the universe is mostly attributed (if the photon were massive the results should be reconsidered) to the cosmological constant.

Taking into account also the redshift of the frequency and the time dilation over the flying path, we get

$$\Delta t_{12}^{DM} = \frac{e^2}{2\pi m_e} \frac{1}{\nu^2} \int_0^z dz' \left| \frac{dt}{dz'} \right| \frac{n_e(z')}{1+z'} \quad (4.28)$$

¹See appendix for an introduction on the Λ CDM model and the derivation of Eqs. 4.26 and 4.27.

4.2.1 DM for the IGM

Inserting Eqs. 4.26 and 4.27 in Eq. 4.28, the delay becomes

$$\Delta t_{12}^{DM} = \frac{e^2}{2\pi m_e} \frac{1}{\nu^2} \int_0^z dz' \frac{n_e(z')}{(1+z') \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \quad (4.29)$$

Thus, the DM associated to the intergalactic dispersion is

$$DM_{IGM} = \int_0^z dz' \frac{n_e(z')}{(1+z') \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \quad (4.30)$$

For a more explicit formula, we address the free electron density of the medium. The mean density can be written as

$$n_e(z) = \Omega_{IGM}(z) n_{e0} (1+z)^3 \quad (4.31)$$

where the last factor accounts for the expansion of the universe over the flying time. Ω_{IGM} is the ionized fraction density of baryon particles in the IGM and n_{e0} is free electronic density in the baryon sector at present time. The expression for n_{e0} reads [233]

$$n_{e0} = \frac{3H_0}{8\pi m_p} \quad (4.32)$$

where H_0 is the Hubble constant at present time and m_p the proton mass.

In our work, we have adopted $\Omega_\Lambda = 0.714$, $\Omega_m = 0.286$ and the reduced Hubble constant $h_0 = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} = 0.69$ [205]. Thus, we obtain

$$DM_{IGM} = \frac{3H_0 \Omega_{IGM}}{8\pi m_p} H_e(z) \quad (4.33)$$

where

$$H_e(z) = \int_0^z \frac{(1+z') dz'}{\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \quad (4.34)$$

Incidentally, this relation has been used in [200] to infer a value for Ω_{IGM} independent from the one given by considerations on the Λ CDM model. We assume that the 24% of the IGM mass is accounted for by helium, that the remaining percentage is due to hydrogen and that the ionized fraction of these quantities is fully and singly ionized. Then we obtain

$$\Omega_{IGM} = 0.049 \pm 0.013 \quad (4.35)$$

in reasonable agreement with the result in [205] which yields $\Omega_{IGM} = 0.041 \pm 0.002$.

4.2.2 Time lag for a massive photon

Our idea is to attribute all or part of Δt_{12}^{FRB} to a massive photon *a la dBP*. FRBs are the perfect tools to test a massive theory. From Eq. 1.96 the difference in velocity between two photons of different frequency is inversely proportional to the frequency squared and directly proportional to the mass m_γ . Because of this functional dependence, low frequency signals - i.e. low energy radiation - are particularly apt to test the massiveness of the photon. Conversely, high-energy data from Gamma Ray Bursts (GRBs) or AGN's have not proven to be competitive in constraining m_γ if compared to experiments performed, *e.g.* on the solar wind.

To quantify the influence of m_γ on a FRB we start by computing the difference in length ΔL traveled by two photons at different frequency, if they are both emitted by a source at cosmological distance z . Calling their difference in velocity Δu , we get

$$\Delta L = H_0^{-1} \int_0^z \frac{\Delta u dz'}{\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \quad (4.36)$$

due to the integration of the velocity over time and the exploitation of Eqs. 4.26 and 4.27, taking into account also the cosmological time dilation. Furthermore, from Eq. 1.96, for two massive photons of energies $E_1 = \nu_1$ and $E_2 = \nu_2$ coming from cosmological distances

$$\Delta u_{m_\gamma} = \frac{m_\gamma^2}{2(1+z)^2} \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) \quad (4.37)$$

and thus

$$\Delta t_{m_\gamma} = \frac{m_\gamma^2}{2H_0} F(E_1, E_2) H_\gamma(z) \quad (4.38)$$

$$H_\gamma(z) = \int_0^z \frac{dz'}{(1+z')^2 \sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} \quad (4.39)$$

$$F(E_1, E_2) = \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) \quad (4.40)$$

4.3 Results

Summing up all the elements we have presented so far, we conclude that the time lag for a FRB at cosmological distance, under the massive photon assumption, is given by the following components

$$\Delta t_{12}^{FRB} = \frac{m_\gamma^2}{2H_0} F(E_1, E_2) H_\gamma(z) + \Delta t_{12}^{DM} + b_{sf} (1+z) \quad (4.41)$$

The function b_{sf} refers to the aforementioned near source dispersion, even if its relevance is marginal for FRBs at cosmological distances. However small, we cannot exclude *a priori* its contribution.

It is straightforward to invert Eq. 4.41 and isolate the photon mass

$$m_\gamma = \left(1.05 \cdot 10^{-14} \text{eV s}^{-\frac{1}{2}}\right) \sqrt{\frac{h_0}{F_{GHz} H_\gamma} (\Delta t_{12}^{FRB} - \Delta t_{12}^{DM} - b_{sf} (1+z))} \quad (4.42)$$

where we have defined for simplicity the function

$$F_{GHz} = F\left(\frac{E_1}{1GHz}, \frac{E_2}{1GHz}\right) \quad (4.43)$$

The most conservative limit occurs when attributing all the time lag to the massiveness of the photon. That is, setting $\Delta t_{12}^{DM} = b_{sf} = 0$. For the FRB 150418 we obtain

$$m_\gamma < 4.6 \cdot 10^{-50} \text{ kg} \quad (4.44)$$

However, this is a coarse estimate. Indeed, it is unwise to completely neglect the dispersion from the IGM. We have already mentioned that in [200] the authors have found $\Omega_{IGM} = 0.049 \pm 0.013$ attributing the entire dispersion that is not of galactic origin to the IGM. In [205], the value for Ω_{IGM} is 0.041 ± 0.002 . If we plug his value into Eq. 4.33, we obtain a DM_{IGM} smaller than the one measured for FRB 150418. Thus, a better try to put an upper limit to the photon mass is to would be to attribute this difference in the DM values to m_γ . In other words, the value of Ω_{IGM} given in [205] is used to compute Δt_{12}^{DM} , which is exclusively attributed to the IGM. Δt_{12}^{DM} is then subtracted to the observed time lag and the difference is attributed to massive effects.

We have decided to work at a 95% CL, which corresponds to 2σ variance. Therefore, in accordance with the values mentioned above, $DM_{IGM} \sim 520(2 \cdot 138) \text{ cm}^{-3} pc$, which corresponds to

$$\Delta t_{12}^{DM} \sim 0.45 \text{ s} \quad (4.45)$$

always with a 95% CL. With the same accuracy

$$\Delta t_{12}^{FRB} \lesssim 0.82 \text{ s} \quad (4.46)$$

In turn, this yields

$$m_\gamma < 3.2 \cdot 10^{-50} \text{ kg} \quad (4.47)$$

at 95% CL.

As we have pointed out, thanks to the low energy of the FRB radiation, these limits are more competitive with respect to the ones yielded by GRBs and AGNs [234]. The results are much in line with those obtained in [48].

4.4 Future improvements

There are large uncertainties on the DM associated to the IGM. Is there a chance for improvement? The redshift for the FRB 150418 has been measured with a

reasonable accuracy, 0.1%. It is unlikely to do much better. For the other FRBs it is very doubtful to obtain z with a similar accuracy. Therefore, the chances to use other FRBs to create a statistic is not plausible at the moment.

Unfortunately, we still have to face a 20% and more uncertainty on the amount of the DM we attribute to the IGM or the galaxies (host and Milky Way). For the singular FRB, is difficult to do better because we have few means to investigate possible inhomogeneities in the IGM, which are the main source of error. However, we are convinced that it would be possible to put a constraint on the combined effect of DM_{IGM} and photon mass whose errors are limited to a mere 10%. To distinguish between the IGM and photon mass contribution, the different functional dependence of the integrals Eqs. 4.27 and 4.39 might be exploited.

We assume that the aforementioned 10% accuracy on the DM measurement is attainable and plot on a (m_γ, Ω_{IGM}) plane the predicted results. We parametrize the curves as follows. For the photon mass, we write Eq. 4.42 as

$$m_\gamma = A\sqrt{B - C} \quad (4.48)$$

where A is a numerical factor that depends on the integral $H_\gamma(z)$; B instead will be written as

$$B = (103.1 \text{ s}) \frac{DM_{IGM}^{obs}}{10^5 pc \text{ cm}^{-3}} \quad (4.49)$$

where Eq. 4.23 has been used. This means that B represents the observed time lag written in terms of the DM , as if it were entirely due to the IGM. Lastly, the factor C is given by

$$C = \Delta t_{IGM} \cdot \frac{\Omega_{IGM}}{\Omega_{IGM}^{\Lambda CDM}} \quad (4.50)$$

and is the fraction of an actual contribution of the ionized plasma effect to the observed time delay relative to the prediction of the standard ΛCDM model for a given FRB.

The results can be seen in Fig. 4.1, where we have furthermore set the ionization fraction to 0.9 but left the parameter Ω_{IGM} free. Hypothetical measurements at $z = 0.1$ are shown in green and mauve, whereas the blue band represents a possible $z = 1.0$ set of data. We don't consider higher redshifts because the FRBs luminosities aren't particularly high. Thus, the measure of higher shifts appears rather implausible. The green band has been determined by taking

$$A = 1.97 \cdot 10^{-14} \text{ eV } c^{-\frac{1}{2}} \quad (4.51)$$

$$DM_{IGM}^{obs} = 83(8.3) \text{ pc cm}^{-3} \quad (4.52)$$

$$\Delta t_{IGM} = 0.086 \text{ s} \quad (4.53)$$

These values have been chosen so that the central value of the green band is the same present in a model where the photon is massless and all the time lag attributed to $\Omega_{IGM}^{\Lambda CDM}$.

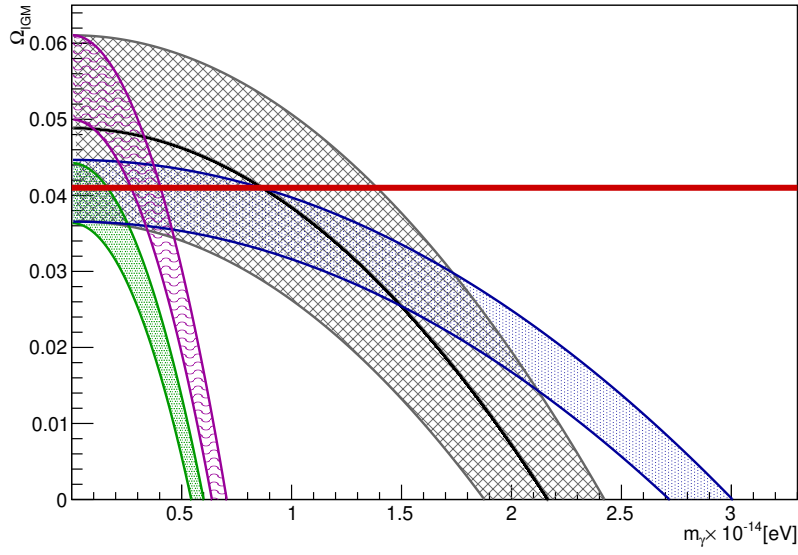


Figure 4.1: In this plot we show on the (m_γ, Ω_{IGM}) plane some bands of hypothetical measures and also the one measured for the FRB 150418. The red band is the Λ CDM expectation value for $\Omega_{IGM} = 0.041 \pm 0.002$. The shaded gray band is the FRB 150418 measure and uncertainties discussed in [200]. The remaining bands are hypothetical 10% accuracy bands that we reckon to be attainable in the near future. The green and mauve bands are for FRBs at $z = 0.1$, the blue one at $z = 1.0$.

Instead, the mauve band has been obtained keeping the upper limit on the Ω_{IGM} set thanks to the FRB 150418 but taking

$$DM_{IGM}^{obs} = 103(10.3) \text{ pc cm}^{-3} \quad (4.54)$$

We can see that working at lower redshifts helps imposing more restrictive limits on the photon mass. If we consider the central value of the green band and impose, very conservatively, that any $\Omega_{IGM} \geq 0$ is acceptable,

$$m_\gamma < 1.1 \cdot 10^{-50} \text{ kg} \quad (4.55)$$

If, instead, we strengthen the requirement to $\Omega_{IGM} = \Omega_{IGM}^{\Lambda CDM}$

$$m_\gamma < 5.3 \cdot 10^{-51} \text{ kg} \quad (4.56)$$

An alternative way to set a limit on the photon mass is given by forgetting any assumption on the Ω_{IGM} and asking, instead, that the 10% accuracy $z = 0.1$ band is consistent with the FRB 150418 constraints. In this case we obtain

$$m_\gamma < 4.5 \cdot 10^{-51} \text{ kg} \quad (4.57)$$

The $z = 1.0$ band is instead obtained by setting

$$A = 4.60 \cdot 10^{-14} \text{ eV c}^{-\frac{1}{2}} \quad (4.58)$$

$$DM_{IGM}^{obs} = 980(98) \text{ pc cm}^{-3} \quad (4.59)$$

$$\Delta t_{IGM} = 0.94 \text{ s} \quad (4.60)$$

We might underline two interesting features here. First of all, we see that such a band is consistent with the mauve one and the expected value for $\Omega_{IGM}^{\Lambda CDM}$, only if the photon mass is in the following range:

$$4.5 \cdot 10^{-51} \text{ kg} < m_\gamma < 7.1 \cdot 10^{-51} \text{ kg} \quad (4.61)$$

This is another way through which we can constrain the value of the light carrier mass. Also, the comparison between the blue band and the one at a lower redshift shows very well how the redshift affects the proportion in which the IGM and the photon mass are relevant. In particular, we see here in practice the aforementioned chance to distinguish between the two effects. Of course, to drastically diminish the uncertainty on the measures, we should lower the ones on DM_{IGM} , maybe exploiting a statistical analysis on a large number of FRBs.

Obviously, we could set a more robust limit if we had more and more certain measurements of the redshift for FRBs. However, an interesting idea is that the process can be inversed. If we accept the value for Ω_{IGM} given by the ΛCDM cosmological model and we rely on the limit on m_γ found through the study of FRB 150418, then it is possible to use the DMs to estimate the redshift of the sources giving rise to the FRB. This would, in turn, yield a distribution which, with some luck, would shed some light on the mysterious FRB origins.

A different way to improve the result is to adopt a statistical approach. From Eqs. 4.27 and 4.39 we clearly see the photon mass contribution becomes more relevant for smaller redshifts because of the power of the factor $(1+z)$ inside the two integrals. Therefore, if given a statistically significant sample of FRBs at different z , for the i -th component of the sample Eq. 4.41 is used, considering the photon mass and the time delay due to the IGM as parameters. In other words, the source frame contribution can be written as

$$b_{sf}^i = \frac{1}{1+z^i} (a_\gamma^i F(E_1, E_2) H_\gamma(z^i) + \Delta t_{12}^{DM\ i} - \Delta t_{12}^{FRB\ i}) \quad (4.62)$$

where

$$a_\gamma = \frac{m_\gamma^2}{2H_0} \quad (4.63)$$

for each FRB in the ensemble. In Eq. 4.62 we must consider both a_γ^i and $\Delta t_{12}^{DM\ i}$ as parameters that will be determined through minimizing the χ^2 of some estimator. In our case, a reasonable estimator is the Root Mean Square (RMS) of the source frame contribution. This is a valid quantity if we can safely assume that the FRBs do have the same origin and, henceforth, the value of the DM due to the plasma surrounding the source isn't too statistically different from the mean of these values over the sample we are considering.

4.5 Other ways to set a photon mass limits

As mentioned before, FRBs are not the only phenomena employable to infer something on the photon mass. We have already pointed out that the advantage of FRBs is to work at low frequencies, where the effect of a massive photon is more visible. However, FRBs are not very numerous. GRBs are a more renown alternative.

4.5.1 Gamma Ray Bursts

Before getting into details about how to extract a photon mass estimation from GRBs data, it is maybe useful to start by recalling a couple interesting facts about them.

As the name suggests, GRBs are intense pulses of γ -rays coming from the cosmos. They were first registered in the 60's by the American Vela satellite. At first, the discovery was classified because the USA thought the Soviet Union was conducting secret nuclear tests. Later, it was shown that a terrestrial or even solar system origin for those bursts was impossible. The results were declassified and published in 1973 [235]. GRBs are among the most luminous objects in the sky, with an energy flux of 10^{44-45} J/s.

Although the emitted energy is similar for most GRBs, the peculiarities of these phenomena are incredible variegated. The light curves² of the the emission

²Light curves are graphs of the luminous intensity of a certain phenomenon plotted *vis a vis* the time at which the intensity is measured.

are not reproduced with accuracy by any model [236]. However, it is possible to classify GRBs on the base on how long the pulse of γ -rays lasts. The GRBs lifespan can range from some fractions of a second, to several minutes.

The GRBs that last around two seconds or less are named short bursts. The origin of these phenomena isn't clear at all [237]. Many of them has been observed in regions where the star formation rate is very low. This rules out a link to massive stars. Actually, the leading hypothesis are that short GRBs originate from mergers of neutron stars binaries or from the mergers of neutron stars with black holes [238–243]. Short GRBs account for almost 30% of the entire population observed so far. The remaining 70% is given by long GRBs. This class comprehends all GRBs lasting more than two seconds. Almost all these phenomena have been localized in galaxies with an intense star production process. In some cases, it was possible to be more specific and link the GRB to a supernova undergoing core collapse. It is because of these data, that long GRBs have been put in correlation with massive star death [244, 245]. It is worth mentioning that among the long GRBs some stand out. Not surprisingly, some astronomers consider a three class classification, as opposed to the twofold presented here. Some of the long GRBs are extremely long (around 10^5 s) and are sometime called ultra-long GRBs. The reason of this distinction relies on the consideration that the ultra-long GRBs have different progenitors. Instead of supernovae, they could be the result of the collapse of blue supergiant stars [246]. We have recorded very few ultra-long GRBs and, hence, this interpretation is open to dispute [247].

Another interesting phenomenon of the GRBs is that most of them display a so-called afterglow. By afterglow we mean an emission at different wavelengths that arrive later with respect to the γ -rays. Such an emission was expected in many models that had been put forward to explain the origin of the GRBs [248]. However, these afterglows were very elusive observationally, because it was hard to observe such longer, incoming wavelengths given that the position of the GRB was not precisely located. In 1997, the Italian-Dutch satellite BeppoSAX managed to finally detect such an afterglow in the x-ray window of the spectrum for GRB 970228, which was used to also give a precise location to the GRB itself [249]. Shortly after, thanks to this precise location, a optical afterglow was measured [250]. Instead, for GRB 970508 an afterglow in the radio window was put in evidence [251]. Today, we know several GRBs for which the afterglow has been measured. For most of these both optical and radio afterglows were present.

The x-ray afterglow is the strongest, but also the shortest afterglow. Not only, almost all GRBs have an x-ray afterglow. It is shown to depend on both time and frequency according to the function

$$f_\nu(t) = \nu^{-\beta} t^{-\alpha} \quad (4.64)$$

with the parameters $\alpha \sim 1.4$ and $\beta \sim 0.9$ determined observationally [252].

Optical and infrared afterglows have been found, instead in about a half of the GRBs that have been localized with precision. Also they display a dependance on time and frequency. The difference is that the factor α takes the value 1.2.

As we have mentioned, the discovery of such afterglows was made difficult by the fact that the optical afterglow is visible only several hours after the GRB itself. Hence, only if the position is determined rapidly the instruments can be directed toward the correct portion of sky.

Last but not least, the radio afterglow is present in almost 50% of the GRBs. 80% of the GRBs that display a radio afterglow also have an optical one. The others are called “dark GRBs”. Radio afterglows are measured several hours, if not days after the burst itself. The nature of dark GRBs is not clear. We mention three hypotheses. A possibility is that the dark GRBs are normal GRBs, with dense molecular clouds that absorb the visible radiation along the line of sight [253]. Another hypothesis is that they are at redshifts bigger than 5 so that the signal is shifted outside the instrument range [254]. This supposition is however questionable because the redshift of some dark GRBs has been set and does not seem to support this interpretation [255]. The last possibility is that the optical afterglow is much feebler and cannot be detected.

As we shall see, the radio afterglow can be exploited to estimate the photon mass.

Before moving on to the radio afterglow, we briefly discuss what mechanism(s) gives rise to such powerful jets. Again, there isn’t a widespread consensus. However, the so called fireball model has acquired some popularity. In the fireball model, a relativistic plasma (Lorentz gamma factor ~ 10) expands from a central source. A relativistic fireball is not necessarily spherical, which allows the characteristic emission in a well defined direction. This model has been developed keeping in mind the possibility that GRBs might be the result of dying stars, which ejects ionized gasses before collapsing.

4.5.1.1 Two limits using GRBs

Given that the massive effects are more relevant at lower frequencies, GRBs are not competitive with FRBs.

We have analyzed GRB071109 and its radio afterglow. An pulse at 8.46 GHz has been detected 0.3 days after the correspondent γ -ray emission [256]. This is the reason why we choose this particular GRB. The others in the catalog were actually detected later, which means that the sensitivity of the measurements was inferior. Unfortunately, the redshift for GRB 071109 is unknown. However, it is reasonable, to pick it in the range

$$z \in [0.1, 5] \quad (4.65)$$

If we choose the value for z that gives us the less restricting photon mass we obtain

$$m_\gamma < 5.0 \cdot 10^{-47} \text{ kg} \quad (4.66)$$

Even without resorting to literature data, we see that the GRB limit is not competitive with that obtained through the FRBs. However, it is worth mentioning that, were we able to use lower frequency observations for GRBs or could we

detect afterglows sooner, the method outlined in this Chapter would be interesting nonetheless. A reason of interest is that this limits involves completely different astrophysical models. And hence it could be, potentially, used to conclude something on what is common and what is not between FRBs and GRBs, trying to divine if they share common features or learning from the differences.

Another method that exploits Eq. 4.62 has been proposed by our collaborators Alexandre Sakharov, Edward Sarkisyan, Nick Mavromatos and John Ellis. We base our exposition on presentation [257]. In what follows, we will neglect the IGM dispersion. Or equivalently, the IGM effect is included in the source frame contribution. This choice is made to be as general as possible and not assume any location for the GRBs. Eq. 4.62 becomes

$$b_{sf}^i = \frac{1}{1+z^i} (a_\gamma^i F(E_1, E_2) H_\gamma(z^i) - \Delta t_{obs}^i) \quad (4.67)$$

where Δt_{obs}^i is the time lag measured for the i -th GRB at a specific frequency.

We look for the parameter a_γ^i that better minimizes the estimator

$$\sigma_{b_{sf}}(a_\gamma^i) = \frac{1}{N_{src}} \sqrt{\sum_{i=1}^{N_{src}} (b_{sf}^i - \bar{b}_{sf})^2} \quad (4.68)$$

being N_{src} the number of sources available and $\bar{b}_{sf}$ the average value obtain over the set of data in consideration. So, $\sigma_{b_{sf}}(a_\gamma^i)$ is the RMS as we had announced before.

Sakharov used data contained in [258] and tabled in Fig. 4.2. The results of this procedure are shown in the Fig. 4.3.

Of course, the confidence in this method depends on the accuracy on the measures of the time delays for every source. To check and improve this stability it is possible to generate toy data sets, pass them through the procedure we have just described and obtain, for each set, an optimized value for the recovery parameter a_γ . This gives a distribution of recovery parameters that can be used to set the 95% CL limit on its value. This is the value that should be then converted into a photon mass.

The toy data sets are created by each of the measured time lag values and throwing around it a normally distributed random set of values, whose dispersion is given by the measurement error bars. The results can be seen in Fig. 4.4.

We can evince from this plot that

$$a_\gamma < 107 \text{ s} \cdot \text{keV}^2 \quad (4.69)$$

which means that, with a 95% CL,

$$m_\gamma < 3.9 \cdot 10^{-41} \text{ kg} \quad (4.70)$$

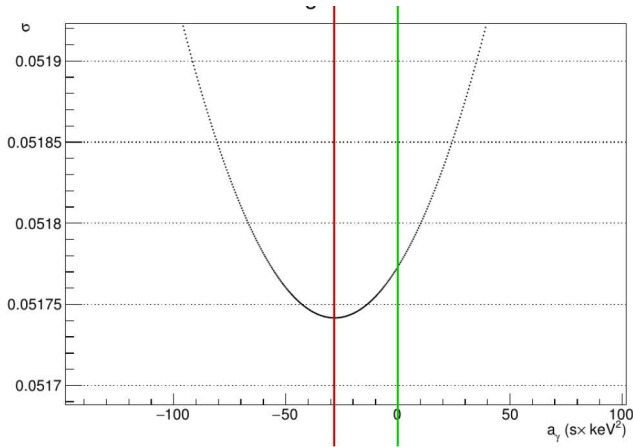
This limit is of course not competitive with the other we have presented exploiting the afterglow of GRB 071109 nor with those yielded by FRBs measurements.

GRB	z	z Refs.	$\Delta t_{\text{total}}^{(E_{\text{high}}-E_{\text{low}})}(\text{s})$
BATSE (64 ms)			
970508	0.835	[24]	-0.059 ± 0.044
971214	3.418	[25]	-0.098 ± 0.045
980329	3.9	[23]	-0.084 ± 0.036
980703	0.966	[26]	0.138 ± 0.053
990123	1.600	[27]	-0.155 ± 0.041
990308	1.2	[28]	0.0188 ± 0.0138
990510	1.619	[29]	-0.0017 ± 0.0143
991216	1.020	[30]	-0.0091 ± 0.0012
990506	1.3060	[31]	-0.0503 ± 0.0075

GRB	z	z Refs.	$\Delta t_{\text{total}}^{(E_{\text{high}}-E_{\text{low}})}(\text{s})$
SWIFT (64 ms)			
050319	3.24	[41]	0.0054 ± 0.0109
050401	2.9	[23]	-0.0135 ± 0.0285
050416	0.653	[23]	-0.1491 ± 0.1075
050505	4.3	[23]	-0.0012 ± 0.0561
050525	0.606	[23, 42]	0.1261 ± 0.0159
050603	2.821	[23]	-0.0032 ± 0.0047
050724	0.258	[43]	0.131 ± 0.1681
050730	3.968	[44]	0.094 ± 0.1361
050820	2.612	[23]	0.033 ± 0.0569
050904	6.29	[45]	0.004 ± 0.0852
050922	2.17	[23]	0.0231 ± 0.0208

GRB	z	z Refs.	$\Delta t_{\text{total}}^{(E_{\text{high}}-E_{\text{low}})}(\text{s})$
HETE (164 ms)			
010921	0.45	[32]	0.0357 ± 0.0585
020124	3.198	[33]	-0.0046 ± 0.0455
020903	0.25	[34]	-0.0150 ± 0.0386
020813	1.25	[35]	-0.1602 ± 0.0794
020819	0.41	[36]	0.222 ± 0.145
021004	2.33	[37]	-0.0402 ± 0.1109
021211	1.01	[23]	-0.0202 ± 0.0639
030226	1.99	[23]	-0.0227 ± 0.0568
030323	3.372	[38]	-0.0148 ± 0.0570
030328	1.52	[23]	0.00825 ± 0.07661
030329	0.168	[39, 23]	0.0037 ± 0.0219
030429	2.66	[40]	-0.0123 ± 0.0965
040924	0.859	[23]	-0.2516 ± 0.0801
041006	0.716	[23]	0.1179 ± 0.1228
050408	1.2357	[23]	-0.0562 ± 0.0989

Figure 4.2: Data for GRBs from three different surveys from [258].

Figure 4.3: RMS of time lags as a function of the parameter a_γ . These results have been presented by Alexandre Sakharov in [257].

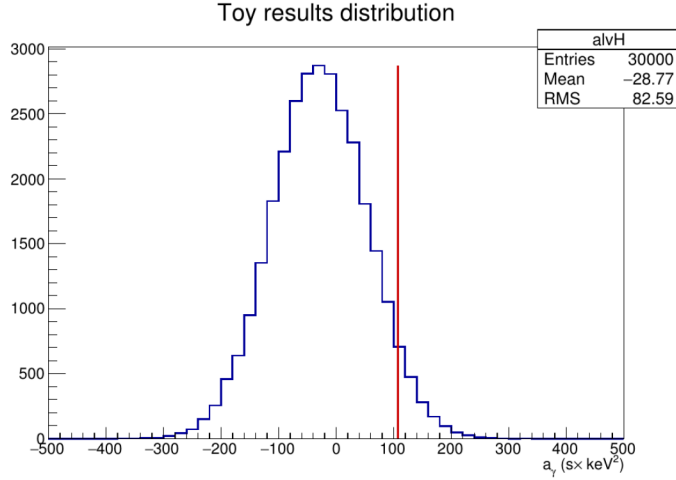


Figure 4.4: Distribution of values for a_γ obtained through the generation of data set for the GRB data at disposal. The vertical line indicates the 95% CL limit and are used to obtain the photon mass value. Also these results were presented in [257].

So, even if it could be improved by using a different spread estimator or by including more sources in the initial data set, we don't reckon this would be an avenue to be stringent on how massive the light carrier is. Nevertheless, the intrinsic interest of this method makes it worth mentioning.

4.6 OLFAR: space based low frequencies investigation

In the previous sections, we have shown the advantages of working with low frequencies. LOFAR [259] and NenuFar [260] represent such opportunity. Herein, we take a step further and present a new space based observatory, the Orbiting Low Frequency Array (OLFAR). This observatory would be particular in many way, not only because of the new frequency range that it will investigate. It will be possibly composed of many nanosatellites built with low cost, COTS³ technology. This is a new, intriguing challenge that tries to exploit a niche in the scientific exploration of the sky and competes against the common, high-budget satellites that are designed specifically for observational duties.

Most of the observation of the radio signals coming from the cosmos is done by Earth based telescopes. For low frequencies, the characteristic wavelength is of the order of tens of meters. This implies that a single disc antenna must be quite large in order to be able to capture a signal. Thus, very frequently, it

³Commercial Off-The-Shelf

is preferred to put in place a large array of smaller antennas whose synthetic aperture⁴ is very large. However, it has some limitations.

In Eq. 4.21 we have introduced the plasma frequency. That represents the frequency under which a signal is reflected off the medium and, hence, cannot propagate through it [261]. The lowest frequency detectable from Earth is related to the plasma frequency of the atmosphere. This value is around 10 MHz, with variations due to the weather conditions and the time of the day. It is thereby evident that there is no possibility to go below this limit from ground observations. Not only: this limit will soon be almost reached thanks to the Low Frequency Array (LOFAR) project [262]. Similarly, the Moon has plasma frequency of some hundreds kHz [263]. This means that hypothetical lunar observatories could push the observation further up the wavelength scale. This is however, a costly project because we would need to install the telescope on the Moon. Also, it wouldn't be the best we could obtain. In fact, the interstellar medium (ISM) has a plasma frequency much lower than that of the Moon atmosphere (around 2 kHz). Therefore it would make little sense to invest a considerable amount of money on a lunar station.

The obvious optimal solution is to use space based telescopes. Such projects have already been proposed. See for instance the Astronomical Low Frequency Array (ALFA) observatory [264] which should be able, potentially, to see down to 30 kHz. As we shall see, projects like OLFAR cannot be competitive with these low values. However, they have some advantages in terms of flexibility and costs, among other things. The limit for ALFA is given by the solar wind plasma frequency at Earth orbit. It has been shown that the plasma density decreases in getting further away from Earth (so much that the probe Voyager was able to measure a kHz signal) [265]. Thus, arrays that could be installed further away might be able to see lower signals.

For completeness, we must mention that the investigation of extragalactic signals presents additional problems. It is known that, at the scale of $h \sim 1$ kpc, the ISM presents a disc of warmer ionized gas. This disc is opaque to frequencies lower than 3 MHz, preventing direct observations beneath this hard limit (although it also helps in mapping the ISM [266]). Even if it is reasonable to think that there are preferred paths that are less dense and thus allow better measurements [263], it is not probable that this window on the extragalactic region will be opened any time soon. Nevertheless, there are some ways around this problem. Absorption and other matter related phenomena play a great role in prevent low frequency signals to reach the Earth. However, this means that there is a tight link between matter and this dispersed waves. Then we can obtain information on the sub-MHz range through observations of the intergalactic matter. Namely, synchrotron absorption and inverse Compton scattering has been already used to put bounds if applied in regions with cosmic rays and magnetic fields.

In any case, for all that concerns direct observations, the best we can do is

⁴An array of antennas work as an equivalent single plate telescope. The synthetic aperture is the aperture that this equivalent single plate would have.

to put in place space based arrays of antennas. The DARIS [267] project was an interesting precursor. It was structured so that a group of nine satellites were to be displaced. Eight of those had little processing power and their role was mainly to register data. They were enslaved to the central unit, which was in charge of processing and of data communication. DARIS showed that technology had advanced enough to make this scenario realistic and, more importantly, implementable with COTS components. It must be added that having a central processing unit and a set of nodes unable to take autonomous decisions increases the risk of a mission failure. In case of a breakdown of the mother spacecraft, the entire network would become useless. From this consideration the idea of OLFAR was born.

4.6.1 A swarm of nanosatellites

Micro-Electronics (ME) and Micro-Systems Technology (MST) have achieved great results in the recent past. Today it is possible to fit billions of transistors on a chip and mechanical systems like resonators, switches or gyroscopes can be fit into few square centimeters, if not millimeters. So it is possible to store a lot of processing power and actuators in a reasonable contained volume. Therefore the idea of exploiting this low cost technology is very natural. Of course, the current satellite with their hyper-specific technology would always outperform any machine assembled using COTS components. However, the contained budget needed to build many of COTS satellites would allow to produce a larger number of them. Such a large number that, in order not to get confused with the classical satellite formations, called swarm.

The similarity goes beyond pure numeric considerations, however. OLFAR, which represent the prototype of this innovative way of exploring the sky through orbital laboratories, is structured trying to emulate the behavior of a swarm in nature. What does that mean, more specifically? Swarms show some peculiar features. Namely robustness and redundancy, large area coverage, lack of hierarchical command structure, limited processing power per unit, self-organization [268]. We'll try to revise most of these features here below.

4.6.1.1 Organization of the swarm

It is normal practice, when building a satellite to aim for redundancy of the systems that are loaded on board, to assure the success of the mission. If the failure of a single system compromised the entire measurement, the risk of sending up the satellite would be too high. In other words, if we have only one or very few highly specialized instruments we want to make sure all of them are robust enough to survive the entire mission time and that the measures are performed even in case some parts malfunction. On the contrary, the strength of a swarm does not lay in the robustness of the single unit.

This does not mean that less care should be put into designing elements that might be able to survive the adversities of an outer space mission. Simply, we want to underline that the positive result of the observation does not depend on

the single element. If one were to fail, the others would be able to carry on with their task(s). This is obviously an advantage with respect to classical satellite formations, especially in terms of design costs because it obviously makes this phase a lot less critical. Also, the fact that a swarm of satellites is spread over a vast region makes the chance of getting damaged by space debris fainter.

Despite using a bunch of identical units, they can be specialized to perform well specific tasks through a reconfiguration. This option should be available for every element of the swarm so that the whole robustness is not affected: the reconfiguration is not permanent and interchangeability is always possible. This procedure requires some care, though. Technically, the process is considered easy but it results in the swarm losing an element. So, if the reconfiguration is triggered in too many parts of the swarm the entire system might become useless. As a consequence, the most reasonable way to use reconfiguration is to switch among a number of secondary functions while leaving the general task of the swarm untouched.

It is possible that the ability to learn (or specify in this case) is not such a wonderful feature for a swarm system. Even though it is physically extremely simple to fit a powerful processing unit on a satellite, it is far less easy to make it perform. Among other problems, solar panels wouldn't provide enough energy for modern processors. Also, these latter tend to heat up, making the cooling of the satellite a very tricky task. Like in nature, however, we should maybe get around these problems by considering that the intelligence is not in the single unit, but, rather, in the entire swarm. This means that the efforts should not be focused on how to fit the most computing power possible in every single element. Instead, the stress must be put on what the primitive behavior required from every component of the swarm is, in order for the whole to be sufficiently intelligent. Even this latter kind of intelligence should not be too enhanced. This means that a swarm is extremely selective in the kind of tasks it performs. It can, *e. g.*, find a specific kind of data very efficiently. If we need another task, it might be better (and even cheaper) to create a different swarm.

What engineers should still research about is the swarm reactions to unforeseen situations. With normal satellites, maneuvers are operated from ground stations but for the swarm this might prove to be tricky.

4.6.1.2 Orbit and launch

Since regular satellites heavily outperform COTS ones it would be useless to station them in orbits where other satellites work. Also, the swarm would add a lot of debris in orbits that are already very dangerous because of floating residues from precedent missions. Also in view of the considerations made concerning how the chance of better measurements increases the more we get far away from Earth, it makes sense to think send the swarms further away. Ideally in places that are difficult or too dangerous to reach for regular spacecrafts. The first possibilities are orbits next to the Moon because further distances would be too risky for the current technological readiness. Incidentally, although the Moon would act as a shield from the most part of the terrestrial radio frequency

interference (RFI), some of it will bend around the obstacle and still affect the measures, even if there is not a direct line of sight to the Earth. Of course, this effect is more enhanced for low frequencies because their big wavelengths make it easier to overcome obstacles. We will say something more about the estimate of this noise later.

This means that we there are three ways to send the satellites into space: dedicated launcher, piggy-back riding via another lunar mission or piggy-back launch to an Earth orbit and then the satellites can reach the Moon themselves.

It goes without saying that a dedicated launcher is economically very demanding and only organizations like ESA and NASA might afford it, but it is questionable if the benefits of the mission would justify such an expense. Also the second option seems unlikely. There are not so many lunar missions so the opportunities would be rather rare. Moreover, it would weight on said mission payload, in terms of added fuel to compensate for the mass in excess. Also, it might be hard to convince someone who sends a rocket to the Moon to bring what might potentially be more debris up with them. Therefore, the last options is the most likely. However, this means that micro-propulsion systems must be mounted on the swarm satellites. There is a lot of research going on on the subject and it has been seen that electric micro-propulsion systems are able to generate long lasting thrusts, able to propel a nanosatellite to the Moon in a matter of months.

4.6.1.3 Power

In order to keep dimension compacts enough to allow an easy launch, the solar panel that equips each nanosatellite cannot be too big. There is also the (maybe more serious) problem of creating a control mechanism for said panels such that it keeps them pointed toward the sun. This problems are even more enhanced if we think of missions in the lunar orbits because they would be shielded from direct sun exposition for long period of times. The proposed solution is actually very simple, though.

An idea would be to hibernate the satellites whenever they are not performing scientific tasks. This would allow to save lot of energy to spend when most needed. This solution has been adopted, e.g., also by the recent Rosetta mission. At this stage we see another advantage of thinking in term of swarms instead of a single unit. The amount of scavenged energy should not be enough to keep every unit active at all time, however it is enough to have, at any moment, a sufficient number of satellites able to perform measurements. That, of course, if the number of units is sufficiently high. The big problem is that the elements of the swarm are unaware of being part of the swarm. Hence, power scheduling might be difficult on an overall level. The problem is overcome by incorporating a sort of “reflex” in the satellites that cause it to hibernate when it receives too much signal power from the rest of the swarm. This would mean that the system won’t operate at the maximum of power, because that cannot be sustained for a long time. However, the system will be permanently operative if the swarm is numerous enough to allow a good rest time to the satellites that need recov-

ery. An emergency system can be added so that the swarm would decide to hibernate as a whole for a predetermined amount of time, if for some reason the overall level drops below a certain threshold. It is interesting to stress that the number of satellites is not necessarily fixed and might be increased over time. As a consequence either the uptime of every satellite can be increased or more satellites are active.

Some energy saving procedure can be applied to the data transmission toward Earth as well. It would be too energy consuming to implement a radio transmitter to send the data to the ground based station. However, if an unit is hit by a radio signal from another member of the swarm, it might be stimulated to emit a coherent signal toward Earth. The result would be, without any further coordination requests, the transmission power increases with the number of satellite available for the transmission. The great challenge, in this setup, is to direct the resulting signal toward the desired point. To reach this goal, the Earth station could help the swarm through a beacon providing feedback.

Similarly, the processing of data faces engineers with problems, but those should be solved by having a sufficient number of active satellites at all times. Maybe, a swarm could even switch between two mode. One devoted to the collection of data, the other to its process.

4.6.1.4 Objective

We have already mentioned that one of the most advantageous features of the swarm is to cover a large area. This implies that the spatial resolution is excellent because this large area is spanned by a great number of detectors. The simplest recording device is, of course an antenna, and therefore radio frequencies are the natural candidates for the first application of such technology. OLFAR will focus on creating a sky map for frequencies below 30 MHz , namely in the $0.1 - 10\text{ MHz}$ window. The corresponding terrestrial amplitude needed would be over 100 km , which is unfeasible also because the aforementioned problems with atmosphere opacity. Reaching the Moon with a swarm of nanosatellites would fill in this hole that we have in the sky exploration. Also, OLFAR is supposed to prove helpful for navigation.

4.6.1.5 OLFAR: limitations and scientific contributions

In this section we will specify what exposed about a swarm of COTS satellites to the OLFAR project.

The expected number of satellites is around 50. According to the principles exposed earlier, these satellite will have a limited “intelligence”. Each unit, therefore, will be able to perform three tasks:

- sense and sample cosmic signals in the assigned frequency window
- process the information via distributed correlation
- send the processed data toward Earth

Thus, there is the need to stock on board hardware for radio observation, distribution and processing and downlinking. To prevent failures, these subsystems will be kept separated and integrated in the miniature spacecraft. On this the research is still ongoing, but it is sure that multiple antennas will have to be hosted in order to handle the data flow into, within and out of the spacecraft.

As far as the orbit of OLFAR is concerned, the options considered are either to station in the L2 Lagrangian point behind the Moon or to revolve around the Moon in orbit with it. The following consideration seem to indicate that the latter is the wisest choice.

We have mentioned before that the RFI might bend around the Moon and affect the measurements. There is no need to worry for artificial radio transmissions and for lighting, because both of those are efficiently shielded out by the Moon itself. Also these signals must go through the atmosphere to escape Earth. This means that they are attenuated by the ionized medium in the ionosphere. This happens for frequencies below 3 MHz . For higher frequencies, the dissipation that would occur in ground wave propagation around the Moon assures that Earth produced waves will not be a significant source of noise. On the contrary, the Auroral Kilometric Radiation (AKR) is more bothersome. AKR is the $100\text{ kHz} - 1\text{ MHz}$ radiation produced by the cyclotron radiation due to electrons that orbit around the Earth following the magnetic field lines. Its total power is estimated to lie between 1 and 10^6 W . Such a radiation is produced at an altitude that varies among one and three times the Earth radius. Hence, it does not propagates through the ionosphere and it reaches the Moon still strong enough to bend around it. These low frequencies are also very much left unaffected by ground wave propagation. Even so, the main source of noise would still be the Galactic Background Radio Noise (GBRN), unless there were a straight line of sight between the detection point and the AKR source. The Moon with its rough terrain and low conductivity would attenuate the AKR signal enough to make it disappear in the Galactic Noise. Unfortunately, because of the altitude at which the AKR is produced, only a small region behind the Moon not reached by it. This reduced the time of a Moon orbit which is left unaffected by the AKR.

The situation of the L2 point is no better. This Lagrangian point is located outside the line of sight free area, which means that the satellites will be directly exposed to the AKR.

As far as noise is concerned, another problem to take into account is the reflection of the AKR. It has been observed that around 10% of the time at 20-40 Earth radii, the AKR is reflected and will reach the backside of the Moon. This reflection affects all orbits higher than 1000 km and therefore, even assuming that OLFAR will orbit around our Moon, it will be operative only 90% of the time.

4.6.2 Scientific contribution

As we have repeatedly stated, the primary purpose of the OLFAR mission will be to open the $0.1 - 10\text{ MHz}$ window to observation.

Δt [s]	m_γ [kg]	n_e [m ⁻³]
10^{-10}	$1.6 \cdot 10^{-55}$	$6 \cdot 10^{-12}$
10^{-8}	$1.6 \cdot 10^{-54}$	$6 \cdot 10^{-10}$
10^{-6}	$1.6 \cdot 10^{-53}$	$6 \cdot 10^{-8}$
10^{-4}	$1.6 \cdot 10^{-52}$	$6 \cdot 10^{-6}$
10^{-2}	$1.6 \cdot 10^{-51}$	$6 \cdot 10^{-4}$
1	$1.6 \cdot 10^{-50}$	$6 \cdot 10^{-2}$
10^2	$1.6 \cdot 10^{-49}$	6
10^4	$1.6 \cdot 10^{-48}$	$6 \cdot 10^2$
10^6	$1.6 \cdot 10^{-47}$	$6 \cdot 10^4$
10^8	$1.6 \cdot 10^{-46}$	$6 \cdot 10^6$

Table 4.1: The equivalent theoretical delays at 1 MHz due to the electron content or massive photon dispersions, and for a typical distance $d = 4$ kpc (representative pulsar distance), computed with respect to the time of arrival at high frequencies.

Among the advantages is the study of pulsars and pulsar timing in an incredibly interesting range, since most pulsars are at their brightest there. Pulsars have been mostly studied in higher frequency windows, especially the 300 – 2000 MHz. This choice was dictated by the quality of data that could be obtained at lower frequencies. The more the wavelength grows, the ISM effect, the effective background sky temperature of the Galactic synchrotron emission and the ionospheric effect take a toll on the quality of data that can be recorded.

OLFAR will completely eliminate the atmosphere disruptive effect. Hence, observers will be provided with data that are affected only by the two remaining effects. The measurement of the DMs at OLFAR frequencies will help to improve the models of pulse shapes and lead to more precise timing. Also - and this is the main reason we are interested in the prospects opened by OLFAR - we can test the ISM.

Specifying Eqs. 4.23 and 3.105 at $\nu_1 = 1$ MHz and $\nu_2 \rightarrow \infty$ for a source distant 4 kpc, we obtain the result shown in the Table 4.1

An example of the results can be seen in Fig 4.5, where we plot four dispersion curves for three different reasonable values of the photon mass and the average electron density estimated at the OLFAR frequency range. Low frequency deviations from these curves might hide some fundamental physics that goes beyond the variation of dispersion.

Another possible area where the OLFAR project might be useful is to shed some light on the phenomenon of superdispersion. This latter is a phenomenon of unsure origin that causes an excess of dispersion at lower frequencies. Superdispersion has been detected by many as shown in [270–274], while others have not found evidence [275,276]. In [277–279] the larger dispersion is at higher frequencies, but for a limited number of sampling frequencies. A possible ex-

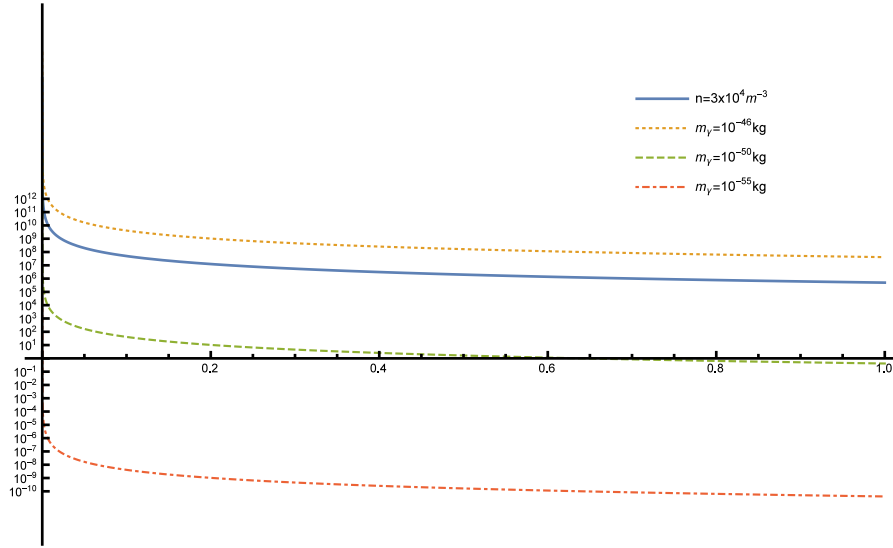


Figure 4.5: For illustration, electron density and dBP dispersions are plotted versus the horizontal and vertical axis, indicating the frequency [MHz] and the delay [s], respectively. The distance is $d = 4 \text{ kpc}$ (representative pulsar distance), an average electron density along the line of sight $n = 3 \times 10^4 \text{ m}^{-3}$ (thick blue line), photon mass $m_\gamma = 10^{-46} \text{ kg}$ (dotted orange line), 10^{-50} kg (dashed green line) and 10^{-55} kg (dotted-dashed red line). The frequency range is $0.1 - 1 \text{ MHz}$ and the delays are computed with respect to the time of arrival at high frequencies. For these values, the dispersion of the massive photon is either larger or smaller than the contribution of the average electron density along the line of sight. The dispersion value at 1 MHz for $m_\gamma = 10^{-46} \text{ kg}$ is $4.11 \times 10^7 \text{ s}$ (around 1.3 years), for 10^{-50} kg is 0.411 s , for 10^{-55} kg is $4.11 \times 10^{-11} \text{ s}$. The dispersion for the electron density is $4.96 \times 10^5 \text{ s}$ (about 6 days). [269]

planation of superdispersion is based on the inhomogeneous properties of the ISM that, by causing multi-path propagation of radio waves where the path depends on the frequency, determines the sampled column density of free electrons being also a function of frequency. Thereby, the DM inconsistencies might be the consequence of imposing a $1/\nu^2$ simple law onto a frequency-dependent and more complex dispersion due to an inhomogeneous ISM. In [273], for the magnetar SGR 17452900, the authors find superdispersion between the L and S bands⁵. They exclude causes as low quality data, profile misalignments, modelling and systematic errors, time variability. On the other hand, the authors observe that frequency dependencies other than $1/\nu^2$ appear to produce DM inconsistencies. We complement their conclusions by commenting that it is not evident to assume that the medium inhomogeneities, and thereby the multi-paths, determine always the same behaviour versus frequency. Whether, this is due to an extra-dispersion $1/\nu^2$ caused by a massive photon cannot be claimed, even theoretically at this stage. For magnetars the presence of the overcritical magnetic field must be considered. The latter demands the analysis of alternative electromagnetism [15] to see if delays can be caused by the large magnetic field. OLFAR might be able to investigate more closely this low frequency limit ranges and find new physics or better explanations.

4.7 Conclusions

In this Chapter we presented some concrete results that exploit actual measurement data. Despite the existence of many experiments to set an upper limit on the mass of the photon, we find the method presented here particularly interesting because it is directly based on measurements that are done with a very high precision by astronomers. Its downside is how the massive effect cannot, *a priori*, be separated from other dispersive contributions unless the source is located at cosmological distance. The analysis of dispersion and superdispersion at very low radio frequencies should be performed in terms of frequency analysis. Better timing at low frequencies should also be pursued. In the second part of the Chapter we have focused our attention on the OLFAR mission and its possible scientific contribution.

The method to set an upper limit on the photon mass is based on time delay measures performed on light at different wavelength coming from FRBs. We have pointed out that there are still many open questions concerning these astrophysical phenomena. So the interest of using our method is twofold. On the one hand, we can investigate the photon propagation. On the other, we might hope to discover something about this recently discovered events. In particular, we could say something more precise on the location of FRBs.

As we mentioned, the downside of the technique we employ is that the dispersion can be attributed to various factors. The three main ones are: near source dispersion, IGM effect and massive photon. For our results, we have not taken into consideration the possible effects of near source dispersion. We have

⁵The L band is the one between 1 and 2 GHz, while the S band is between 2 and 4 GHz.

written the time delay in terms of the DM. This latter can be furthermore put in relation to the photon mass. Inverting the formula, we have the photon mass in terms of the measured time delay.

We focused our attention on FRB 150418, for which the redshift of the hosting galaxy has been determined at $z = 0.492 \pm 0.008$ [200]. The measured time delay for two photons at frequencies $\nu_1 = 1.2 \text{ GHz}$ and $\nu_2 = 1.5 \text{ GHz}$ respectively is $\sim 0.8 \text{ s}$. The most stringent limit corresponds to attributing all the time delay to the massiveness of the photon. In this case, we obtain that

$$m_\gamma < 4.6 \cdot 10^{-50} \text{ kg} \quad (4.71)$$

A more reasonable estimate would be reached by crediting some of the effect to the IGM. That can be done by using the value for the IGM density fraction given in [205]. This value yields an inferior DM with respect to the commonly accepted one, found in [200]. We considered the difference between the two as the photon mass contribution and obtained

$$m_\gamma < 3.2 \cdot 10^{-50} \text{ kg} \quad (4.72)$$

with 95% CL. Both these results are much more competitive with respect to those yielded by GRBs or AGNs because the photon mass in a dBP theory is proportional to the inverse square of the frequency and the low frequency signals from FRBs are favored. The results are in agreement with the conclusions in [5, 48], too.

We are convinced that these limits can be made even more robust in the future and that not much can be done concerning the measurement for a single event. The impossibility to properly estimate the IGM anisotropies prevents us from getting a significant improvement on the DM_{IGM} error bars. Instead, we show that a statistical approach allows to increase our precision. The idea is to compute an estimator on a given set of data. As such estimator, we have chose the RMS of possible contribution to the DM coming from next to the source. The origin of all FRBs should be similar. So it is logical to assume that the conditions are roughly the same around all of them. The estimator will depend parametrically on the photon mass and by minimization of the χ^2 the best fitting value for m_γ is computed. With this technique the upper limit becomes $1.1 \cdot 10^{-51} \text{ kg}$.

Concerning the OLFAR project we have discussed what the advantages and disadvantages are, in comparison with a high-budget, specialized satellite. On one hand, the COTS technology cannot compete in terms of performance with specifically-designed units. On the other, the limited costs allow to send a great number of satellites. We presented the basis of the swarm formation that should allow to create a niche market for astronomical observation performed via nanosatellites. Another strong point of OLFAR, in comparison with ground based detector is its location in outer space. This prevents any disruption from the atmosphere. The wide area covered by the swarm means that the spatial resolution will be excellent, which will turn into substantial improvements in pulse modelization. Through OLFAR the IGM can be tested. Another realm where OLFAR could shed some new insight is superdispersion.

CONCLUSIONS

In this thesis we have analyzed alternatives to Maxwell's theory and their possible effects on light propagation. Despite the undeniable success that the accepted theory of electromagnetism has, there are good reasons to go beyond it. Over the past century, there have been improvements in the understanding of all the other fundamental forces. Instead, the electromagnetic theory has been left unchanged. Although this does not mean that the theory is not valid, fundamental physics is called to explain why such a progress didn't occur. Moreover, the transition from classical to quantum electrodynamics is not straightforward and requires artificial techniques that might be avoided with slight changes in the electromagnetic theories. Last but not least, the photon is the only massless particle belonging to the Standard Model. There is no exhaustive explanation for this fact and fundamental physics is once again burdened with the task of providing an answer to this question or to provide alternative models.

One of the most important aspects of this work is that it has touched several branches of physics. This is valuable because it shows how traversal the consequences of non-Maxwellian theories can be. We present non-linear theories where the deviation occurs at classical level, assuring a modification of the fields of a charge. We introduce a non-linear effective Lagrangian that reproduces the tree level quantum effects without the need to go through the quantum formalism. We talk about a massive photon and try to relate its mass to a mechanism arising from the symmetry principles.

In Chapter 2 we adapt a method that has been successfully used to simplify the computation of the radiation field of an accelerating Maxwellian charge to the case of the non-linear theory of Born-Infeld. Although this theory has been solved in particular cases, the result for the general case is unknown. Our attempt constitutes a new approach to compute the radiation and, in perspective, the self-force of an accelerating charge. Especially the study of the radiation reaction would be interesting in Born-Infeld. In fact, this non-Maxwellian theory smooths the fields at the origin and we expect a much simpler version of the self-force expression. Unfortunately, the complexity of the solution and the time constraints prevented us from obtaining an explicit result, but the author and his collaborators intend to continue working on it. The second part of the Chapter is instead devoted to the analysis of the frequency shift that the light propagating to spatial infinity from an overcritical Heisenberg-Euler dipole suffers. Our computation shows that we should expect a polarization dependent

effect; for one polarization we have a redshift, for the other a blueshift. We express our result as a function of the magnetic field and of the azimuthal angle. We notice that the absolute value of the shift increases as the field strength grows. In terms of the angle, we have a specular behavior between the two polarization, instead: one has a maximum at the pole and a minimum at the equator, while the other has a minimum at the pole and a maximum at the equator. Our work can be considered a toy model heading toward the description of light propagation from a magnetar. Taking values compatible with the observations on magnetars (that is a star with two solar masses and a 10 km radius), we show that our frequency shift is only one or two orders of magnitude smaller with respect to the gravitational one.

In Chapter 3 we study how an effective photon mass can derive, in the context of Extended Standard Models, for the spontaneous Lorentz symmetry breaking. We assume a supersymmetric environment and show four - paired by two according to them being CPT-odd or CPT-even - different Lagrangians that spontaneously break Lorentz symmetry. This symmetry violation occurs at high energies so, at testable scales, the effects are very small. Nevertheless, we focus precisely on consequences that are measurable, namely the dispersion of light. For the k_F -term CPT-even model, we highlight a non-Maxwellian but still massless behavior, where the group velocity is diminished by a tiny factor that depends on the propagation direction and on the components of the Lorentz violating background tensor. For the Carroll-Field-Jackiw CPT-odd model we observe both from the analysis of the dispersion relations and of the time delays for photons at different frequencies that photons should be massive. This is an effective mass that depends on the direction and on the modulus of the Lorentz breaking background vector. We show that the upper limit on the photon mass is 10^{-55} kg , according to estimates of the background vector order of magnitude performed in laboratory experiments [197], This result is in line with the official Particle Data Group limit [147], as well as with those proposed by the local research group using ESA data from Cluster [48] or Fast Radio Burst (FRB) time delay [196]. Furthermore, we also give an explanation why the effective mass should be proportional to the modulus of the background vector. We show how, writing the Lagrangian in a non-manifest covariant way, we can obtain a term *à la* de Broglie-Proca.

In Chapter 4 the attention has been drawn to more observational aspects and profited of a new type of source to set an upper limit on the photon mass. With data from FRB 150418, we have measured the time delay for two photons at frequencies $\nu_1 = 1.2\text{ GHz}$ and $\nu_2 = 1.5\text{ GHz}$. The time delay is put in correlation with the Dispersion Measure (DM) that is the integral along the path followed by the photon from the source to the observer over the electron density of the medium through which it propagates. The experimental result for the DM is too large to attribute it only to the galactic electron plasma effect. Normally, such exceeding dispersion is attributed to the Intergalactic Medium dispersive effect. We have instead assumed that it is completely or partly due to a de Broglie-Proca massive photon. If we ascribe the entire DM to the massive photon, we obtain an upper limit $m_\gamma < 4.6 \cdot 10^{-50}\text{ kg}$. Instead, if

we accept that the density fraction for the Intergalactic Medium in the universe is the one measured through the cosmic microwave background [205], we can attribute only the difference between the expected dispersion for this plasma and the one actually measured to the massiveness of the photon. The ensuing limit is $m_\gamma < 3.2 \cdot 10^{-50} kg$.

Part of this work has been presented to the July 2015 Marcel Grossman Meeting in Rome, whose proceedings are under print [269]. The remaining has been published in peer reviewed journals [196, 199, 269]

Appendix A

Units and dimensions

Although they have been later inserted in a coherent theoretical context, it is worth recalling that all the equations at the base of the electromagnetism are based on experiments. Accordingly, proportionality constants assume arbitrary values. We consider the base units of length $[L]$, mass $[M]$, time $[T]$ and current $[I]$. The latter is the variation of the charge with respect to time. This trivially implies that the ratio between charge and current has the dimension of time, which leads to Eq. 1.24.

If Eq. 1.24 is dimensionally correct, the Gauss law takes the form

$$\vec{F}_{Gauss} = k_1 \frac{q_1 q_2}{r^2} \vec{n} \quad (\text{A.1})$$

where the quantity has already been defined in Chapter 1. With respect to Eq. 1.19, we have specified the proportionality constant and set it to k_1 . The value and dimensions of such a constant can be determined via Eq. A.1 if the dimension and value of the elementary charge are set independently. Alternatively one can choose k_1 arbitrarily and a definition for the charge follows. Both solutions are viable as long as the product $k_1 q_1 q_2$ has the dimension of $[ML^3T^{-2}]$.

The electric field is defined as the force per unit of charge¹. So

$$\vec{E} = k_1 \frac{q_1}{r^2} \quad (\text{A.2})$$

¹This is not strictly the most general definition possible. It would be more correct to consider the field numerically proportional to the force per unit charge and the proportionality constant a universal constant. The subtlety lies in the fact that nothing obliges to choose a dimensionless parameter. It could be possible to choose a dimensional one that renders the field of different units with respect to the force per unit charge. However, $\vec{E}$ is the first derived quantity for which we define the dimensions. It would only be in defining other derived quantities dimensions that a different choice might prove easier for computations or more physically insightful. Moreover, the entire system must be self-consistent and a different choice of this constant would imply a change in the dimensions of all the physical quantities related to the electric field. This is why there is no loss of generality in using this more restrictive definition for the time being.

For static magnetic fields, Ampere's law links the force to the currents I_1 and I_2 flowing in two wires at distance d

$$\frac{d\vec{F}_{\text{Ampère}}}{dl} = 2k_2 \frac{I_1 I_2}{d} \vec{n} \quad (\text{A.3})$$

Here, l represents the length of the wires and it is not assigned. Since Maxwell's equations must be consistent, the definition of the dimensions for the current and the electric field reveal something about the dimensions of k_2 . In fact, from Eq. A.1,A.3, it is seen that

$$\left[\frac{k_1}{k_2} \right] = [L^2 T^{-2}] \quad (\text{A.4})$$

This ratio has the dimensions of a velocity squared. Its numerical value is experimentally very close to the speed of light, thereby

$$\frac{k_1}{k_2} = c^2 \quad (\text{A.5})$$

Furthermore, the magnetic induction B is proportional to the force per unit of current, similarly to what occurs for the electric field. We call such a constant β . In the case of the wire, we have

$$\vec{B} = 2k_2 \beta \frac{I}{d} \quad (\text{A.6})$$

From Eqs. A.1-A.3,A.6 it is seen that

$$\left[\frac{E}{B} \right] = \left[\frac{L}{T\beta} \right] \quad (\text{A.7})$$

With this new set of parameters, Maxwell's equations can then be written as

$$\vec{\nabla} \cdot \vec{E} = 4\pi k_1 \rho \quad (\text{A.8})$$

$$\vec{\nabla} \times \vec{B} = 4\pi k_2 \beta \vec{j} + \frac{k_2 \beta}{k_1} \frac{\partial \vec{E}}{\partial t} \quad (\text{A.9})$$

$$\vec{\nabla} \times \vec{E} + k_3 \frac{\partial \vec{B}}{\partial t} = 0 \quad (\text{A.10})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (\text{A.11})$$

where k_3 is the constant that links, in Faraday's law, the electric and magnetic fields. Unsurprisingly, k_3 is related to β . Indeed, combining Eq. A.9 and Eq. A.10 we get

$$\nabla^2 \vec{B} - k_3 \frac{k_2 \beta}{k_1} \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad (\text{A.12})$$

System	k_1	k_2	β	k_3
Electrostatic	1	c^{-2}	1	1
Electromagnetic	c^2	1	1	1
Gauss	1	c^{-2}	c	c^{-1}
Heaviside-Lorentz	$\frac{1}{4\pi}$	$\frac{1}{4\pi c^2}$	c	c^{-1}
SI	$\frac{1}{4\pi\epsilon_0}$	$\frac{\mu_0}{4\pi}$	1	1

Table A.1: We present the four more common system used to define the arbitrary constants in Maxwell's equations. For all of them, the base units are length, time and mass. However, for the first four they are measured in centimeters, grams and seconds respectively. The SI system uses meters, kilograms and seconds, plus the current with the Ampère as unit.

working in absence of sources. This is the propagation equation of a wave traveling at the speed of light. Hence

$$k_3 \frac{k_2 \beta}{k_1} = c^{-2} \Rightarrow k_3 = \frac{1}{\beta} \quad (\text{A.13})$$

Given this premise, we can appreciate from where the various systems of units for Maxwell's equations come from. In view Eqs. A.5,A.13 of the four possible parameters of the theory, only two need to be chosen arbitrarily, allowing us different solutions. In Tab. A.1, we summarize the most frequent ones.

The electromagnetic and SI units differ only by a power of ten, multiplying k_1 and k_2

$$\frac{1}{4\pi\epsilon_0} = 10^{-7} c^2 [ML^3 T^{-4} I^{-2}] \quad (\text{A.14})$$

$$\frac{\mu_0}{4\pi} = 10^{-7} [MLT^{-2} I^{-2}] \quad (\text{A.15})$$

On top of the three base quantities, the SI system needs also a fourth one, the current with the Ampère as unit.

The Gauss and Heaviside-Lorentz system differ only for a numerical factor 4π . In such systems, k_3 has the dimensions of the inverse of a velocity. Because of this and Eq. A.10, the electric and magnetic field have the same dimension.

In Tab. A.2 Maxwell's equation and the Lorentz force per unit charge in the various systems are displayed.

Furthermore, in the text we have decided to employ natural units. The advantage of this choice is that, unlike what happens to rationalized constants that depend on the properties of some prototype object or particle - which are

System	ϵ_0	μ_0	Maxwell's equations	Lorentz force per unit charge
Electrostatic	1	c^{-2}	$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ $\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t}$ $\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ $\vec{\nabla} \cdot \vec{B} = 0$	$\vec{E} + \vec{v} \times \vec{B}$
Electromagnetic	c^{-2}	1	$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ $\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t}$ $\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ $\vec{\nabla} \cdot \vec{B} = 0$	$\vec{E} + \vec{v} \times \vec{B}$
Gauss	1	1	$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ $\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$ $\vec{\nabla} \cdot \vec{B} = 0$	$\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}$
Heaviside-Lorentz	1	1	$\vec{\nabla} \cdot \vec{E} = \rho$ $\vec{\nabla} \times \vec{B} = \frac{\vec{j}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ $\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$ $\vec{\nabla} \cdot \vec{B} = 0$	$\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}$
SI	$\frac{10^7}{4\pi c^2}$	$4\pi \cdot 10^{-7}$	$\vec{\nabla} \cdot \vec{E} = \rho$ $\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t}$ $\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ $\vec{\nabla} \cdot \vec{B} = 0$	$\vec{E} + \vec{v} \times \vec{B}$

Table A.2: Definitions of ϵ_0 and μ_0 , Maxwell's equations and Lorentz force per unit charge in the various unit systems.

arbitrarily chosen -, they only depend on free space properties. Among all sets of natural units, we have chosen to work with the Planck units. This approach was originally proposed by Max Planck in 1899 and it consisted in setting all five universal constant to unity. Then

- the speed of light $c = 1$
- the gravitational constant $G = 1$
- the reduced Planck constant $\hbar = \frac{h}{2\pi} = 1$
- the Coulomb constant $\frac{1}{4\pi\epsilon_0} = 1$
- the Boltzmann constant $k_B = 1$

Adopting Planck units means substituting the base units for the fundamental quantities with others related to the Planck scale. In other words, for switching from SI units to Planck ones, the meter is replaced by the Planck length as unit to measure distances. The required changes are listed in Tab. A.3.

The Planck units are very similar (but conceptually very different) from the Gauss or Lorentz-Heaviside units. Like in these two cases, $\frac{1}{4\pi\epsilon_0}$ is set to unity.

Unit	Expression	SI value
Planck length	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616199(97) \cdot 10^{-35} m$
Planck mass	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176470(51) \cdot 10^{-8} kg$
Planck time	$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}}$	$5.39106(32) \cdot 10^{-44} s$
Planck charge	$q_P = \sqrt{4\pi\epsilon_0 \hbar c}$	$1.875545956(41) \cdot 10^{-18} C$
Planck temperature	$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	$1.416833(85) \cdot 10^{32} K$

Table A.3: Table of the base unit in the Planck natural system

Thus, Maxwell's equations read

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (\text{A.16})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (\text{A.17})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{A.18})$$

$$\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t} \quad (\text{A.19})$$

Despite the apparent simplicity of the natural units, they must be used with care. As P.S. Wesson said “Mathematically, it is an acceptable trick which saves labour. Physically, it represents a loss of information and can lead to confusion.” To help the reader, we propose here a simple example of how they must be used. Let's take the gravitation law:

$$F = G \frac{m_1 m_2}{r^2} \quad (\text{A.20})$$

The first step is to inset the corresponding expression for G in terms of the new unis

$$F = \left(\frac{F_P l_P^2}{m_P^2} \right) \frac{m_1 m_2}{r^2} \quad (\text{A.21})$$

where F_P is the Planck force defined by

$$F_P = G \frac{m_P^2}{l_P^2} \quad (\text{A.22})$$

and represents the unit of force because it attracts two particles having the Planck mass and separated by the Planck length. Therefore,

$$\frac{F}{F_P} = \frac{\left(\frac{m_1}{m_P}\right) \left(\frac{m_2}{m_P}\right)}{\left(\frac{r}{l_P}\right)^2} \quad (\text{A.23})$$

So, if we choose F_P as unit for the force, m_P for the mass and l_P for the length, one can write

$$F [F_P] = \frac{m_1 [m_P] m_2 [m_P]}{(r [l_P])^2} \quad (\text{A.24})$$

In practice, one does not go through this process each time. Eq. A.22 takes such a form in virtue of having normalized G to one.

Appendix B

QED and Feynman diagrams

In the quantum context all observable quantities are operators that act on a purely mathematical space, called Hilbert space. A detailed explanation of what an Hilbert space or an operator is goes beyond the purposes of this text. The magnetic and electric field can be written in terms of operators; more specifically, in terms of creation and annihilation operators (see footnote 9) . The first creates a photon with a defined momentum, the other destroys it.

The results of the complex interactions between particles and fields can be visually represented very easily. That is the purpose of a Feynman diagram.

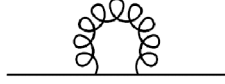
These representations have been introduced by Nobel Laureate Richard Feynman concerning just the electromagnetic theory. Nevertheless, they are applicable to all kind of fields. To simplify the matter, we don't dwell on other examples than those coming from QED. With this limitation, the three basic actions to be represented are

- a charged fermion moving in space and time
- a photon moving in space and time
- an electron emitting or absorbing a photon

These are the only actions that need to be taken into account because EM fields are generated by charges and composed by photons. Therefore charged particles are the sources and photons the mediators of the interaction. There is no other element intervening in the process.

The translation of this into a graph occurs as follows. The image is composed by points (called vertexes) and lines. Each vertex represents a point of interaction between the particles involved in the process. Each line represents a particle. By definition, a solid line indicates a fermion whereas a wavy line represents a photon. There is no preferred direction to draw a Feynman diagram, but, in general, one tries to keep on the x axes the time and on the y axis the space. If one accepts this convention, the system finds itself in the initial state on the left and in the final state on right. Furthermore a fermion in the initial

Figure B.1: One loop QED contribution showing the production and re-absorption of a virtual photon during the propagation of an electron.



state is represented with an arrow entering the vertex, whereas a photon with the arrow pointing away from it. Viceversa in the final state.

This might seem very abstract, but it's actually quite easy from the operational point of view. To better grasp the meaning of these rules, let's apply them to one of the cases presented in our text. We repeat here Fig. 1.1

In this case, we have a fermion entering the vertex to the left. At the vertex, the fermion emits a photon. For a certain amount of time they propagate separately, until they reunite at the time of the vertex to the right. Here the photon is absorbed again by the fermion, which then continues to propagate. The fermion line is straight because the particle never deviates.

If we repeat the operation with Fig. 1.2, we see that, in this case, we have a fermion propagating. At the level of the first vertex it emits a two fermions (in our case an electron and its antiparticle to assure charge conservation during the process). These two fermions propagate for a certain time. Then they collide at the second vertex and, as a result, produce another photon.

A complete description of the interaction between two charges is given by summing all the possible contributions that arise in such ways.

Such graphs should not be taken too "literally". Our representation of particles moving along straight or curved lines does not mean that they actually follow those trajectories. Likewise, the wavy lines we use for photons do not imply that they are more of a wave than fermions. We represent the motion as if it were stationary, but nothing assures us that the particle velocity remains constant. We do not know the details of what happen between vertex and vertex. We can only describe the results of a transition from the initial to the final state.

Appendix C

Λ CDM model

The Λ CDM model is one of the many possible parametrizations of a cosmology involving the Big Bang. The precise description of this model goes beyond the purposes of this work. However, it is widely accepted because, for its supporters, it describes reasonably well observed features of the universe such as the Cosmic Microwave Background, the large scale structure distribution, the abundance of elements created via nucleosynthesis and the expansion of the universe.

The name of the model derives from Einstein field equations in the form

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \quad (\text{C.1})$$

where the constant Λ was originally introduced by Einstein to obtain a static universe. The great scientist defined this idea his biggest mistake because Hubble's measurements proved that the universe was expanding. However, the equation was exhumed after the acceleration of the universe was measured. For such an acceleration a peculiar and unknown kind of energy is held responsible: dark energy. It has the same effects of a fluid with negative pressure whose effect are well incorporated by Einstein's equation with a cosmological constant in it.

The CDM part refers to the introduction of the Cold Dark Matter to account for the missing mass that has been proposed to explain the gravitational effects like the flat rotation curves of galaxies, the light lensing of galaxy cluster and the enhanced clustering of galaxies. The dark matter does not emit electromagnetic radiation and interacts only gravitationally (and eventually through weak force) with the “normal” matter. It is supposed to be cold (i.e. its velocity is supposed to be much smaller than the speed of light), dissipationless and collisionless. Dark energy and dark matter would account for 95% of the universe energy-mass density.

The expression in Eq. 4.26 has been obtained by adopting the Friedman-Robinson-Walker metric

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (\text{C.2})$$

where we have used spatial spherical coordinates. The curvature is null since observationally the universe is found to be flat, on cosmological distances; $a(t)$

is an unknown function of time referred to as the scale factor. The Hubble constant is [102]

$$H(t) = \frac{\dot{a}(t)}{a(t)} \quad (\text{C.3})$$

from which it ensues that the scale factor in term of redshift is

$$a(t) = \frac{1}{1+z} \quad (\text{C.4})$$

The fundamental equations that drive cosmology, the Friedmann equations, read [102]

$$H^2 = \frac{8\pi\rho + \Lambda}{3} \quad (\text{C.5})$$

$$\dot{H} + H^2 = -\frac{4\pi}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad (\text{C.6})$$

with ρ being the density and p the pressure of the universe at a specific time in its evolution. The first equation is essentially derived from the 00 component of the Einstein's equations and the second is obtained from the first together with the trace of Einstein's field equations. Furthermore, energy conservation yields

$$\frac{d}{da}(\rho a^3) = -3pa^2 \quad (\text{C.7})$$

Choosing an equation of state $p = p(\rho)$, it is possible to determine the dependence of the density with the scale factor. This density tells us how relevant every single component of the universe is at a certain era. Now, given that we have taken the universe as flat, we are left with three possible components: radiation, non-relativistic matter and the cosmological constant. Of course, we cannot draw conclusions on the cosmological constant from Eq. C.7. However, for the non-relativistic matter, we have that $p \ll \rho$. Hence $\rho \sim a^{-3}$. Instead, for radiation, the pressure is $p = \frac{\rho}{3}$ which yields $\rho \sim a^{-4}$. The dominant component of the universe is the non-relativistic matter. Therefore, defining the quantity called critical density, as

$$\rho_c = \frac{3H^2}{8\pi} \quad (\text{C.8})$$

and the density fraction

$$\Omega_X = \frac{\rho_{X0}}{\rho_c} \quad (\text{C.9})$$

(where, of course, ρ_{X0} indicates the density of a specific component at present time), it is immediate to rewrite Eq. C.5 as

$$H = H_0 \cdot \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \quad (\text{C.10})$$

To finally obtain Eq. 4.26, we write the present age of the universe as an integral over the scale parameter

$$t_0 = \int_{a=0}^1 dt = \int_0^1 \frac{da}{\dot{a}} = \int_0^1 \frac{da}{aH} = H_0^{-1} \int_0^1 \frac{da}{a \cdot H(z)} \quad (\text{C.11})$$

where in the last step we have used the notation of Eq. 4.27. Then a change of variable is performed to express a in term of the redshift

$$t_0 = H_0^{-1} \int_0^\infty \frac{dz}{(1+z) \cdot H(z)} \quad (\text{C.12})$$

Therefore, the age of the universe at any time is given by

$$t(z) = H_0^{-1} \int_z^\infty \frac{dz}{(1+z) \cdot H(z)} \quad (\text{C.13})$$

Eq. 4.26 follows simply taking an infinitesimal variation of t .

Publication List and Curriculum Vitae et Studiorum

List of publications:

1. G. Acquaviva, L. Bonetti, G. Cognola and S. Zerbini, “Ray tracing in FLRW flat space-times”, *Phys. Rev. D*, 88, 124024 (2013), DOI:10.1103/PhysRevD.88.124024
2. G. Acquaviva, L. Bonetti, S. Zerbini and L. Vanzo, “Vacuum fluctuation of conformally coupled scalar field in FLRW space-times”, *Phys. Rev. D*, 89, 084031 (2014), DOI:10.1103/PhysRevD.89.084031
3. L. Bonetti, S. P. Bergliaffa and A. D. A. M. Spallicci, “Electromagnetic shift arising from the Heisenberg-Euler dipole”, *Proceedings of the Fourteenth Marcel Grossman Meeting on General Relativity* edited by Massimo Bianchi, Robert T Jantzen, Remo Ruffini, World Scientific, Singapore, 2016
4. M. Oltean, L. Bonetti, A. D. A. M. Spallicci. and C. F. Sopuerta, “Entropy theorems in classical mechanics, general relativity, and the gravitational two-body problem”, *Phys. Rev. D*, 94, 064049, (2016), DOI: 10.1103/PhysRevD.94.064049
5. L. Bonetti, J. Ellis, N. E. Mavromatos, A. S. Sakharov, E. K. G. Sarkisyan-Grinbaum and A. D. A. M. Spallicci, “Photon mass limits from fast radio bursts”, *Phys. Lett. B*, 757, 548 (2016), DOI:10.1016/j.physletb.2016.04.035
6. L. Bonetti, L. R. dos Santos Filho, J. A. Helayël-Neto and A. D. A. M. Spallicci, “Massive photons from Super and Lorentz symmetry breaking”, arXiv:1607.08786 [hep-ph] (2016), accepted for publication in *Adv. Space Res.*
7. M. J. Bentum, L. Bonetti, A. D. A. M. Spallicci, “Dispersion by pulsars, magnetars, fast radio bursts and massive electromagnetism at very low

radio frequencies”, arXiv:1607.08820 [astro-ph] (2016), accepted for publication in Phys. Lett. B

Curriculum vitae et studiorum:

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List of Figures

1.1	Feynman diagram showing the one loop QED contribution to the electron self-energy. Going from left to right, the diagram shows the electron evolving freely, producing a virtual photon at the first node. The photon is reabsorbed at the second node and, finally, the electron continues its propagation.	16
1.2	Feynman diagram showing the one loop QED contribution to the photon self energy. Reading from left to right, the diagram shows the photon propagating and split into a particle-antiparticle pair at the first node. The pair annihilates at the second node reforming the photon that continues its propagation.	17
2.1	Spacetime in Rindler coordinates. Region I is the Rindler wedge. Region III the anti-Rindler wedge. Regions II and IV are the Milne and anti-Milne wedges respectively.	46
2.2	The upper and bottom panels display the dependence of $^{(1)}z$ and of $^{(2)}z$ with κ , respectively. [130]	64
2.3	Plot of $^{(1)}z$ (left) and $^{(2)}z$ in terms of $u = \cos \theta$, for $\lambda = 1, 10, 100$ (continuous, dashed, and dotted line respectively). In the left plot, the EMS is negative and decreases with the field. In the right plot the opposite occurs. [130]	65
3.1	Spin $\frac{1}{2}$ fermions and gauge bosons in the SM	75
3.2	We show the order in which the symmetries are supposed to break: the order is not certain and in our work we adopted that of [143], although others are possible. At Planck scale they are all exact symmetries. The breaking of Lorentz symmetry (LOS) is the first and happens above the energy scale of the Great Unified Theory (GUT). At $10^{11} - 10^{13} \text{ GeV}$ we have the breaking of SUSY. At the observable level we find the models presented in this work and that carry reminiscence of such symmetries being broken.	76

3.3	Time delay for Class 1 as a function of θ , for a frequency range between 0.1 and 1 MHz at the distance of 4 kpc . We have chosen a low frequency range in order to stay in the regime at which current experiments like OLFAR and LOFAR work. The distance has been chosen at 4 kpc because it is the average distance at which a pulsar is located and a lot of time delay measurements are performed on pulsars. [199]	92
3.4	Group velocity for a wave circularly polarized and right-handed, plotted in function of the frequency in the range 1 MHz –1 GHz that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10. . . .	94
3.5	The group velocity corresponding to $\tilde{\omega}_+$ plotted as a function of the frequency in the range 1 MHz –1 GHz , that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10.	96
3.6	The group velocity corresponding to $\tilde{\omega}_-$ plotted as a function of the frequency in the range 1 MHz –1 GHz , that corresponds to the wave number in the $\hat{z}$ direction divided by the modulus of the background $\tilde{k}_3$ ranging from 1 to 10.	97
4.1	In this plot we show on the (m_γ, Ω_{IGM}) plane some bands of hypothetical measures and also the one measured for the FRB 150418. The red band is the Λ CDM expectation value for $\Omega_{IGM} = 0.041 \pm 0.002$. The shaded gray band is the FRB 150418 measure and uncertainties discussed in [200]. The remaining bands are hypothetical 10% accuracy bands that we reckon to be attainable in the near future. The green and mauve bands are for FRBs at $z = 0.1$, the blue one at $z = 1.0$	125
4.2	Data for GRBs from three different surveys from [258].	131
4.3	RMS of time lags as a function of the parameter a_γ . These results have been presented by Alexandre Sakharov in [257].	131
4.4	Distribution of values for a_γ obtained through the generation of data set for the GRB data at disposal. The vertical line indicates the 95% CL limit and are used to obtain the photon mass value. Also these results were presented in [257].	132

- 4.5 For illustration, electron density and dBP dispersions are plotted versus the horizontal and vertical axis, indicating the frequency [MHz] and the delay [s], respectively. The distance is $d = 4 \text{ kpc}$ (representative pulsar distance), an average electron density along the line of sight $n = 3 \times 10^4 \text{ m}^{-3}$ (thick blue line), photon mass $m_\gamma = 10^{-46} \text{ kg}$ (dotted orange line), 10^{-50} kg (dashed green line) and 10^{-55} kg (dotted-dashed red line). The frequency range is $0.1 - 1 \text{ MHz}$ and the delays are computed with respect to the time of arrival at high frequencies. For these values, the dispersion of the massive photon is either larger or smaller than the contribution of the average electron density along the line of sight. The dispersion value at 1 MHz for $m_\gamma = 10^{-46} \text{ kg}$ is $4.11 \times 10^7 \text{ s}$ (around 1.3 years), for 10^{-50} kg is 0.411 s , for 10^{-55} kg is $4.11 \times 10^{-11} \text{ s}$. The dispersion for the electron density is $4.96 \times 10^5 \text{ s}$ (about 6 days). [269] 140
- B.1 One loop QED contribution showing the production and re-absorption of a virtual photon during the propagation of an electron. . . . 154

List of Tables

4.1	The equivalent theoretical delays at 1 MHz due to the electron content or massive photon dispersions, and for a typical distance $d = 4$ kpc (representative pulsar distance), computed with respect to the time of arrival at high frequencies.	139
A.1	We present the four more common system used to define the arbitrary constants in Maxwell's equations. For all of them, the base units are length, time and mass. However, for the first four they are measured in centimeters, grams and seconds respectively. The SI system uses meters, kilograms and seconds, plus the current with the Ampère as unit.	149
A.2	Definitions of ϵ_0 and μ_0 , Maxwell's equations and Lorentz force per unit charge in the various unit systems.	150
A.3	Table of the base unit in the Planck natural system	151

Bibliography

- [1] D. H. Delphenich, “Nonlinear electrodynamics and QED”, *Grundlage einer Theorie der Materie*, 2003. arXiv: hep-th/0309108.
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Electromagnétisme non-maxwellien en astrophysique et applications spatiales à très basses fréquences

Résumé :

L'univers est largement observé avec les signaux électromagnétiques – supposés maxwelliens – bien que la fenêtre d'onde gravitationnelle vient juste d'être ouverte. La cosmologie est depuis longtemps confrontée à un univers à 96% noir et inconnu. Donc, il est pertinent pour la physique fondamentale de débiter une vérification théorique et expérimentale de l'électromagnétisme et de son messenger, le photon. La thèse aborde les théories non-maxwelliennes, en se focalisant sur les effets vérifiables. Nous traitons d'une série d'applications et évaluons le degré d'utilité des théories électromagnétiques alternatives. Nous revoyons les théories les plus importantes, qu'elles soient non-linéaires ou massives. Pour la première catégorie, pour l'analyse de la force propre, nous considérons la radiation d'une charge de Born-Infeld uniformément accélérée ; dans la théorie d'Heisenberg-Euler, nous trouvons un décalage en fréquence induit par un dipôle magnétique surcritique, associé à un magnétar. Le photon étant la seule particule sans masse dans le Modèle Standard (MS), nous explorons les extensions de MS. En effet, pour la catégorie massive, dans un contexte supersymétrique, nous trouvons des comportements non-maxwelliens survenant dans des cas paires et impairs de symétrie CPT (Charge-Parity-Time Reversal) : soit une propagation à vitesse sous-lumineuse ou un comportement massif dispersif proportionnel au vecteur qui provoque la rupture de la symétrie de Lorentz. Plus loin, nous déterminons une limite supérieure de la masse de 3.2×10^{-50} kg en analysant les Fast Radio Bursts. Les différences des temps d'arrivée d'un photon massif (à la de Broglie-Proca) s'expriment comme l'inverse de la fréquence au carrée, comme les photons traversant un plasma. De là provient notre intérêt pour la radioastronomie à basse fréquence au sol ou dans l'espace. Un domaine inexploré cachant des pulsars se situe en-dessous de 15 MHz, et sera accessible par un essaim de nano-satellites.

Mots clés : électromagnétisme non-maxwellien, photons, astrophysique

Non-Maxwellian electromagnetism in astrophysics and in space applications at low radio frequencies

Abstract:

The universe is largely observed with electromagnetic signals – supposedly Maxwellian – though the gravitational wave window was just unbarred. Meanwhile, cosmology is since long confronted with an universe for 96% dark and unknown. Thus, it is pertinent for fundamental physics to start a theoretical and experimental verification of electromagnetism, and its photon messenger. The thesis discusses non-Maxwellian theories, focusing on testable effects. We deal with a range of applications and assess how alternative electromagnetic theories are helpful. We revise the most notable theories, either non-linear or massive. In the former class, for analysis of the self-force, we consider the radiation of an uniformly accelerating Born-Infeld charge; in the Heisenberg-Euler theory, we find a frequency shift induced by an overcritical magnetic dipole, associated to a magnetar. Being the photon the only massless particle in the Standard Model (SM), we explore SM extensions. Indeed for the massive class, in a supersymmetric context, we find non-Maxwellian behaviors arising in even and odd cases of Charge-Parity-Time Reversal (CPT) symmetry: either subluminal propagation or massive dispersive behavior proportional to the Lorentz symmetry breaking vector. Further on, we determine an upper mass limit of 3.2×10^{-50} kg by analyzing Fast Radio Bursts. Massive (à la de Broglie-Proca) photon arrival time delays go with the inverse of frequency squared, as photons passing through plasma. Thereby, our interest for low frequency radio-astronomy on ground or in space. An unexplored domain hiding pulsars lies below 15 MHz, and it will be accessible by a swarm of nano-satellites.

Keywords : non-Maxwellian electromagnetism, photons, astrophysics

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