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Impact of cathode material on performance and plasma behavior of a high-current vacuum arc thruster

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Chapter I

Introduction

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1 Vacuum arcs in space propulsion

1.1 Electric propulsion

Space propulsion is based on the principle of conservation of momentum. To set a spacecraft in motion, a mass of propellant is expelled at a given velocity. The ejected mass produces a thrust force in the opposite direction, thereby propelling the spacecraft. From this principle, two paradigms emerge. The most well-known, largely because of its visibility to the general public, is chemical propulsion used in launch vehicles such as Ariane 6, Falcon 9, Soyuz, and others. In this case, two propellants, a fuel and an oxidizer, are burnt and expelled at a velocity of about 2-5 km/s [1]. This form of propulsion is designed to expel a large mass, with mass flow rates on the order of hundreds of kg/s, at a relatively modest velocity [2, 3]. It is called chemical propulsion because the thrust is primarily generated from the energy released by combustion reactions. This type of propulsion is crucial for overcoming Earth gravitational force at ground level, where very high thrust levels are required.

Once in orbit, free from the effects of gravity, another method consists in expelling a small mass of propellant at very high velocity. This is the second paradigm, in which electric propulsion falls. By ionizing the propellant, the latter can be expelled at high speeds, on the order of several tens of km/s [4]. The energy used for ionization is electrical energy, which can be directly generated in orbit using solar panels. The thrust produced by electric thrusters is small, ranging from the μN scale [5] to a few N [6]. However, in the (nearly) total absence of friction, applying a small force over a long duration is equivalent to applying a large force over a short period. More importantly, the low mass flow rate combined with the high exhaust velocity results in reduced propellant consumption [4], enabling access to distant orbits or, alternatively, providing a substantial gain in payload capacity.

Electric propulsion is an active field of research, and an increasing number of industrial actors are adopting this form of propulsion, driven by the trend toward satellite miniaturization and the rise of large constellations. It is also a technologically diverse domain, encompassing a wide variety of concepts that offer different performance levels, each with its own advantages and drawbacks [4]. The various thruster technologies are primarily distinguished by the way in which the propellant is ionized and expelled. Among the most widespread are Hall thrusters, valued for their high thrust and efficiency [7–9]; gridded ion engines, known for their high specific impulse [10, 11]; and electrodeless thrusters such as helicon devices [12, 13] or ECR thrusters [14, 15].

1.2 Vacuum arc thrusters

In this context, vacuum arc thrusters (VATs) stand out as a rather unique technology in several aspects. As their name suggests, these devices operate by generating an electric arc between two electrodes. In vacuum and without any gas introduced between the electrodes, there is no medium to ionize. Under the effect of the locally high electric field, the electrode where the lowest potential is applied, namely the cathode, is vaporized and ionized. These plasma emission sites at the surface of the cathode are named cathode spots. The plasma ejected from the spots generates a thrust in the opposite direction. Hence, VATs are the only technology aside from pulsed plasma thrusters (PPTs) that directly uses solid fuel without any fluidic

system or propellant tank. The propellant is the electrode itself, and could be made out of any metal.

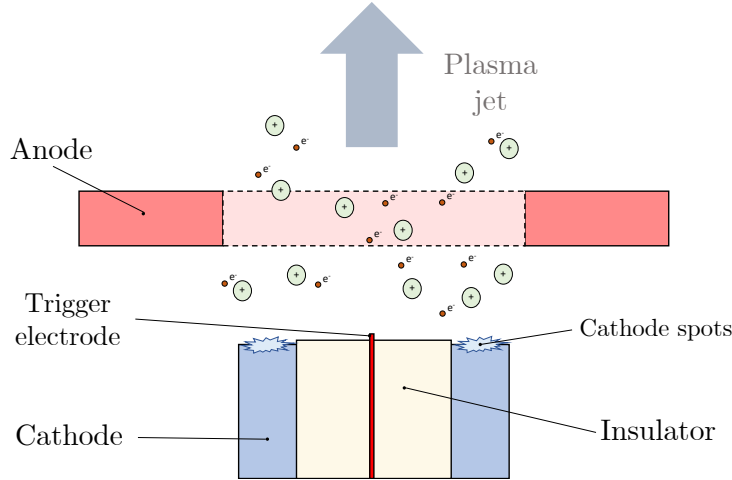


Figure 1.1: Simplified schematics on the VAT working principle. A cross-section view of the PJP is represented.

Like PPTs, VATs must be pulsed to prevent excessive melting of the electrode, which could alter its geometry and cause inoperability of the thruster. The goal is to directly ionize the solid metal while minimizing its liquid phase. Other technologies such as field emission electric propulsion (FEEP) also use metal as propellant [16]. However, the electrode geometry and propellant ionization mechanisms in FEEPs make this technology quite different from VATs in terms of plasma properties. The thruster technology whose plasma behavior most closely resembles to the one generated by VATs are PPTs, where the propellant is generally PTFE [17, 18].

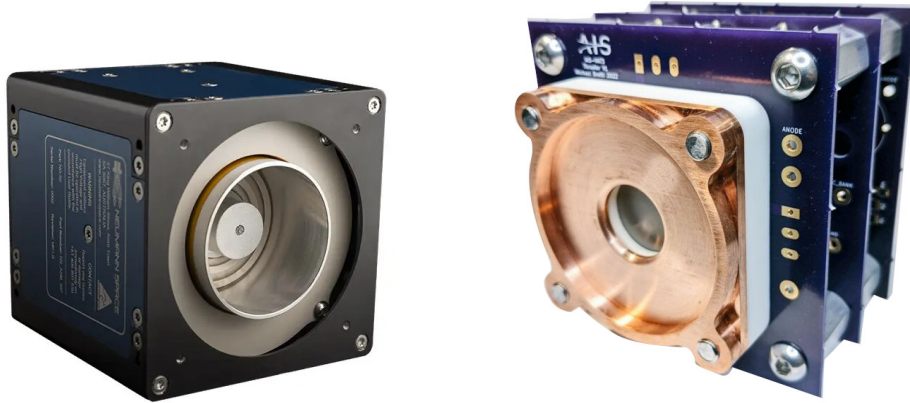
Both those two technologies are usually powered by a capacitor bank, which is discharged at a given operating frequency $f_{\text{operating}}$. The vacuum arc initiation is, in most VATs, assisted by an auxiliary electrode that provides a high voltage pulse to trigger a breakdown [19–21]. An alternative approach consists in adding a conductive coating to the cathode–anode insulating surface, and restricting the coating–cathode interface to only a few points. At these contact points, the very high power density leads to vaporization and ionization of the conductive layer, thereby initiating the arc. VATs using this triggering principle are commonly referred to as “triggerless” VATs. The main advantage of this technique is straightforward: it removes the need for an additional electrode and a high-voltage transformer. Its limitation, however, lies in the need for precise control of the thickness of the conductive layer. During thruster operation, the layer is gradually reformed by sputtered cathode material being redeposited onto the insulator. If the layer becomes too thick and the number of contact points increases excessively, arc initiation eventually fails, as the current flows entirely through the coating layer without producing a flashover. Hence, this approach tends to limit the operational lifetime of the thruster.

Another approach consists in triggering the vacuum arc using a pulsed laser that ablates the cathode surface [22–24]. The cathode material is vaporized, allowing the arc to propagate. However, in practical terms, this method is not implemented in VATs, as it requires integrating a laser powerful enough to erode the cathode surface. This adds weight and complexity to the thruster, thereby weakening one of its main

advantages: its simplicity and robustness.

Due to their inherent simplicity, VATs are particularly appealing to academic teams engaged in the development of electric propulsion systems. This simplicity also makes VATs the most easily miniaturized technology. The pioneers in this miniaturization effort are the George Washington University team with their micro-cathode VAT, under development since 2008 [25–29]. The Technion-Israel Institute of technology have been developing their own VAT for micropropulsion as well, meant to equip a 2U cubesat [30, 31], the cubesat unit U referring to a cube of 10 cm length. One may also cite the University of Southampton and the Cube-de-Alps project, which aims at developing an all-printed VAT [32, 33]. An increased effort in the development of VATs by Chinese universities has also been observed since 2019 [34–39].

Beyond academia, several companies are also engaged in the development of VATs, including Magdrive (UK), HyperNova Space (RSA), Neumann Space (AUS) [21] and Applied Ion Systems (USA). The thrusters developed by those two last manufacturers are presented in figure 1.2. The French company COMAT is also developing a VAT, the PJP, which is the thruster investigated in this thesis. The PJP and Neumann’s thruster are relatively similar in terms of discharge current range and electrode voltage.



(a) Neumann Drive, from *Neumann Space*TM (b) AIS-VAT3 from *Applied Ion Systems*TM

Figure 1.2: Two vacuum arc thrusters recently developed by the companies *Neumann Space*TM and *Applied Ion Systems*TM.

2 Fundamentals of the vacuum arc process

The physics of the vacuum arc is rather difficult to grasp, as it is a highly non-stationary and violent phenomenon, whose spatial and temporal evolution is extremely fast. Since the 70s, extensive efforts have been devoted to characterize the vacuum arcs, encompassing theoretical analyses, experimental investigations, and numerical simulations [40]. Indeed, vacuum arcs are widely employed in plasma processing, and more particularly in surface treatment such as etching [41] and coating [42]. They are also found in a lot of metallurgical processes [43] and in soldering [44]. Some other applications include miniature X-ray sources for medicine or

lithography [45], or even ion sources for particle accelerators [46]. Three major historical models can nevertheless be distinguished, associated with the works (but not solely) of Gennady A. Mesyats, Isak I. Beilis and André Anders. This section aims to briefly present the three main models in a qualitative manner. It is also worth highlighting the contributions of Mikhail S. Benilov [47–49], Burkhard Jüttner [50–52], Raymond L. Boxman [53], Ian G. Brown [54, 55] and J.E. Daadler [56, 57], whose work has significantly contributed to the understanding of vacuum arcs and, more broadly, electrical discharges. References are provided for the reader interested in exploring the theoretical framework and analytical developments in greater depth.

2.1 Ecton model

The ecton process has been theorized by Mesyats et al. in the 90s [58], although the individual phenomena composing the ecton had been discovered a few years earlier [59, 60]. The ecton can be regarded as an explosive electron emission (EEE), also referred to as an “electron avalanche” to emphasize the violent and cascading nature of the phenomenon. Ectons form the basis for the creation of plasma emission zones on the cathode surface, namely the cathode spots. Out of this model, Mesyats defines three main temporal stages in the development of the vacuum arc. These stages are illustrated in figure 1.3, and can be described as follows:

1. **The breakdown** corresponds to the very first moments where current starts to flow between the electrodes. Through field emission, electrons are emitted and current can be extracted from the asperities of the cathode surface. At this stage, the asperities can be regarded as microscopic protrusions with strong electric fields from which electrons are emitted. The extracted current is rather small, but sufficient to initiate conduction between the two electrodes. This step is extremely brief, occurring on the timescale of a few ns [51].
2. **The spark** appears at the moment when EEE occurs. Under the effect of intense Joule heating at the microscopic protrusions, the latter explode and the cathode material is vaporized and ionized. This results in the formation of dense plasma clouds, referred to as “plasmoids” [61], near the cathode surface. The spark is a transient state that lasts only for a short time period of about 1–10 μ s, during which the plasma is highly unstable.
3. **The arc** becomes stabilized. The plasma reaches a sufficient conductivity to sustain a stable current. At this stage, the apparent motion of cathode spots is observed, as they extinguish and ignite all across the cathode surface to maintain the current [62]. From this point onward, the arc can be considered to be in a steady state, and it may be sustained as long as the energy input remains sufficient.

This model is primarily intended to describe the initial stages of vacuum arc formation. Subsequent models, such as the ones presented hereafter, are more focused on the arc itself and the mechanisms by which it is sustained. The ecton model laid the foundation for understanding cathode spots, and constituted the earliest framework to conceptualize the vacuum arc as a continuous sequence of micro-explosions.

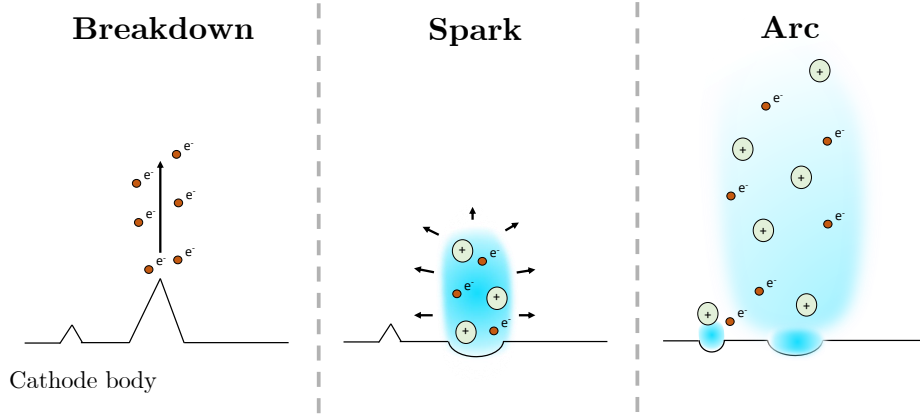


Figure 1.3: Steps of vacuum arc ignition according to the ecton model.

2.2 Kinetic model

Building on earlier models, Isak Beilis developed a kinetic model focusing on the spatial evolution of plasma properties above the spot. This kinetic approach assumes a stable arc maintained by thermal and kinetic equilibrium, rather than by constant micro-explosions. The plasma is subdivided into different regions with distinct properties, as illustrated in figure 1.4 according to the description made by Beilis [63]. The numbers in the figure indicate the boundaries between the different regions.

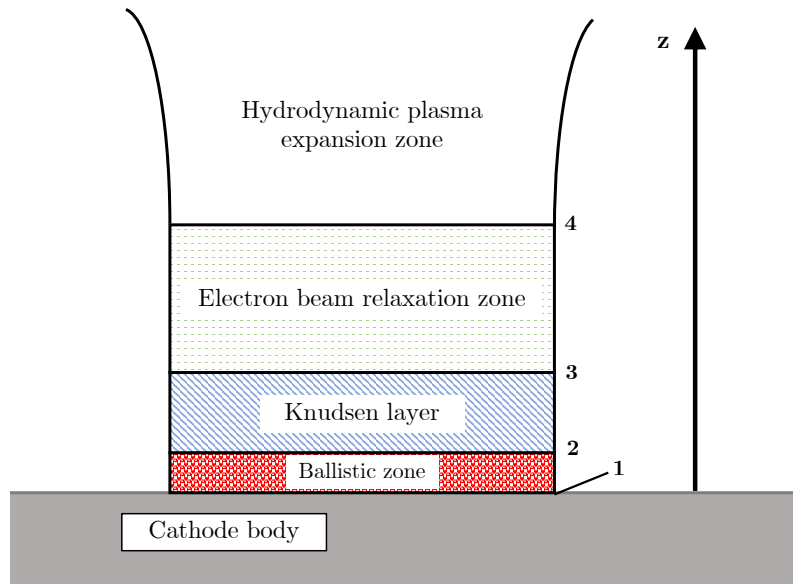


Figure 1.4: Kinetic model proposed by I. Beilis et al. Figure adapted from [63].

1-2 : The ballistic zone , or space-charge zone, is characterized by an intense electric field. In this region located in the direct vicinity of the cathode surface, electrons are accelerated toward the plasma and ions toward the cathode. Ion bombardment contributes to the heating and the erosion of the cathode

surface. The thickness of this layer is on the order of a few electron mean free paths, i.e., roughly several tens of nm.

2-3 : The Knudsen layer is, for the heavy species, a transition zone toward equilibrium. Beilis divides the overall plasma flow into two main regions: the kinetic region, located before boundary 3 (ballistic + Knudsen layer), and the hydrodynamic region located beyond boundary 3. The Knudsen layer is named as such because in this region the Knudsen number satisfies $K_n \approx 1$. This layer is characterized by a bimodal velocity distribution of the heavy species. On one side are the so-called outgoing particles, including evaporated ions and neutrals, which show relatively high thermal velocities. On the other side, some particles return toward the cathode after undergoing collisions in the plasma. Consequently, the ion velocity distribution function (IVDF) is the sum of two Maxwellian distributions: one corresponding to the outgoing particles and the other to the returning particles. The Knudsen layer is essential for explaining the transition between the ballistic and hydrodynamic regions. Once the boundary 3 is reached, the heavy species are at equilibrium. However, it should be noted that after this boundary, two populations of electrons coexist: the plasma electrons, which follow a Maxwellian distribution, and the beam electrons, which are faster and follow a shifted half-Maxwellian distribution. The thickness of this layer is on the order of a few mean free paths of the heavy species, i.e., approximately 10–100 nm.

3-4: The electron beam relaxation zone is in quasi-equilibrium state. Here, the electrons lose their energy through both inelastic and elastic collisions. The two electron populations (hot and cold) tend toward forming a single Maxwellian distribution at boundary 4. The thickness of this region is generally on the order of approximately 1 μm .

>4 : Hydrodynamic plasma expansion . Here is the quasi neutral region, which can be considered at equilibrium, and where the plasma is in supersonic expansion. At boundary 4, the electron beam has completely merged in the bulk plasma through collisions, and the electron current is only defined by the plasma electric conductivity.

To summarize this classification, we have the ballistic region, which corresponds to the space-charge zone; the Knudsen layer, which is a transition zone toward equilibrium for the heavy species; and the electron beam relaxation zone, which is a transition zone toward equilibrium for the electrons. The details of the modeling of these different regions, including the velocity distribution functions of the various species, as well as their densities and temperatures, are provided in the different works of Beilis[64, 65].

2.3 Fractal approach

The fractal approach is the most recent of the three approaches presented in this chapter and was developed primarily by André Anders et al. [66]. In this model, Anders puts forward the idea that cathode spots are in fact fractal, self-similar objects, that do not exhibit any characteristic time or length scale. This stands in contrast with traditional stationary models, which assume homogeneous and stable

spots. This idea is based on the analysis of arc voltage noise [66], as well as on observations of the craters left on the cathode surface after vacuum arc operation [67].

In brief, the noise in the arc voltage follows a power law in the form $1/f^\alpha$ with $\alpha \approx 2$. This power law is observed across several orders of magnitude, up to 10^8 Hz, without any dominant frequency emerging. The fact that $\alpha \approx 2$ makes it possible to relate this noise to a Brownian noise. Assuming that fluctuations in the arc voltage are linked to the behavior of cathode spots, in accordance with the ecton model, it is then suggested that the successive ignitions and extinctions of multiple spots on the cathode surface follow a Brownian motion. To develop this idea further, the absence of any cutoff frequency in the power law further supports the concept that cathode spots have no characteristic size or timescale. In particular, the fractal model proposes that each cathode spot is in fact composed of smaller spots, which themselves consist of even smaller spots. In other words, the spots are made of self-similar sub-spots. This self-similarity was notably observed at COMAT, see figure 1.5, while analyzing the surface asperities on the PJP cathode after triggering arc discharges. These microscopic observations, made prior to this thesis, sparked my curiosity and ultimately led me to focus my research more closely on this fractal description of the arc.

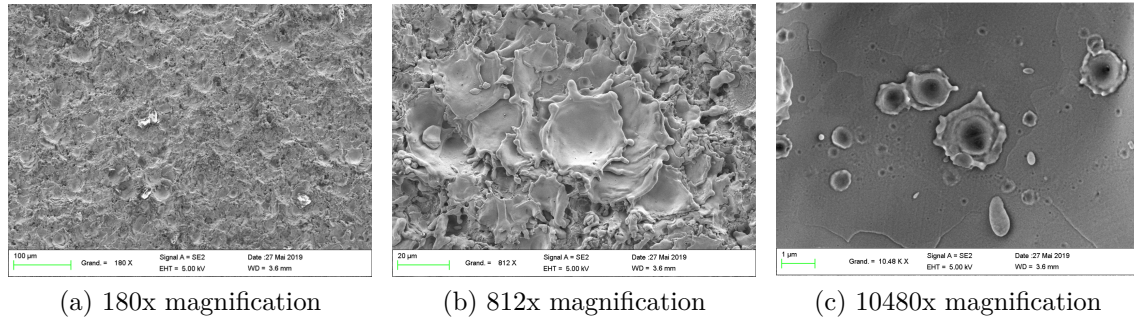


Figure 1.5: Microscope images of craters left on the cathode surface, for different magnifications (Cu cathode). Images provided by COMAT and reproduced with permission.

The fractal approach could also be named multi-scale approach, since the number of observable spots depends on the scale of observation. It has already been proposed in the literature to classify spots into different types according to their size [68, 69]. The macro-spots, in the size of several μm as in figure 1.5b, which themselves are composed of numerous microspots the size of nm, as in figure 1.5c. Clusters of macro-spots are called “group spots”. These three distinct size classes exhibit slightly different behaviors and properties, allowing, among other things, the extraction of characteristic times associated with their respective scales. This segmentation makes it then possible to break down the fractal model into several observational scales. This approach is particularly interesting, as it intrinsically accounts for the possibility of scale-dependent behaviors.

3 Research objective and outline

High-current vacuum arcs have been extensively studied in the context of electrical contacts and circuit breakers [70–73]. This research effort was yet primarily focused

on studying the plasma stability and the electrical properties of the discharge. On the other hand, the various VATs developed worldwide tend to operate at low current, with relatively long pulse duration. The study of high-current, short-duration vacuum arcs in the context of space propulsion presents then a significant research potential due to the limited number of studies in this field. The main objective of this thesis is to establish a link between plasma behavior and the performance of high-current VATs through various experimental investigations. This approach is expected to improve the understanding of the underlying physics and to serve as a basis for the future development of more efficient and optimized VATs.

The structure of this thesis is as follows:

Chapter II provides a complete description of all the instruments and experimental apparatus used in this work, as well as a presentation of the PJP thruster from COMAT.

Chapter III reports on a series of experiments meant to characterize the PJP plume by means of electrostatic probes. This chapter introduces different models of Langmuir probe analysis, providing access to local plasma properties such as the electron density, electron temperature, and plasma potential. Additional measurements reveal the ion ejection velocity for two cathode materials, namely titanium and copper.

Chapter IV presents long exposure photograph of the cathode spots patterns formed during the PJP discharges, for different cathode materials. The images are analyzed through the prism of the fractal approach, and the relationship between material properties and the characteristics of the fractal patterns is discussed.

Chapter V delves into the dynamics of fractal pattern formation using high-speed camera footage. The way in which cathode spots develop on the cathode surface during the PJP pulses is observed. The propagation trajectories are tracked, and the apparent spot propagation velocity is determined.

Chapter VI presents thrust and impulse bit measurements, obtained using different methods and for several cathode materials. This chapter establishes a connection between the findings at microscopic scale and the overall performance of the thruster.

Chapter VII presents numerical simulations of the inter-electrode gap, obtained with a hybrid PIC-Monte Carlo model. The generation of multiply charged ions and the influence of the input parameters on the model outputs is discussed.

Chapter VIII summarizes the main conclusions of this work and outlines possible directions for future research.

Chapter II

Experimental setup and diagnostics

Contents

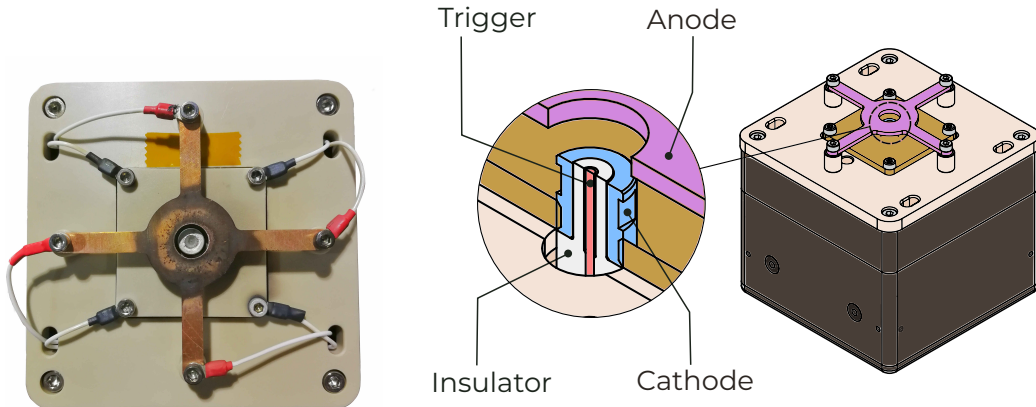
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1 Plasma Jet Pack

The Plasma Jet Pack (PJP) is a high-current Vacuum Arc Thruster (VAT) developed by COMAT, a French company based in Toulouse. Several PJP versions have been developed to date, for different needs and different satellite integrations [74]. The PJP version under study in this work is the H2020 version, designed to be easily dismantled which allows testing of different electrode configurations. This version was studied and matured as part of the European H2020 PJP project, involving German, Czech, and French partners, and led to publications in various international conferences and scientific journals [75–77].

The PJP generates a high-current vacuum arc through the cyclic discharges of a capacitor bank. The repetition frequency can reach 2 Hz at maximum, for a power consumption of 30 W. The cathode to anode voltage is continuously set to 250 V. A triggering system applies a few kV pulse on the cathode surface for 1 μ s. This pulse allows ignition of the plasma through an explosive process [58]. The cathode material is locally vaporized and ionized, on surface regions called cathode spots. After the trigger is switched-off, the arc is forced to propagate to the anode through the existing metal vapor. This second arc discharge allows roughly 3 kA to flow between the two electrodes with the present configuration. Overall, the energy released over 30 μ s into one single PJP discharge reaches about 3 J.

Figure 2.1a shows the front view the PJP H2020. The 4-legs electrode is the annular copper anode. It is highlighted in purple in figure 2.1b. The cathode corresponds to the grey center-mounted hollow cylinder, colored in blue on the detailed view. The cathode is screwed onto a copper plate directly wired to the capacitor bank. The gap in the axial direction between the cathode and the anode is approximately 5 mm. The trigger electrode, colored in red in figure 2.1b, is a tiny metal tip placed exactly on the thruster axis of symmetry and separated from the cathode with a boron nitride insulator.



(a) PJP thruster: front view.

(b) PJP schematics with electrode detail

Figure 2.1: PJP thruster front view (a) and detail of the electrodes (b).

Measurements presented in this work have been conducted with a copper anode and cathodes made out of various materials, see next chapters. The PJP discharge current (I_d) waveform is given in figure 2.2 for the 4 main metallic cathodes used in this work, namely: titanium, copper, iron and aluminium. The traces are averaged

over 100 pulses for each cathode material, and are given here as an indication of the mean amplitude. Those traces are however representative of single discharge waveforms. The I_d waveforms actually vary very little from one pulse to the next for a given material, and keep almost the same smoothness as the averaged one. In figure 2.2, one can see that the Fe curve shows a slight inflection around $5\mu\text{s}$ because the thruster experienced operational issues in this dataset. In any case, the discharge current reaches 2.5 to 3.5 kA on average. The PJP target performance and characteristics are given in table 2.1 for the inclined reader.

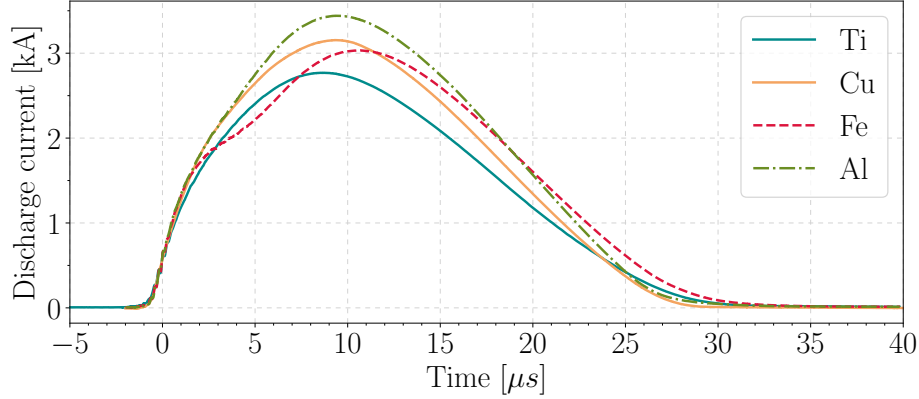


Figure 2.2: Averaged discharge current waveforms for different cathode materials. Traces averaged over 100 single PJP discharges.

Table 2.1: PJP thruster target performance and characteristics.

Power	30	W
Thrust to power	10	$\mu\text{N}/\text{W}$
Average thrust at 30 W	300	μN
Specific impulse	2500	s
Total impulse	400	N s
Overall mass	1	kg
Overall volume	1	U

2 EPIC-2 vacuum chamber

Experiments have been performed in the EPIC-2 vacuum chamber. This tank is a stainless steel cylinder 54 cm in diameter and 104 cm in length, for an approximate capacity of 240 L. Two 2200 L/s (N_2) magnetically levitated Edwards STP-iS2207 turbomolecular pumps are placed on top of the chamber, see figure 2.3. Those pumps are evacuated through a $110\text{ m}^3/\text{h}$ dry primary pump, the Edwards GV110. This pumping arrangement permit to maintain a background pressure of 10^{-6} mbar during thruster operation. The pressure is monitored with a Pfeiffer PBR260 Pirani/Bayard-Alpert pressure gauge.

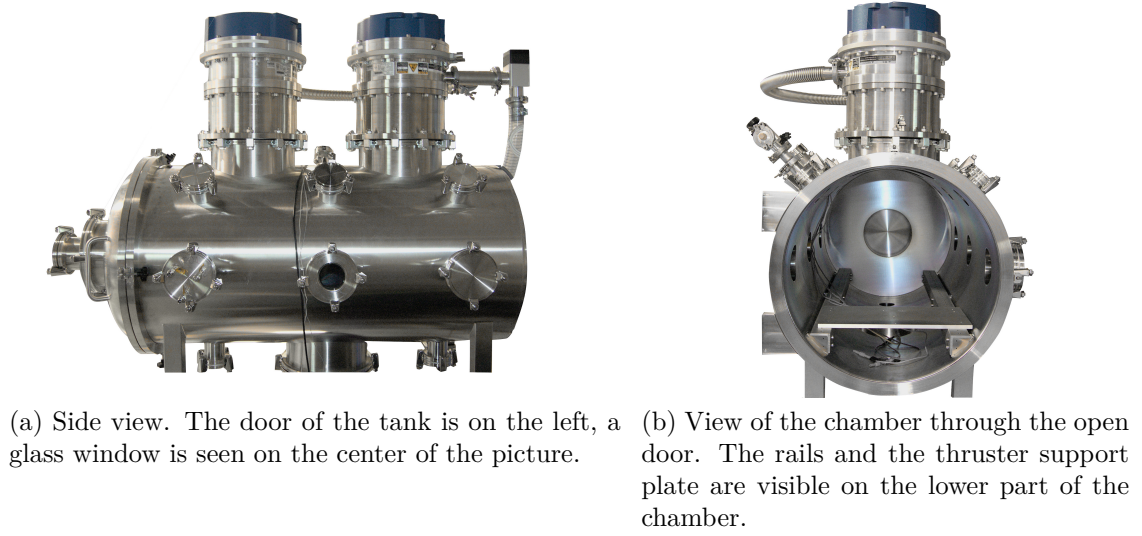


Figure 2.3: Photographs of the EPIC-2 vacuum chamber. The two STP-iS2207 pumps are seen on the top of the chamber.

EPIC-2 is also equipped with several electrical feedthroughs, for both in-situ diagnostics and thruster power supply. Those feedthroughs are mostly composed of BNC or SubD connectors. Two Kodial (Borosilicate 7056) glass windows allow visual inspection of the thruster during operation. The feedthroughs and windows can be interchanged, provided that the opening diameters are standardized according to the ISO-K or ISO-F norms. The sealing of the flanges is ensured by FPM O-rings, held in place with clamps or clamping rings, depending on the flange diameter.

Inside the chamber, the PJP is mounted on a square fixed to a plate, the latter being screwed on rails. This mounting allows adjustments of the distance to the windows or to instruments, depending on the experiment to be performed. For investigating the plasma plume by means of electrostatic probes, the PJP is placed far back in the tank to let the plume expand as freely as possible. When it comes to cathode surface imaging, the PJP is positioned closer to the front window in order to reduce the distance to the lenses and sensors that are placed outside the chamber.

3 Langmuir probe

The Langmuir probe (LP) is a widely used diagnostic in plasma physics. The theory developed by Mott-Smith and Langmuir in 1926 permit to estimate the local electron properties and the local potential in a plasma [78] by inserting an exposed metal tip into it. The current flowing through the probe depends on the voltage applied to the latter, and the analytical properties of this Intensity vs Voltage curve permit to apply models that gives access to the local properties of the plasma. Acquisition of a Langmuir probe I-V characteristic curve is usually performed by smoothly sweeping the probe bias voltage while recording the current [79–83]. In the context of this work, since the plasma is pulsed over a very brief period of time, the method for acquiring I-V curves has been adapted. This method is detailed in chapter III.

The LP is here a cylindrical tungsten rod, 0.6 mm in diameter, connected to the biasing circuit by a wire connector. To prevent a large portion of the rod and the

wire connector from collecting ions, most of the rod is insulated from the plasma by a ceramic tube. The exposed part of the probe is 14 mm long. Since the discharges are very brief, there is no strong thermal constraint on the probe. The wire connector is then covered with a heat-shrink tube, see figure 2.4, and the end of the ceramic is sealed with boron nitride paste. The LP was biased using a Tenma 72-13360 power supply, its maximum bias voltage being 60 V. The current collected by the probe was measured with a $50\ \Omega$ Pearson 2878 current probe indicating an output ratio of 0.1 V/A.

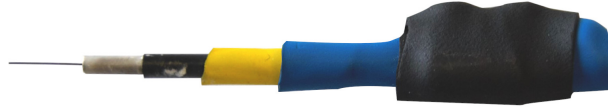


Figure 2.4: Langmuir probe used in this work. The exposed tip is seen on the left, the colored PVC parts are different layers of heat-shrink tubes used for proper electrical insulation.

4 Time-of-Flight technique

Time-of-flight (TOF) refers to the measurement of the time it takes for a particle to travel a certain distance. In this context, the particles are ions moving through a plasma. Assuming the ions follow a straight-line path, the method involves measuring the ion current at two separate locations using a set of electrostatic probes [84, 85]. Since the plasma is here pulsed, the detected current increases and then drops back to zero. The recorded signals are typically not smooth but exhibit identifiable features. By measuring the time difference, Δt , between corresponding features in the two signals, the ion velocity can be determined. Note that while the ion current waveforms may appear complex, their overall shape remains consistent from pulse to pulse, with reproducible features. To ensure the accuracy of this method, probes should first be negatively polarized to solely capture ions. The bias voltage is set here to -35 V, which is enough to repel the overwhelming majority of electrons in the plasma jet [86]. Note that the probes should be equally biased to avoid the presence of an electric field in between. They must also be far from each other for their plasma sheath and wake to not interact. Ultimately, the first probe that the plasma jet encounters should disrupt the flow as little as possible.

In that respect, a ring-shaped probe (RP) has been developed. This probe consists in a 15 mm diameter loop formed with a 0.6 mm in thickness tungsten wire. This geometry allows to collect ions without disturbing the central cone of the plasma jet. Further downstream ions are collected by a 6 mm in diameter planar probe (PP). The choice of making the RP diameter more than two times bigger than the PP diameter is to ensure that the ring does not screen the planar probe. Probes are positioned 4 cm apart and placed at different positions in the plasma plume of the VAT as explained in the next sections. A photograph of the set of probes is presented in figure 2.5a.

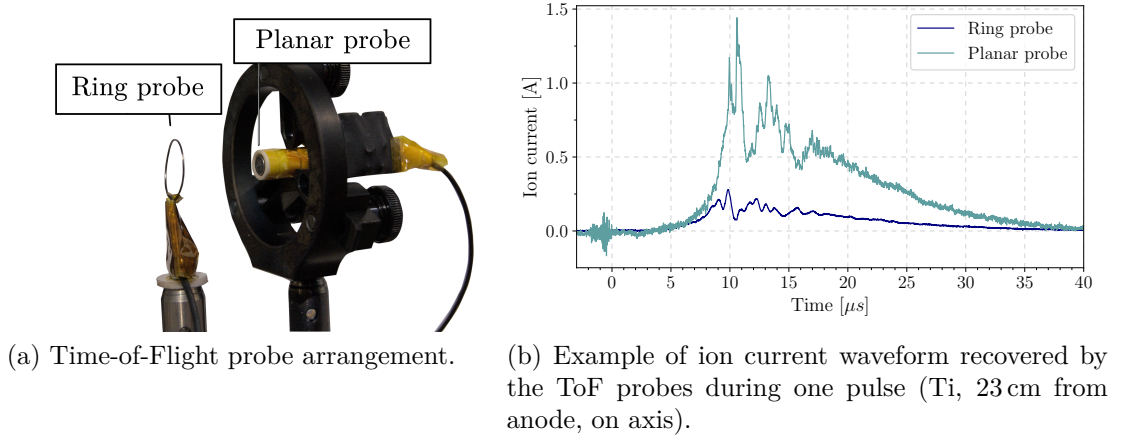


Figure 2.5: Time-of-Flight probes and typical current traces.

It is particularly important in that measurement setup that the current probes used to measure the current flowing through the electrostatic probes are strictly similar. This is the reason why two calibrated $50\ \Omega$ Pearson 2878 current transformers are placed in the circuit to monitor the evolution in time of the ion current. They can measure a maximum current of 400 A and their bandwidth spread from 10 Hz to 70 MHz, which is sufficient to recover the structures of interest in the ion current waveforms. The signal from the two probes are read from a Teledyne Lecroy 6104A oscilloscope. The power supply used to bias both the ring probes and the planar probes was a Tenma 72-10480. A schematic of the experimental setup is presented in figure 2.6. Figure 2.5b shows an example of ion current waveforms recorded with the ToF setup with a Ti cathode. As previously explained, signals are complex with well defined peaks visible on the two traces. The difference in amplitude is due to the difference in probe collection area.

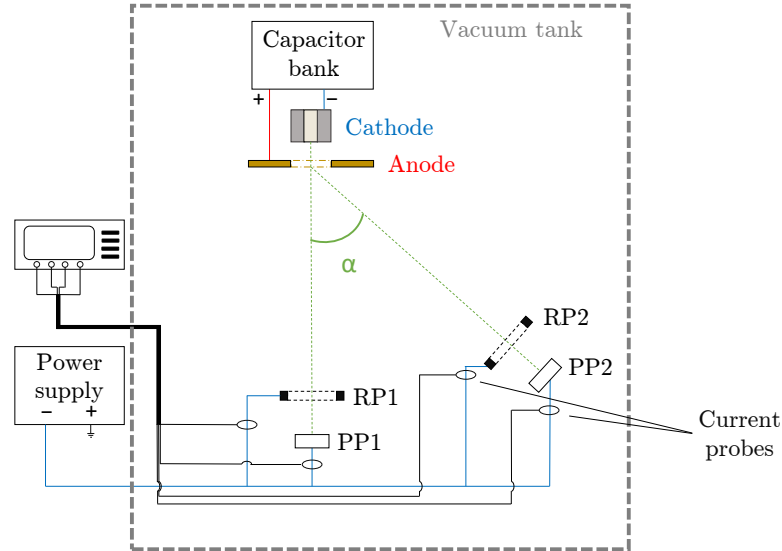


Figure 2.6: Layout of the experimental setup for measuring the ion velocity in the plasma jet of the PJP with the ToF technique. Arc triggering system not represented. Not to scale.

5 Photography and fast imaging

5.1 Long exposure photographs

The camera used to image the cathode surface during thruster operation is a Nikon D40. This camera features a 6Mpx CMOS sensor, and has been equipped with a Sigma 180 mm F2.8 macro lens to enable detailed observation of the smallest areas of the cathode. In order to further increase the magnification ratio, a Kenko TELEPLUS MC7 AF 2x lens as well as two extension tubes have been added as schematized in figure 2.7. The exposure time has been set to 1 s so that all the light emitted during the discharge is received by the sensor. In this way, one can extract the properties of all cathode spots ever ignited on the cathode surface during a complete PJP pulse. As spots emit a large amount of light, the camera ISO has been set to the lowest value available, i.e. 200, thus limiting the sensor light sensitivity. The camera captures footage of the thruster from outside the chamber, through one of the glass windows. To achieve an appropriate exposure, a neutral density filter with an attenuation factor of 10 has been mounted ahead of the lenses, as illustrated in figure 2.7. In order to avoid variations in the camera alignment while manually operating the shutter, the sensor has been triggered by means of an infrared remote control synchronized with the PJP duty cycle. The same setup and procedure apply for all cathode materials. After changing of the cathode, the thruster and the optical system were realigned to warranty sharp high-resolution images. In total, 410 photographs have been taken in this work. For each cathode material, the dataset includes about 50 pictures, a large enough number to ensure correct outcomes of a statistical analysis.

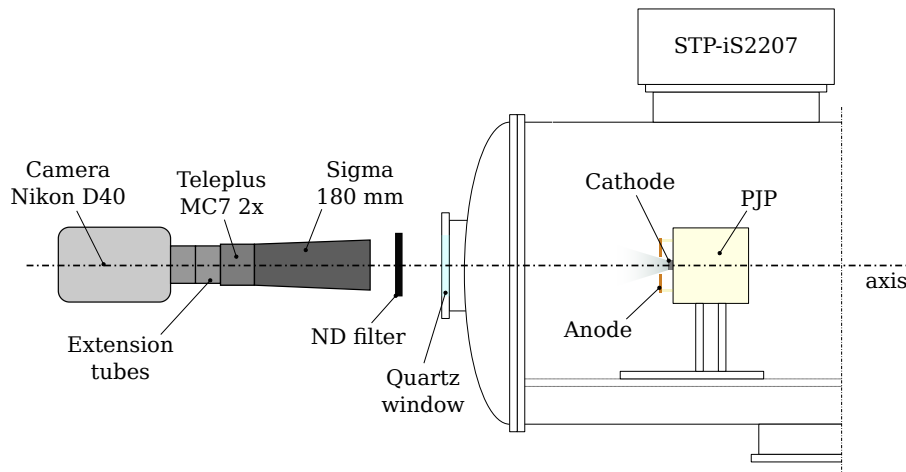


Figure 2.7: Schematic of the experimental setup used for taking long exposure photographs of the cathode spots. Not to scale.

5.2 Fast camera

The fast camera used in this work is the Phantom TMX7510. The TMX7510 features a CMOS sensor with an acquisition frequency of up to 1.7 million frames per second (fps). However, at this rate, the image resolution has to be lowered to 256×32 px. To

ensure all the cathode surface is contained in the image and that no spot initiates out of the frame, the image resolution is set in this work to 256×128 px, unless otherwise stated. This resolution leads to an acquisition frequency of 450,000 fps, which represents roughly one picture every $2.2 \mu\text{s}$. With these settings, 13 photographs are captured throughout a single PJP discharge.

In a similar manner as for taking long exposure photographs, the camera is placed outside the chamber and captures images through a window. To enhance observational capabilities, a Nikon 200 mm lens is mounted on the camera. This enable zooming on the cathode surface, so that it fills the entire camera field of view. For the same reason, the PJP is placed 30 cm away from the window, which is the closest possible position. Overall, the camera lens is positioned 90 cm away from the cathode, on a stainless steel stand as depicted in figure 2.8. Due to the large amount of light emitted during the discharges, 2 neutral density filters are installed in front of the lenses. These filters have a respective optical density of 1 and 2, ensuring that the camera sensor is not saturated. Saturation would result in a loss of information in the acquired images.

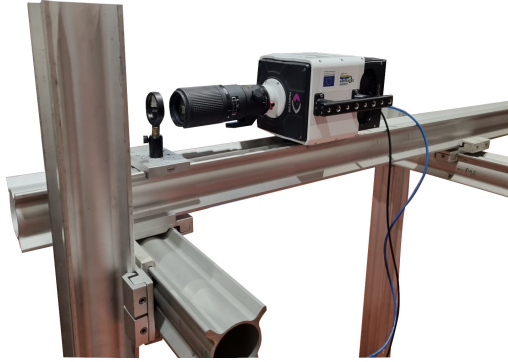


Figure 2.8: Assembly of the TMX7510 with the 200 mm lens. A neutral density filter set is placed in front of the camera.

6 Thrust balance

6.1 Mechanical design

A thrust balance, also named thrust stand, is a mechanical device used to measure the thrust generated by a thruster. In the context of plasma thrusters, these devices must be sensitive enough to recover forces in the order of magnitude of mN or even sub mN. The operating principle of a thrust balance is as follows: the thruster fires and induces a force on the pendulum, which then moves around its axis of rotation. The displacement of the pendulum relative to its equilibrium position is measured, allowing the force induced by the thruster to be determined. This assumes, of course, that the balance has been calibrated prior to the measurements, meaning that the relation between the force applied and the pendulum displacement is known. Thrust balances can take several forms, but the main types used in electric propulsion are hanging pendulum [87–89], inverted pendulum [90–92], torsional pendulum [93–96], null balance [97, 98] or double pendulum [99].

In this work, a torsional thrust stand has been developed from scratch to measure the thrust level generated by the PJP. The torsional thrust balance offers several advantages, especially when it comes to retrieve small impulsive forces:

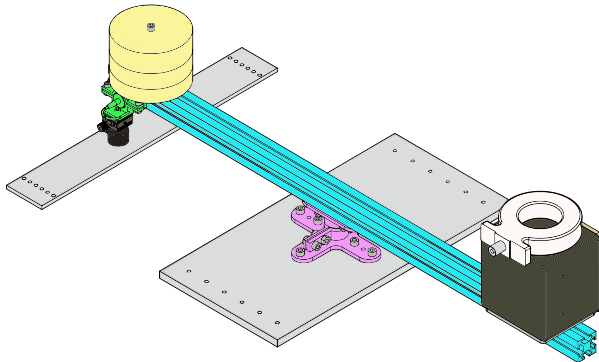
- A torsional thrust balance allows to obtain an oscillating movement from a dirac of force. This spreads the phenomenon to be observed over time, facilitating the development of the acquisition chain. It primarily makes it easier to select the adapted displacement sensor, by reducing the acquisition frequency required to measure a usable signal.
- In EPIC-2, this architecture allows to maximize the torque. Since the chamber is cylindrical, a torsional pendulum architecture takes advantage of the entire length of the chamber, not solely its diameter. By doing so, the thrust-induced displacement of the balance is increased, thereby enhancing its sensitivity.
- If the thruster is placed at one end of the balance and the displacement sensor at the other end, the electromagnetic noise induced by the engine on the displacement sensor is reduced. In the case of hanging pendulum or inverted pendulum architectures, the displacement measurement is generally performed almost at the same location as the point of thrust application [88].

The thrust stand developed as part of this work consists essentially in a Bosch profile bar mounted on a flexural pivot. Bosch profiles offer a good balance between stiffness, strength, and weight, enabling the development of a direct-type balance, meaning the thruster is directly mounted on the balance. Its lightweight design results in less inertia, making the balance easier to set in motion. The length of the bar is 80 cm, i.e. almost the length of the vacuum chamber, and is shown in blue in figure 2.9a. The flexural pivot is a Automotion R4000.I-10 showing a torsional spring rate of 5.45 Nmm/°. The pivot is held in place by the parts highlighted in pink in the figure. The main challenge in direct thrust measurements is to minimize the stiffness of the thruster's power cables. Indeed, electrical cables can create additional resistance that opposes the pendulum's movement, making it more difficult to detect small forces. To address this, the cables used to power and control the PJP were routed through connectors aligned with the axis of rotation, see figure 2.9b. This minimizes the torque applied by the cables on the balance. The cables used here are flexible, multi-strand wires with silicone housing.

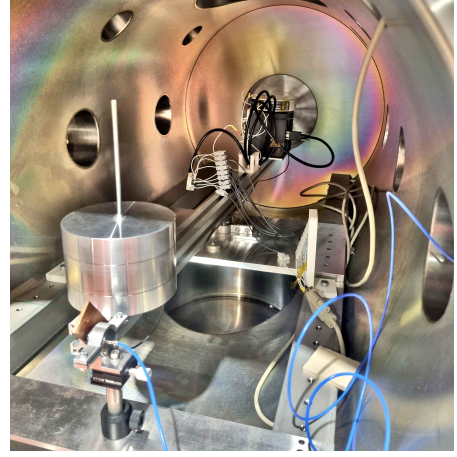
To ensure that the barycenter of the balance is aligned with the axis of rotation, a counterweight system is installed on the other side of the balance. A first weight counterbalances the PJP, and a second weight counterbalances the current probe measuring the discharge current (the white loop on top of the PJP in figure 2.9a). The set of counterweights is represented in yellow in the schematic.

6.2 Displacement sensor

To avoid disturbing the movement of the pendulum, the displacement sensor must be contactless. This is why strain gauge-style sensors are generally not used for balances aiming to detect low thrusts, in the sub-mN range. Similarly, since the balance operates in vacuum, sonic sensors are not suitable. The chosen displacement sensor for the balance presented in this work is a capacitive sensor. Capacitive sensors are



(a) Schematic of the thrust balance.

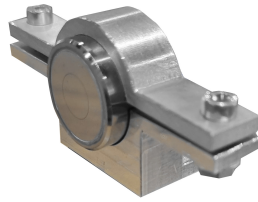


(b) View of the thrust balance when placed inside EPIC-2.

Figure 2.9: Torsional thrust stand developed in this thesis.

more precise than inductive sensors and less expensive than optical sensors, making them the ideal compromise between precision and cost. Additionally, capacitive sensors can be operated with a high sampling frequency, which is of interest when it comes to measure rapidly changing displacements.

Here, the sensor is a Micro-epsilon CSH2-CAM1.4, presenting a $[0-2]$ mm measuring range. The sensor is shown in green in figure 2.9a and measures the distance relatively to a copper target. The assembly can be seen at the bottom left of figure 2.9b. The displacement measurement is encoded from 0 V to 10 V by a DT6112/LC unit, with an acquisition frequency of up to 20 kHz. The dynamic resolution of the complete system (sensor + unit) is 0.03% of the measurement range at 20 kHz, which corresponds to $0.6 \mu\text{m}$. The signal is decoded by the unit and read on a Teledyne LeCroy HD06104A oscilloscope to achieve the best possible temporal resolution. The 2 components of the acquisition chain are shown in figure 2.10.



(a) CSH2-CAM1.4 sensor mounted on its support.



(b) DT6112/LC encoding and decoding unit

Figure 2.10: Capacitive sensor components

6.3 Calibration system

Calibration has to be performed to establish the relationship between the applied force and the resulting displacement of the pendulum. To this end, a system capable of applying a known, variable force on the pendulum is required to enable the construction of a sensitivity curve. Commonly used methods include electrostatic

actuators [94, 96], movable mass apparatus [88], and voice coils [99]. Mechanical calibration with movable masses generally offers a good precision and is quite simple to implement. However, its implementation on a torsional pendulum is challenging as such masses are inherently unsuited to the dynamics of this type of balance. For this reason, voice coil calibration was selected.

The voice coil consists of a fixed coil and a permanent magnet geometry installed on the pendulum. By passing current through the coil, a Lorentz force is generated that interacts with the magnetic field $\mathbf{B}$ of the magnet. The force produced by a voice coil is given by [100, 101]:

$$F_{em} = B_m N \pi r_c I_{\text{coil}} \quad (2.1)$$

where B_m is the magnetic flux density at the coil location, N is the number of coil turns, r_c is the average radius of the coil and I_{coil} is the electrical current passing through the coil. In this work, the moving part is composed of a pure iron core, a ring-shaped permanent magnet, and an aluminum housing. The coil is made out of a 1.32 mm copper wire (16 AWG) wrapped around an aluminium support.

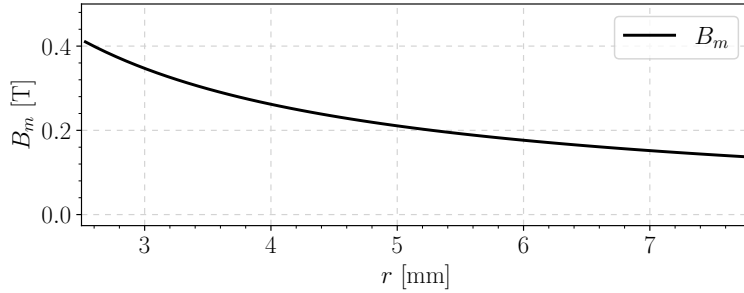
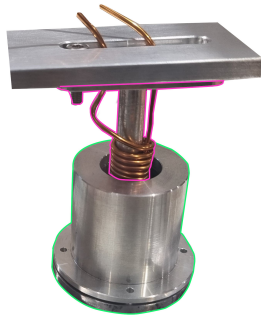


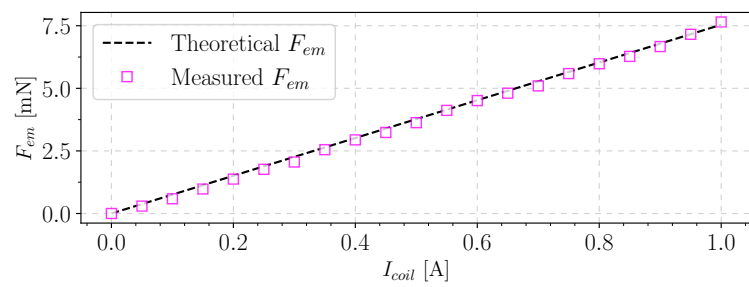
Figure 2.11: Norm of the magnetic flux density B_m over the inner radius of the moving part. The left boundary of the graph is the iron core, and on the right is the ring-shaped magnet.

A 2D-axisymmetric simulation of B_m has been performed, taking into account the properties of the magnet and the dimensions of the moving part of the voice coil. A radial profile of B_m inside the gap of the mobile part is shown in figure 2.11. The coil contains 17 turns and shows an average radius of 5 mm, meaning that B_m from the magnet is about 0.2 T at this location. A photograph of the voice coil is shown in figure 2.12a.

However, it is to note that when the voice coil is mounted on the balance, the coil section was only partially immersed in the magnetic circuit. This was done to allow a greater range of balance motion and to enable the application of smaller forces, closer to the thrust level of the PJP. N in equation 2.1 refers to the number of coil turns immersed in the region where the magnetic flux passes. In the case of the measurements presented in this thesis, N lies between 2 and 3. The theoretical curve of the electromagnetic force F_{em} as a function of the current passing through the coil is shown in figure 2.12b, for the voice coil as mounted on the balance. This force was then experimentally measured as a function of the applied current, using a precision balance. The calibration data is plotted over the theoretical model. A strong correspondence is observed between the measurements and the theoretical model, with a highly linear behavior of the force produced by the voice coil.



(a) Voice coil developed in this work. The fixed part is highlighted in pink and the moving part is in green.



(b) Calibration curve of the voice coil

Figure 2.12: Voice coil developed in this work and calibration curve.

Chapter III

Electrostatic probes in the plasma jet

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1 Langmuir probe theory

1.1 General considerations

What makes the Langmuir probe (LP) so widely used is its simplicity of construction. LP measurements basically involves placing a conductive surface in a plasma and monitoring how the collected current changes in response to the applied voltage. The resulting curve is known as the I–V characteristic, see figure 3.1. It allows plasma properties to be retrieved. To achieve this, one must apply physical models that relate the collected current to the intrinsic properties of the plasma. The shapes of the conductive surface where the LP theory apply are the following: spherical, planar or cylindrical, the ideal type of probe being the spherical one. However, manufacturing a perfectly spherical probe from refractory metals (such as tungsten or tantalum), of which the use is mandatory with respects to the temperatures at play, is extremely complex. This is why the probe used in this study is a cylindrical probe. It still provides very good results while its manufacturing is easy, and its operation is well-documented in the literature.

The first plasma property, the floating potential, is relatively easy to determine as it is defined as the potential at which $I_i = I_e$. In other words, it is the point on the I–V curve where the ion and electron contributions to the current are balanced, i.e. the bias voltage for which the total current $I = 0$ A. The plasma potential V_p is the potential where I starts to deviate from its exponential growth, where $\frac{dI}{dV_b}$ reaches its maximum. At this point of the voltage sweep, all electrons can reach the probe regardless of their energy: the probe bias voltage neither repels nor accelerates them, and no sheath forms around the probe [102].

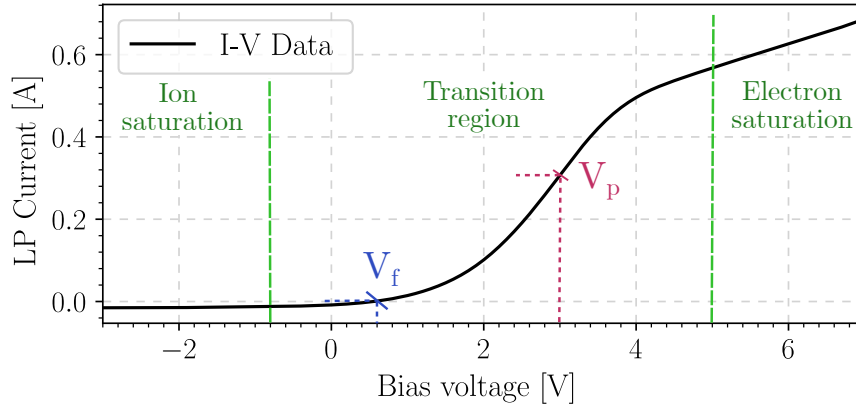


Figure 3.1: Typical LP characteristic recorded in the PJP plume, with regions identified.

The electron temperature T_e can be retrieved once V_f and V_p determined, using the so-called method of potentials. According to this method, T_e follows from the relation:

$$T_e|_{\text{potential method}} = \frac{V_p - V_f}{\ln \left(\sqrt{\frac{m_i}{2\pi m_e}} \right)} \quad (3.1)$$

where m_i is the ion mass and m_e is the electron mass. However, a more accurate method known as the slope method allows the determination of T_e from the derivative of the electron current curve:

$$T_e|_{\text{slope method}} = \left[\frac{d \ln I_e}{dV_b} \right]^{-1} \quad (3.2)$$

where I_e is the electron current to the probe and V_b the probe bias voltage. One point to note here, however, is that this method assumes a Maxwellian electron energy distribution function (EEDF) [103]. The difficulty here lies in estimating the electron current i.e. the electron contribution to the total current collected by the probe. This requires, if one approaches the problem from the opposite direction, determining the ion current to the probe. And this is precisely where the core challenge of LP data analysis emerges: identifying a theory that accurately describes ion collection.

1.2 Ion collection models

The common point among all ion collection models is that they assume a collisionless plasma with very low density. This immediately highlights the main limitation of the LP analysis: the models are, in theory, not well suited to plasmas encountered in electric propulsion. The necessary conditions for the strict applicability of the models presented hereafter are as follows [102]:

Quasi-neutral plasma: $n_i = n_e$

Collisionless: $\lambda_d \gg r_p$ with λ_d the Debye length and r_p the probe radius.

Isotropic and homogeneous

Electrostatic: $\frac{dB}{dt} = 0$

Non magnetized: $r_{L,e} \gg r_p$ with $r_{L,e}$ the electron Larmor radius.

Cold ions: $T_i \ll T_e$

Electrons at thermal equilibrium : Having a Maxwellian velocity distribution function (VDF)

These conditions must be met for LP analysis models to be valid. Only the Druyvesteyn method does not require the assumption of a Maxwellian electron velocity distribution function. However, this method needs very smooth, low-noise I-V curves as it involves analysis on the second derivative of the data. This method will not be addressed here, as it is not applicable to the curves obtained with the PJP. The three LP ion collection theories presented hereafter are based on the analysis of the ion saturation part of the I-V curve.

Orbital motion limited

The most widely used theory for LP analysis is the Orbital Motion Limited theory (OML). Originally developed by Langmuir and Mott-Smith in 1926 [78], this model assumes an infinite radius of the sheath surrounding the probe. It comes that the ions are attracted from a large distance, and that the collected current is ruled by the orbital motions of the charged particles. In practice, a large sheath implies a low plasma density and the OML theory should be inaccurate for collisional plasmas.

Despite its computational simplicity, this theory seems to lead to consistent results even when it is not theoretically applicable [103, 104], making it very popular. The equation ruling the LP collected current, according to Mott-Smith and Langmuir in their original work [78] is the following:

$$I_{e,i} = 2\pi r_p L_p e n \frac{2}{\sqrt{\pi}} \sqrt{\frac{k_B T_{e,i}}{2\pi m_{e,i}}} \left(1 + \frac{|e(V_b - V_p)|}{k_B T_{e,i}} \right) \quad (3.3)$$

with r_p the probe radius, L_p the probe length, k_B the Boltzmann constant, $m_{e,i}$ the mass of the species (ion or electron), e the elementary charge, V_b the probe bias voltage, V_p the plasma potential and $T_{e,i}$ the temperature of the species. When the probe is cylindrical, the probe collecting area is $A_p = 2\pi r_p L_p$. Focusing on the ion saturation part of the I-V curve, the current collected by the probe is purely I_i as the electrons do not have sufficient energy to overcome the potential barrier created by the bias voltage. If $T_i = 0$ is assumed, the OML equation then reads :

$$I = A_p e n \frac{\sqrt{2}}{\pi} \sqrt{\frac{|e(V_b - V_p)|}{m_i}} \quad (3.4)$$

Once V_p retrieved with the $\frac{dI}{dV_b}$ method, the only unknown constant in equation 3.4 is the density n . With experimentally measured values of I , a linear regression on the ion saturation part of the I-V curve gives a straight line of which the slope is n .

Allen-Boyd-Reynolds

Chronologically, the first theory to incorporate a sheath model was developed by Allen, Boyd, and Reynolds (ABR) in 1957 [105]. This theory was the first to solve Poisson's equation over a domain extending from the probe surface to infinity. Essentially, the potential $V(r)$ is calculated from $r = r_p$ to $r = \infty$. The problem was simplified by initially assuming $T_i = 0$, meaning that ions are attracted purely radially toward the probe with no initial velocity. This implies that there are no orbital motions in this theory. Originally developed for spherical probes, the ABR theory was later adapted for cylindrical probes by Chen in [106]. Considering a cylindrical probe centered at $r = 0$, with ions starting from rest at infinity (where $V = 0$), Poisson's equation reads:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = -\frac{e}{\epsilon_0} \left(\underbrace{\frac{I}{2\pi r} \left(\frac{-2eV}{m_i} \right)^{-\frac{1}{2}}}_{n_i} - \underbrace{n_0 \exp \left[\frac{eV}{k_B T_e} \right]}_{n_e} \right) \quad (3.5)$$

According to the analytical development explained in [79], one can normalize r to λ_d by introducing the variables $\xi = \frac{r}{\lambda_d}$, and $\kappa = -\frac{eV}{k_B T_e}$ to ease readability:

$$\frac{\partial}{\partial \xi} \left(\xi \frac{\partial \kappa}{\partial \xi} \right) = \underbrace{\frac{eI}{2\pi k_B T_e} \sqrt{\frac{m_i}{2\epsilon_0 n}}}_{\text{Normalized probe current } J} \kappa^{-\frac{1}{2}} - \xi \exp[-\kappa] \quad (3.6)$$

Since this theory involves complex differential calculations, the equations were linearized by Chen in [107]. It is observed that both ξ and J depend on the density

$n = n_i = n_e$. One way to limit the occurrence of n in the expressions is to analyze in terms of $J\xi$. The linearized equations ultimately yield the expression for the ion current in the form:

$$I_i = 0.327(J\xi_p)T_{eV}\frac{L_p}{\xi_p}\sqrt{\frac{n_{11}}{A}} \quad (3.7)$$

where T_{eV} is the electron temperature in eV, n_{11} the density in units of 10^{11} m^{-3} and A the atomic number of the atom. From there, the expression can be fitted to the ion saturation part of the I-V curve, i.e using modern optimization libraries in python. It should be noted, however, that since the ABR theory neglects angular momentum and ion orbiting around the probe, it tends to underestimate the density.

Bernstein-Rabinowitz-Laframboise

In reality, though, ions present a finite T_i and are only collected by the probe when their angular momentum is small enough to be trapped by the probe, due to orbital motion. In order to solve this problem, Bernstein and Rabinowitz developed in 1959 a theory [108] that takes into account the ion temperature, assuming mono-energetic ions. Laframboise later extended their work by making the model parametrisation for Maxwellian ion distributions at temperature T_i [109].

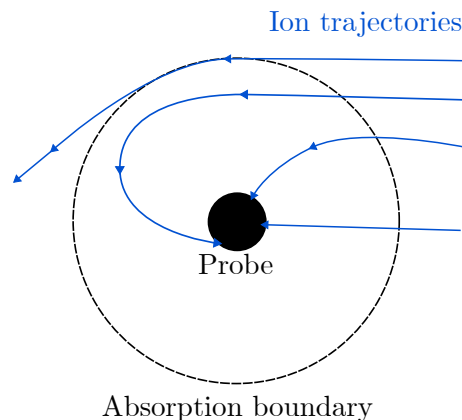


Figure 3.2: Absorption radius around the probe, depending on the bias voltage.

This so-called BRL (Bernstein-Rabinowitz-Laframboise) model takes then into account the ion angular momentum, as well as the existence of an absorption radius around the probe. Depending on the probe potential and the ion angular momentum, some ions are trapped and collected by the probe when they cross this virtual boundary (see figure 3.2). Hence, the BRL is the most complete theory presented in this study. The foundation and development of the theory are long and complex, and will not be detailed here. However, the reader can find the theoretical background in references [107–112], which span over 40 years since the theory was first formulated. What is important to retain is that the BRL model requires the parametrization of curves to be fitted to the data. These curves, known as Laframboise curves, take the form:

$$\frac{1}{i^4} = \frac{1}{(A\eta^B)^4} + \frac{1}{(C\eta^D)^4} \quad (3.8)$$

where A, B, C and D are ξ -dependent parameters calculated using functions from reference [107], and with

$$\eta = -\frac{e(V_b - V_p)}{k_B T_e} \quad \text{and} \quad i = \frac{I_i}{enA_p} \left(\frac{k_B T_e}{2\pi m_i} \right)^{-\frac{1}{2}} \quad (3.9)$$

The way these three models are applied to the experimental data in this study are detailed in the following section.

2 Acquisition method and model solving

To obtain the I-V curves, a sweep of the bias voltage is usually performed on the probe and the collected current for each voltage step is measured [113, 114]. Using modern acquisition systems (e.g. from Impedans or Hiden manufacturers), this operation takes on the order of milliseconds. The evolution of a vacuum arc is fast and stochastic [56], which makes data acquisition and analysis challenging. To see the evolution of the plasma parameters over time, a minimum of one full sweep per μs would be required as the PJP discharge lasts roughly for 30 μs . It is then impossible to retrieve the evolution of plasma parameters using the conventional method. An alternative method is proposed here to properly recover the change in time of the LP I-V curve in the course of the average PJP discharge [115].

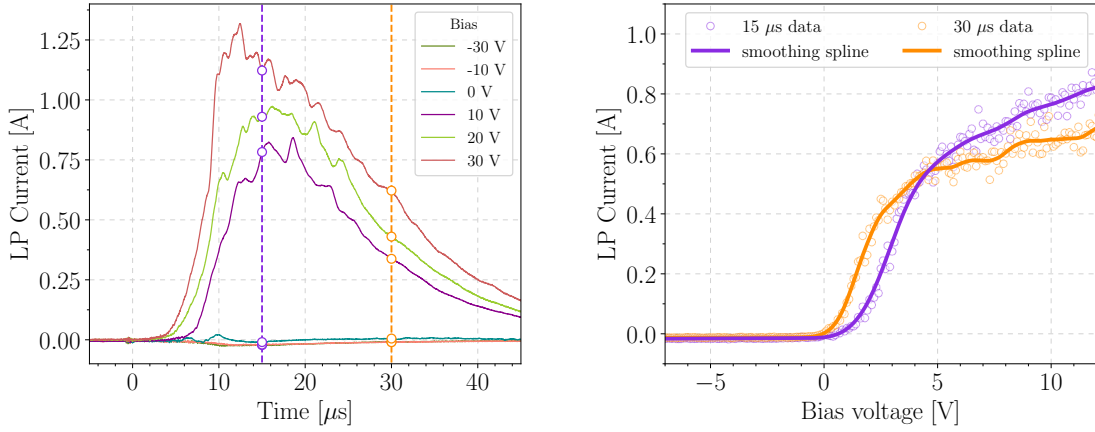
In this study, bias voltage has gradually been increased from -60 V to 60 V. For each voltage step, n pulses are triggered and each trace is recorded and averaged. This allows to depict an averaged behavior of the vacuum arc, as the probe current waveforms strongly vary from one pulse to another. Once all waveforms are acquired over the selected voltage range, a home-made Python program converts the time-current-voltage matrix into a current-voltage-time matrix. In other words, the evolution in time of the probe current for a given voltage is replaced by the change in current as a function of the applied voltage for a given time, or in short the I-V curve for a given time [115]. An example on how I-V curves are reconstructed is given in figure 3.3. As the data is noisy and scattered, a cubic spline interpolation is made on the datasets to ease visualization and analysis.

Once the I-V curves are plotted, it is possible to apply the LP analysis models. Here, the first parameter sought is the plasma potential V_p , which is the voltage where the maximum of the derivative $\frac{dI}{dV_b}$ is reached. The first order derivative of the 15 μs I-V curve interpolation spline is plotted in figure 3.4. The determination of V_p is useful for two reasons : first, it indicates a local quantity of interest for the characterization of the plasma behavior. Second, it permits to roughly estimate the electron density n_e as :

$$I(V_p) \approx I_e(V_p) = n_e e A_p \sqrt{\frac{k_B T_e}{2\pi m_e}} \quad (3.10)$$

where e is the elementary charge, A_p the probe collecting area, k_B the Boltzmann constant, T_e the electron temperature and m_e is the electron mass. For this first estimation one should fix a rather arbitrary value of T_e : 2 eV is chosen here.

This estimation is important as it sets the initial conditions of the optimization algorithm that fits the models to the dataset. For the determination of n_e , one has to analyze the ion saturation branch of the I-V curve. This part is the one where



(a) LP traces for various bias voltages. Time $t = 15 \mu\text{s}$ is shown in purple and $t = 30 \mu\text{s}$ is in orange. (b) I-V curves at $t = 15 \mu\text{s}$ and $t = 30 \mu\text{s}$. A closer look is taken at the transition region, as it represents an area of particular interest.

Figure 3.3: Reconstruction of I-V curves for different t . Current is measured 20 cm from the anode. PJP equipped with a Ti cathode.

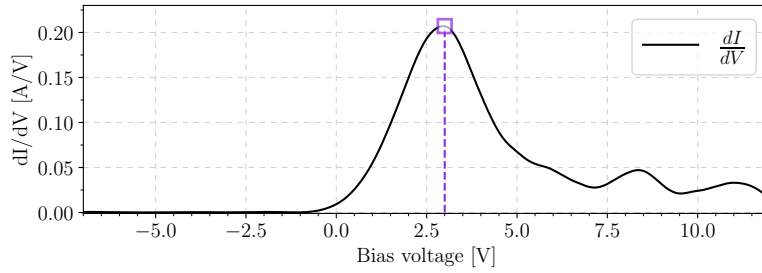


Figure 3.4: Derivative of the $15 \mu\text{s}$ I-V curve spline from figure 3.3b. The bias voltage where $\frac{dI}{dV_b}$ is maximum gives the plasma potential V_p , here 3 V.

the probed current shows a linear behavior in the negative values, see figure 3.1. To fit the models to the experimental data, values of I are squared thus becoming positive and showing a more linear behavior, easing the fitting process [107]. The way n_e is retrieved through the models is as follows: BRL, ABR and OML models are contained into separate python functions. A Levenberg-Marquardt optimization algorithm minimizes the deviation of those functions compared to the datasets by adjusting (among other optimization variables) the electron density n_e . If the starting point of this algorithm is too far from the actual n_e , the results would not converge to a consistent value. This is why, to get back to the point of V_p determination, it is important to get a correct approximation of n_e via V_p prior to the model evaluation.

Figure 3.5 shows the evaluation of OML, ABR, and BRL models on the ion part of the I-V curve. The differences between the models can clearly be seen, as OML shows a straight line while the BRL presents a knee as it approaches the floating potential V_f . It is noted that the BRL diverges at strongly negative voltages, beyond -30 V . This is quite normal because the theory is not adapted for plasmas with a ratio of probe radius to Debye length $\xi = R_p/\lambda_d > 3$ (here $\xi \approx 30$). The collected ion current here is slightly too large at negative voltages for the BRL to apply

correctly. However, the knee at V_f fits the data very well. The ABR model, on a visual aspect, is a compromise between the two. It fits better than the BRL at very negative voltages and includes a small knee near V_f , making it visually more accurate than the OML. Nonetheless, the quality of the curve fitting to the measured data does not inherently reflect the models accuracy in predicting the true value of n_e [107]. This specific point will be addressed later.

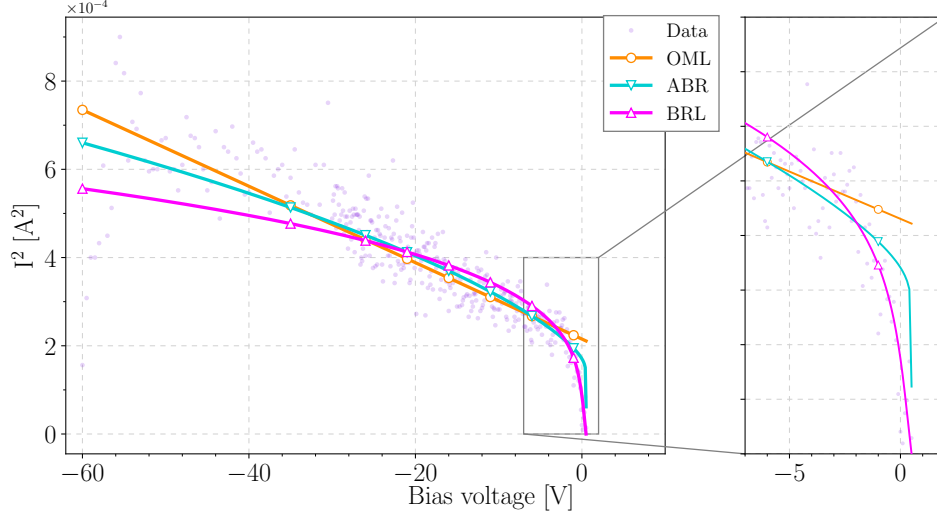


Figure 3.5: OML, ABR and BRL fit to the ion saturation part of the I-V curve.

By subtracting the ion current I_i from the probed current between V_f and V_p , and assuming a Maxwellian distribution of electrons, one can retrieve the electron temperature T_e by fitting the electron current curve. This is where the BRL and ABR models come at play once again: a good prediction of the ion current in the transition region means a better approximation of the electron current I_e . Figure 3.6 shows the electron current between V_f and V_p as corrected from the total current by the models. The expression of the electron current, assuming a Maxwellian distribution, is then:

$$I_e = n_e e A_p \sqrt{\frac{k_B T_e}{2\pi m_e}} \exp \left[\frac{e(V_b - V_p^e)}{k_B T_e} \right] \quad (3.11)$$

where n_e is the electron density previously estimated, V_b is the probe bias voltage and V_p^e the plasma potential, which is let as an optimization variable in this part of the analysis. By fitting this expression of I_e to the corrected data, one can obtain values of T_e according to the two models.

The value of V_p^e given by the Maxwellian fit differs from the V_p given by the $\frac{dI}{dV_b}$ method. The causes of these discrepancies are not really consensual but seem to be more of an analytical nature, in the sense that the models are more sensitive to perturbations near V_f [107]. This is especially true since the models have been developed with the assumption of a non collisional and low density plasma, which is not the case here. For this reason, the values of V_p given in the rest of this study are the values given by the maximum of $\frac{dI}{dV_b}$. The application of the different models allows then to retrieve, in this precise order:

1. The plasma potential V_p

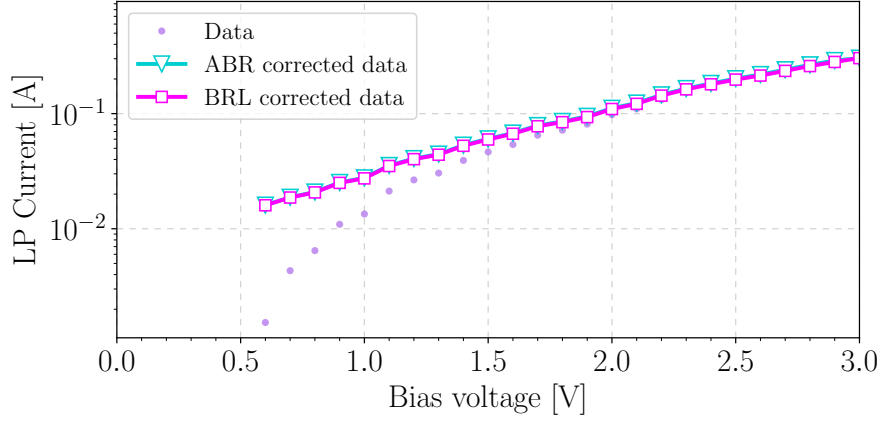


Figure 3.6: Electron current in the transition part of the I-V curve, as estimated by the ABR and BRL model

2. The electron density n_e
3. The electron temperature T_e

Influence of the model

The outputs of the different models for the $15\mu\text{s}$ I-V curve are presented in table 3.1. With the choice of determining V_p via the derivative of the I-V curve, the main difference between the models lies in the estimation of n_e . The BRL model consistently yields higher density values than the OML, while the ABR model gives lower values than the OML. In particular, ABR density values are almost an order of magnitude lower than those from BRL and OML. Despite visually fitting the data well on the graph, it provides density values that are too low compared to the experiments [107]. Chen in [107] even states that it should not be applied to cylindrical probes. Therefore, in the remainder of this study, only the results from the OML and BRL models will be analyzed. References [79, 107] show that, when comparing LP values to interferometry measurements, the actual electron density lies between the values given by OML and BRL. It also appears that the geometric mean between the BRL and ABR values yields an estimate close to reality. In our example, this would indicate a density of $2.9 \times 10^{18} \text{ m}^{-3}$. However, simply taking the average of two models lacks of solid physical foundation, we will therefore favor a separate analysis of the results from each model. In any case, the average between ABR and BRL tends to be close to the results of the OML model, as can be seen in table 3.1. Ultimately, the OML model appears to be particularly (and surprisingly [103]) reliable given its simplicity, even in cases where it theoretically should not apply, such as in RF plasmas or at relatively high electron/ion densities.

Regarding the values of T_e there is no difference between the two models that include a sheath theory, so the values presented hereafter will be those given by the BRL and ABR model. The knee it introduces to characterize the ion current I_i close to V_f makes it more precise for estimating the electron current I_e in the transition region, thus making it more accurate for fitting to a Maxwellian distribution.

Table 3.1: Outputs of the different models for the 15 μ s, 20 cm I-V curve.

Model	OML	ABR	BRL
V_p [V]	3.0	2.0	2.8
n_e [m ⁻³]	1.1×10^{18}	8.5×10^{17}	5.0×10^{18}
T_e [eV]	0.5	0.9	0.9

3 Langmuir probe experimental results

3.1 Plasma potential

As previously mentioned, the plasma potential is the first value of interest that can be retrieved by analyzing the LP curves. Here, V_p is calculated as the maximum of $\frac{dI}{dV_b}$. It is therefore based solely on the I-V curve and does not depend on the models. It represents the average electric potential in the quasi-neutral volume of the plasma. In the case of the PJP, the evolution of the plasma potential over time is shown in figure 3.7, at three different distances from the anode plane along the centerline.

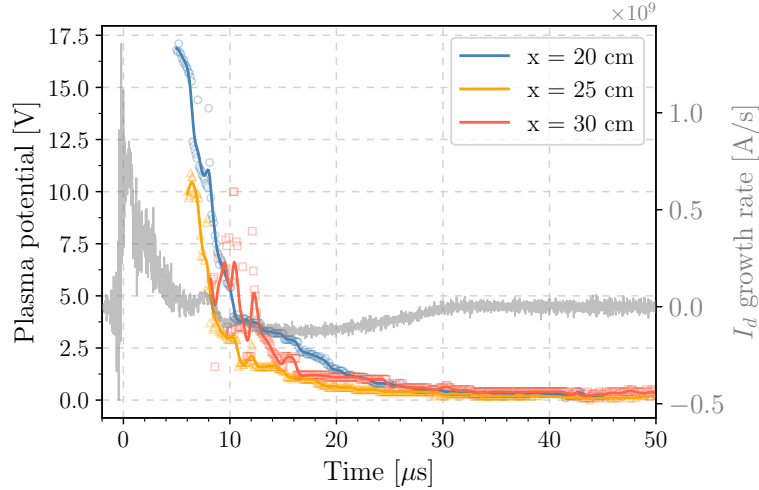


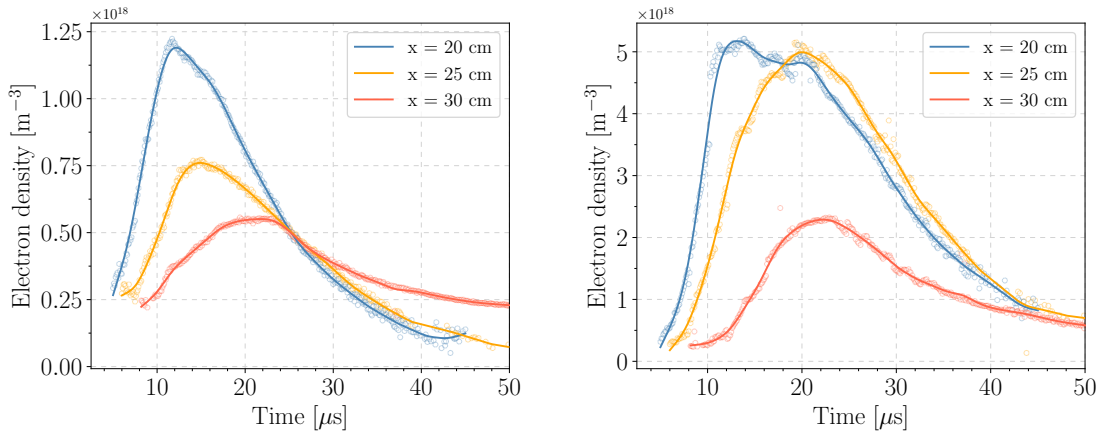
Figure 3.7: Evolution of the plasma potential V_p at different locations in the plume. The discharge current growth rate is shown in gray.

During the initial moments when the plasma reaches the probe, high values of V_p are observed, up to 17 V. This peak in plasma potential is very brief, lasting only a few μ s, and decreases very rapidly. Plotting the growth rate profile of the discharge current $\dot{I}_d = \frac{dI_d}{dt}$ (in gray in figure 3.7), one can notice a strong correspondence in shape between the evolution of $\dot{I}_d$ and the temporal evolution of V_p in the plume. Under a high current growth rate, electrons are strongly accelerated due to their high mobility, while ions take longer to be set in motion. This results in the formation of an electric field within the plasma, which is then greater when the growth rate is high. The plasma potential trace therefore follows the growth rate profile, taking into account the time shift necessary for the plasma to reach the probe. By calculating the ratio of the anode-probe distance to the time shift, the plasma ejection velocity can be estimated at approximately 40 km/s. This is of the same order of magnitude

as the ion velocity found using the ToF method, discussed in section 4. Before these points of high V_p , there is too little current collected by the probe to correctly apply the models. The blank data area at the beginning of the acquisition simply indicates an impossibility of data processing.

3.2 Electron density

Figure 3.8 shows the evolution of electron density over time at three different locations in the plume, 20, 25, and 30 cm respectively. The evolution is shown according to both the OML and BRL models. It is clear that the density profile follows the shape of the discharge current: a sharp rise, a plateau, and then a slower decay. The descent is slower than that of the discharge current, which could indicate plume-related phenomena or saturation effects in the bias power supply that delay current evacuation at high positive voltages (this can be seen in figure 3.3a). Regardless of the model used, the electron density reaches the order of 10^{18} m^{-3} , which is high at such a distance from the cathode. Surprisingly, the BRL model shows the same peak density between 25 cm and 30 cm, which likely results from analytical issues as previously discussed. From the shape of these profiles, it appears once again that the OML model is more reliable despite its simplicity. Indeed, the resulting profiles seem to have greater physical relevance, showing a clearer resemblance to the discharge current and a peak density that decreases with the inverse square of the distance. This behavior is analog as a free expansion in a low pressure medium.



(a) Evolution of n_e in the plume according to the OML model.

(b) Evolution of n_e in the plume according to the BRL model.

Figure 3.8: Time evolution of the electron density n_e at different locations in the plume, according to the OML and BRL models.

In fact, these high density values are expected, considering that the density should decrease in the plume according to the relation [52, 116]:

$$n_e = C \frac{I_d}{r^2} \quad (3.12)$$

where I_d is the discharge current, r the distance to the cathode spot (where the plasma is created) and C is a constant related to the ion erosion rate specific to the cathode material, in the order of $10^{13} \text{ A}^{-1} \text{ m}^{-1}$ for titanium [117, 118].

Given the high current, a high electron density was anticipated. Similar orders of magnitude are reported in reference [119], closer to the cathode but in a vacuum arc with a lower discharge current. To aid visualization, figure 3.9 shows the evolution of the density along the axis according to equation 3.12. When superimposing the peak density in the plume from the LP measurements, it can be seen that values are in good agreement with the theory. The curve forms an asymptote near the cathode, but its actual maximum value is expected to reach the order of 10^{23} – 10^{25} m^{-3} near the cathode spot [119, 120].

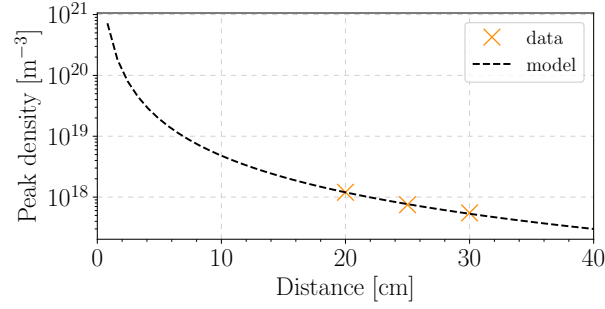


Figure 3.9: Theoretical decrease of the electron density along the jet axis.

3.3 Electron temperature

Since the models yield very similar values of T_e , figure 3.10 shows the time evolution of the electron temperature according to the BRL model. The evolution of T_e closely resembles the one of V_p , which is expected given that these two quantities are closely related as explained in section 3.1. A peak between 4 and 6 eV is observed at the beginning of the discharge, which then decreases to stabilize to a value around 0.4 eV. The value of T_e inside the steady state arc is known to be about 2–4 eV [120]. This would suggest that electron cooling occurs between the anode and the measurement point in the plume, which is not particularly unexpected as the jet expands freely. There is no external magnetic field so there is little chance that the electrons are heated by any mechanism within the plasma plume [121].

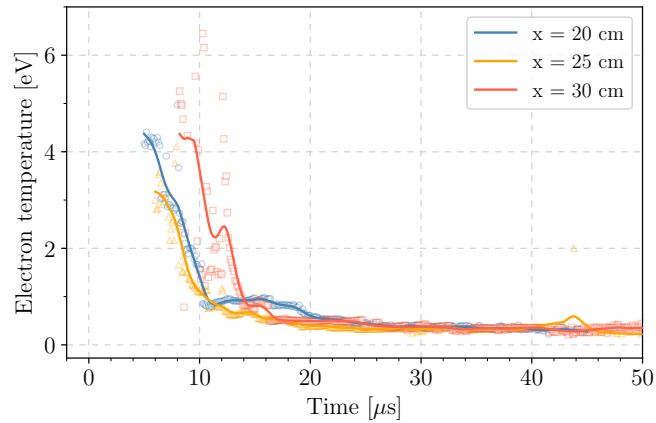


Figure 3.10: Time evolution of the electron temperature T_e at different locations in the plume. BRL model.

4 Ion exhaust velocity

4.1 Time-of-flight traces

Measurement of the ion velocity is based in this work on the time-of-flight method. It involves placing two probes at different locations in the vacuum chamber and measuring the time it takes for particles to travel the distance between the two probes. Here, the particles are first captured by a ring-shaped probe (RP) and then by a planar probe (PP). Both signals are recorded for each pulse of the PJP, and the time of flight is extracted during post-processing. At first, a cross-correlation analysis between the RP and the PP signals was considered to retrieve the ion velocity. However, first results showed that the two waveforms are not strictly similar, leading to inaccuracy while using cross-correlation. Therefore, a more favorable approach is to identify the different peaks and determine their respective velocity. In other words, it is necessary not to process the waveform as a whole, but to separately analyze its components in order to get an accurate representation of the velocity distribution [122]. With the current experimental arrangement, raw traces are quite noisy and exhibit many peaks. To ease peak identification, the signals are filtered through a 1 MHz cutoff frequency low-pass filter. Then, a normalization in magnitude is carried out to ease the comparison. Eventually, the peaks are identified with a custom-built algorithm. The different peaks are identified on the RP waveform according to fixed conditions: minimal amplitude, width, time between the different peaks and prominence, i.e. relative amplitude with respect to the surroundings. An array containing the time of occurrence and the amplitude of all peaks is created. Peaks with similar amplitude are then sought in the PP waveform, within a range defined by the cross-correlation time shift. Doing that, the results are more precise when it comes to identify the same peak on the RP and the PP. If no corresponding peak is found on the PP waveform, or if the shape of the peak is too different, the ToF is not calculated to avoid miscalculations.

Result of the signal filtering and subsequent peak identification is shown in figure 3.11. The time $t=0$ refers to the moment the oscilloscope is triggered, when the discharge current reaches 500 A. This value is set to avoid recording the electromagnetic noise associated with the activation of the PJP high-voltage trigger. The amplitude of this noise can exceed the maximum value of I_i at the probes, which induces an error while normalizing the waveform.

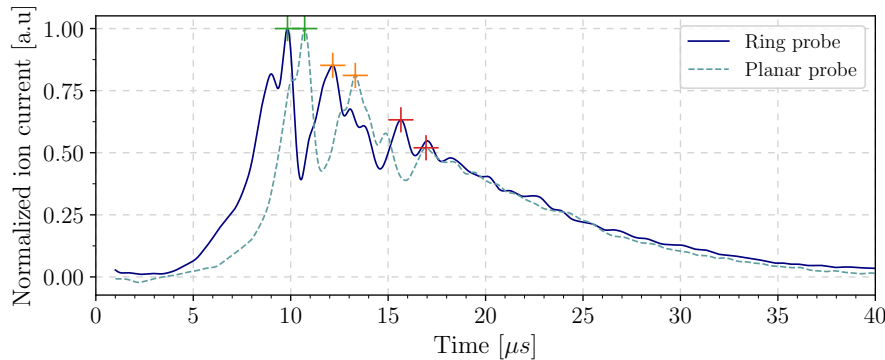


Figure 3.11: Filtered and normalized ToF traces. Colored crosses indicate the 3 main populations accounted for the velocity calculation.

Although the waveforms recovered from the ToF probes slightly vary from one PJP discharge to another, the appearance of at least 2 peaks is systematically observed. In this study, we focus on the 3 main peaks that emerge from the ion current waveform, the third one being sometimes missing. These peaks are probably different ion populations, generated at different times during the discharge. The vacuum arc appears to move across the cathode surface and this apparent motion takes place via local micro-explosions [68]. It is possible that the different peaks result from the ionization of larger volumes from the cathode material, in the course of the vacuum arc motion. The possibility that these peaks are ion populations with different charges cannot be ruled out either, although it is more unlikely. In the case of a purely electrostatic acceleration, assuming that all ions are created at the same time, the ratio between velocities should be approximately equal to the ratio between charges, which is not seen here, as we will see later. Multiply charged ions are generated in large quantities in vacuum arcs [123, 124]. Here it seems more likely that ions with different charges are created all throughout the 30 μ s of the discharge. With this assumption, ions with a higher charge state generated at the end of the discharge would strike the probes at the same moment as slower ions created earlier, making the precise identification of the charge impossible with the current method. The peaks observed in figures 3.11 and 3.16 are probably not solely due to a difference in charge then. Thus, the mechanisms leading to their observation remain uncertain.

In figure 3.11, the first identified population is indicated in green and presents a velocity of 45 km/s. Then the second population (orange) shows a 34 km/s velocity whereas the third one (red) reveals the lowest velocity, about 31 km/s. Hereafter in the study, the first, second and third peak refers to the respective time of occurrence and not to the magnitude in ion current. This example is representative of the vast majority of waveforms obtained, in the sense that the population that appears first is faster than the second, itself faster than the third one.

4.2 Titanium

Following this method, the ToF probes were placed at several positions within the plume along the PJP centerline. At each position, 100 discharges were triggered. This allows information to be gathered on both 1-the evolution of velocity as a function of distance from the exit plane and 2-the distribution of velocities for a given position within the plume. Measurements cover the [9-32] cm range. Below 9 cm, the ion current density in the jet was too high thus saturating the probes. This made data acquisition impossible with the current ToF setup, despite the very small ion collection area. One way to overcome this limitation is to manufacture even smaller probes, which was not possible during this test campaign. Another option to measure the exhaust velocities close to the anode, where the plasma density is high, is to use laser diagnostics such as laser induced fluorescence spectroscopy (LIF) [9, 125, 126]. Beyond 32 cm the study was not continued as no significant evolution was observed along the axis, as shown in figure 3.13.

Figure 3.12 shows histograms of the speed distribution of the different populations, for the PJP equipped with a titanium cathode. For these examples, the ToF probes were placed 23 cm or 32 cm downstream of the anode (distance anode-RP). The mean velocity for each population is given in table 3.2. The probability density

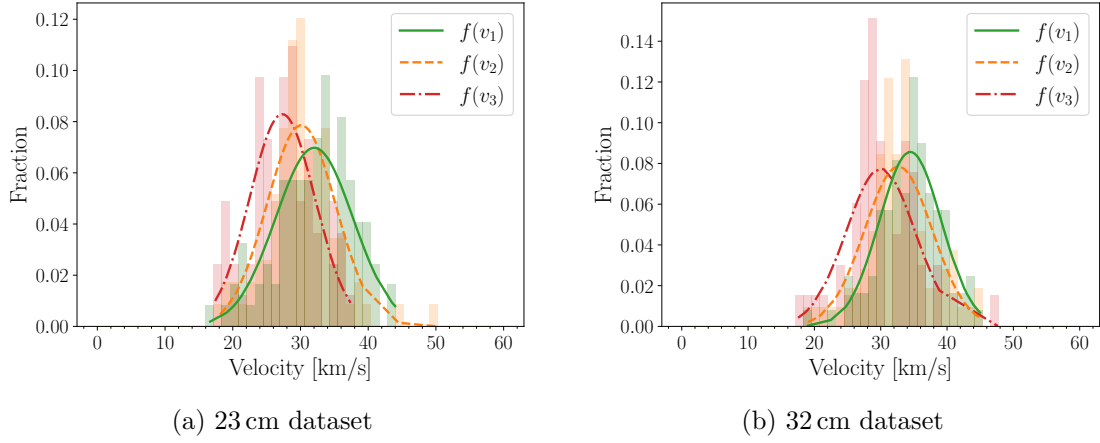


Figure 3.12: Titanium ion speed distribution, for two different locations downstream of the anode exit plane.

function is plotted for each ion population to ease visualization, as distributions can reasonably be represented by a Gaussian function. Titanium ion velocities spread from 17 km/s to almost 50 km/s, which is higher than what is usually found in the literature [127]. This high ion velocity in the plume of the PJP has already been noted with a nickel-chromium cathode in [86].

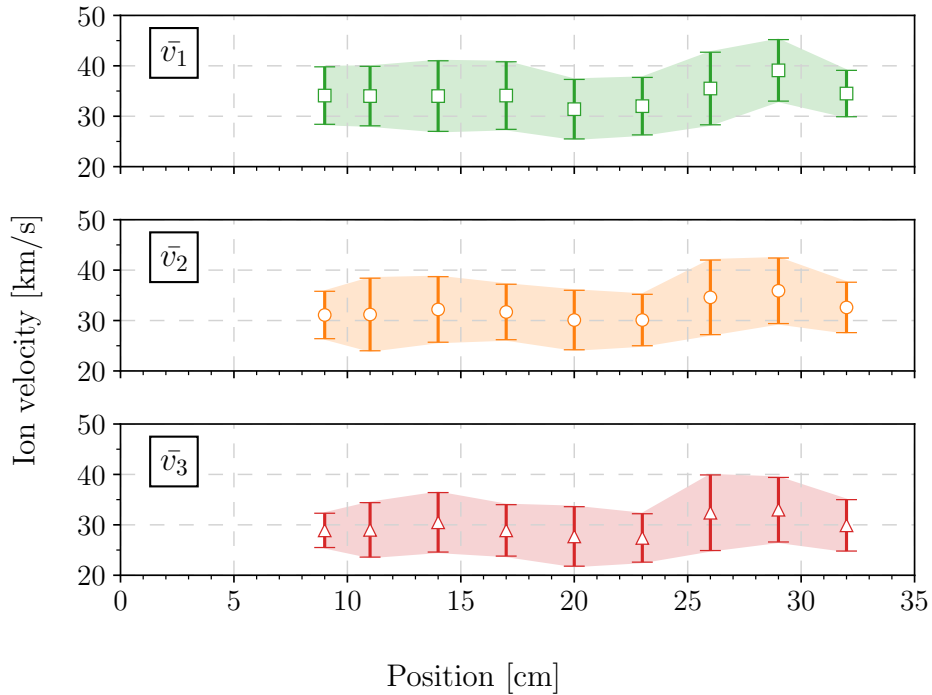


Figure 3.13: Mean Ti ion velocity along the plasma jet axis. The subscript refers to the ion population.

Figure 3.13 shows the evolution of the titanium ion mean exhaust speed, as well as the associated standard deviation for each position. $\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$ represent respectively the mean velocity of the first, second and third population in the waveform,

as exemplified in figure 3.11 and 3.12. Numerical data is given in table 3.2 for the inclined reader. $\bar{v}_1$ for titanium varies from 31 to 39 km/s with a standard deviation between 5.6 and 7.5 km/s. The rather high standard deviation exemplifies the uniqueness of each arc, making it necessary to acquire data over a very large number of pulses to extract a global behavior. These velocity measurements correspond to a mean kinetic energy ranging from 214 to 286 eV, which is way higher than in other works [65, 127–129].

Table 3.2: Values of mean Ti ion exhaust speed.

Distance [cm]	v_1 [km/s]	v_2 [km/s]	v_3 [km/s]
9	34.1	31.1	28.9
11	34.0	31.2	29.0
14	34.0	32.2	30.5
17	34.1	31.7	28.8
20	31.4	30.1	27.7
23	32.0	30.1	27.4
26	35.5	34.6	32.4
29	39.1	35.9	33.0
32	34.5	32.6	29.9

4.3 Copper

A similar analysis was conducted on the PJP equipped with a copper cathode. The main difference was that during the measurements along the axis, two points ($x=11$ cm, $x=14$ cm) had to be removed due to issue with thruster operation. Figure 3.14 shows the datasets obtained at 23 and 32 cm downstream of the anode plane. Once again, the ordering of the populations is observed, with the first one being the fastest. The average exhaust speed of Cu ions is higher than that of titanium ions, with average velocities $\bar{v}_1$ ranging from 39 to over 45 km/s over the measurement range. This is quite interesting given that copper ions are heavier and, therefore, should be less mobile than titanium ions. Further insights are discussed in section 4.6. Additionally, the standard deviations in velocity are roughly similar than the ones of titanium ions. The distributions exhibit clear Gaussian shapes, although the speeds of the third population $\bar{v}_3$ are slightly more dispersed.

Numerical data is given in table 3.3 for the complete range of measurements, and evolution of the velocity is shown in figure 3.15. These velocity measurements correspond to a mean kinetic energy ranging from 498 to 660 eV.

4.4 Influence of the ejection angle

Studying the ion velocity as a function of the ejection angle with respect to the plasma jet axis provides information on the homogeneity of the plume and on the way the plasma expands. Two similar arrays of ToF probes were then placed on-axis and off-axis at different angles from the centerline. Measurements presented in this section are taken 20 cm away from the cathode center. Following the method presented previously, examples of titanium and copper ion current waveforms are plotted in figure 3.16.

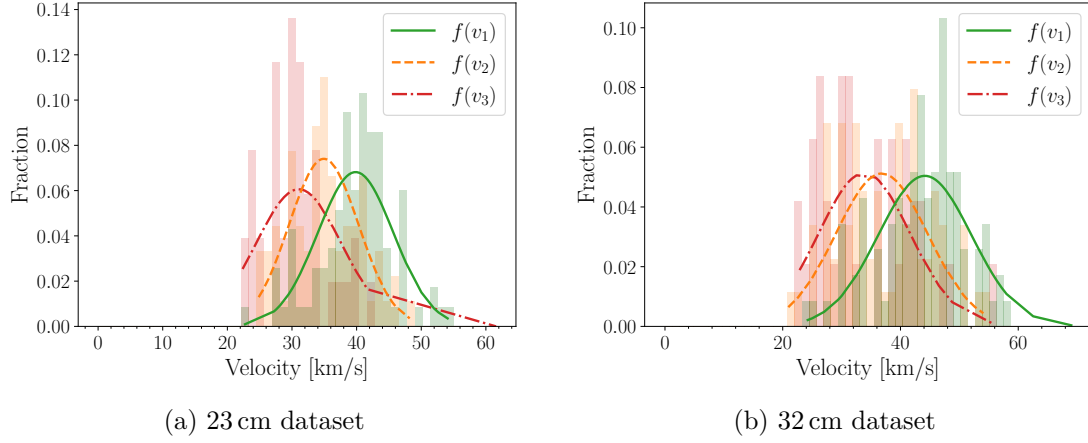


Figure 3.14: Copper ion speed distribution, for two different locations downstream of the anode exit plane.

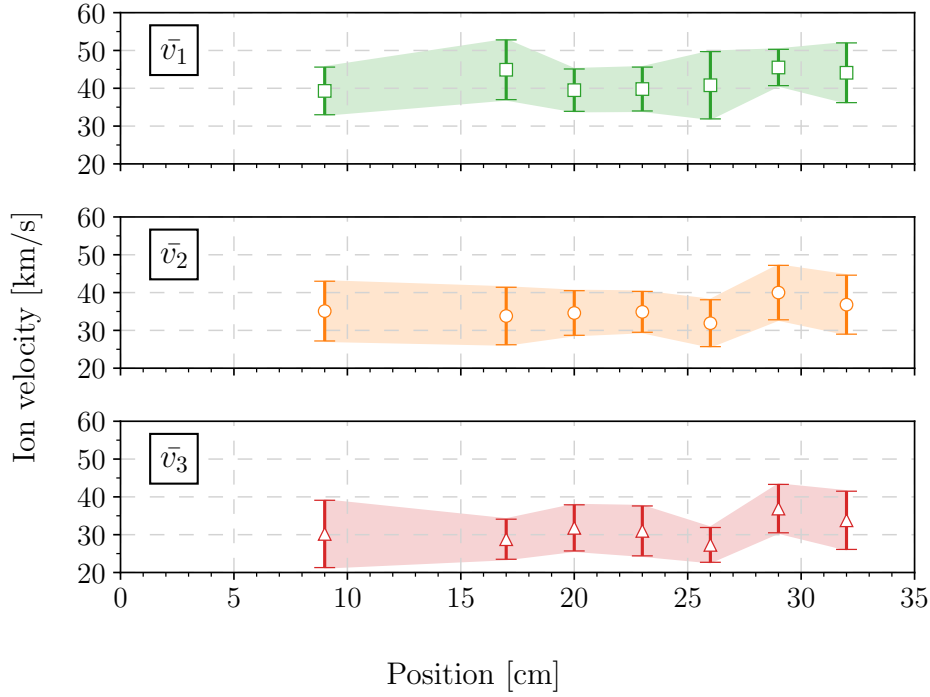


Figure 3.15: Mean Cu ion velocity along the plasma jet axis. The subscript refers to the ion population.

Waveforms recorded off-axis show several peaks but their occurrence in time is different compared with on-axis measurements. In figure 3.16a, for titanium, a ion current peak appears first on the set of probes placed at 45° (around $8 \mu\text{s}$) and then the current rises on-axis. In the example shown in figure 3.16b, where the PJP is equipped with a copper cathode, the situation is reversed. Measurements at 45° show a single main peak, whereas many peaks and a broad profile are observed on-axis. Overall, ion current waveforms recorded off-axis show similar global properties with respect to the on-axis waveforms: their typical duration is about $40 \mu\text{s}$ and several peaks are observed. The occurrence in time of those peaks varies significantly from

Table 3.3: Values of mean Cu ion velocities.

Distance [cm]	v_1 [km/s]	v_2 [km/s]	v_3 [km/s]
9	39.3	35.1	30.2
17	44.9	33.8	28.8
20	39.5	34.6	31.8
23	39.8	34.9	31.0
26	40.8	31.9	27.3
29	45.5	40.0	36.9
32	44.1	36.8	33.8

one pulse to an other. Moreover, in the course of a given discharge I_i waveforms are never the same depending on the observation location. In particular, see figure 3.16, the maximum in ion current is not reached at the same moment for the two sets of probes, even though they are at the same distance from the anode. This highlights both the anisotropy and the significant inhomogeneity of the plume.

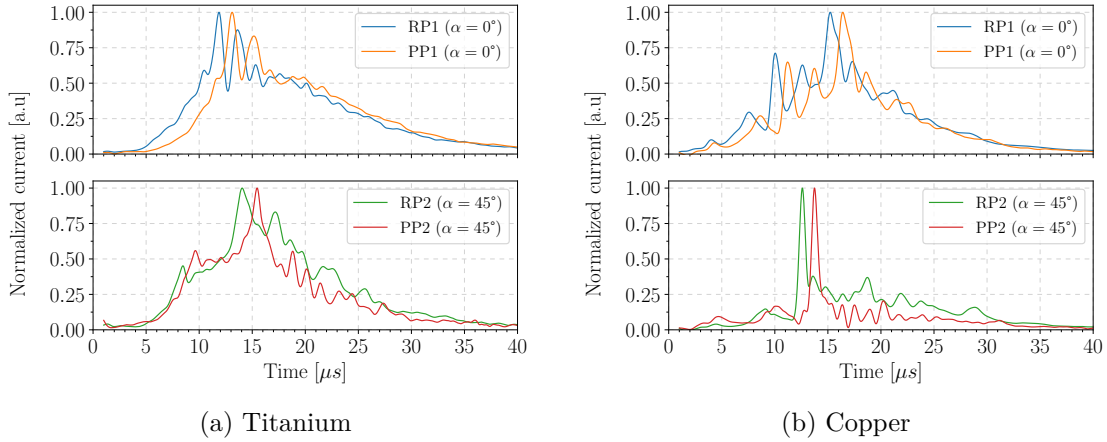


Figure 3.16: On- and off-axis ion traces (single shot) for a Ti (left) and a Cu (right) cathode.

Measurements at larger angles, see figure 3.17, present the same pattern. From time to time, ions are even faster at 60° from the centerline than on-axis. It is however noted that quite frequently the plasma does not reach the probes placed at 60° whereas the signal is measured on the axis. This shows the high divergence of the plasma plume. It also indicates that the ejection direction is not the same depending on the pulse, that direction being therefore not aligned with the thruster centerline. The explanation lies in the circular shape of the cathode exposed surface. The latter has a disk-shaped surface that allows the cathode spots to develop radially. It was observed in [130] that the spots do not cover the entire cathode surface during a pulse, as they tend to develop in branches that are not symmetrical relative to the disk center. This certainly leads to some directivity when ejecting the cathode material in the form of a plasma.

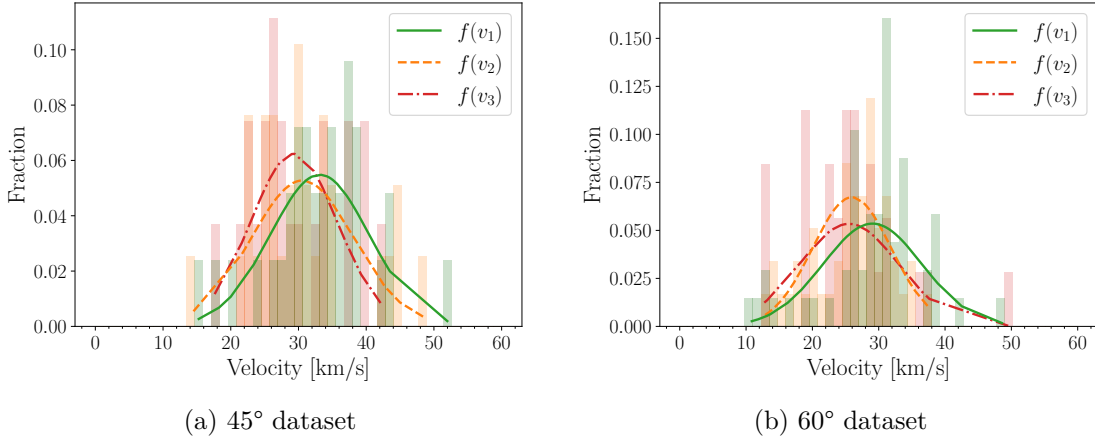


Figure 3.17: Ti ion velocity distribution 20 cm from the anode plane for different angles.

4.5 Fast ions

In total, about 16000 ToF traces have been analyzed in the frame of this work. Among them, one Ti population and four Cu populations show a velocity higher than 80 km/s. These fast populations have subsequently a very low probability of occurrence. Figure 3.18 shows the fastest population recorded with the ToF setup over the whole dataset. The measured velocity reaches 160 km/s for Cu ions which corresponds to a specific impulse of 16310 s. As a comparison, the highest measured velocity for the Ti ions dataset is 85 km/s.

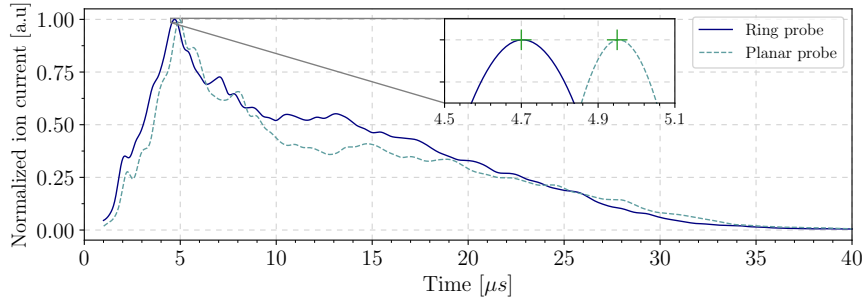


Figure 3.18: ToF traces showing the fastest ion population observed. Cu cathode, 9 cm away from the anode.

Those fast ions are probably multiply-charged ions, that are known to be produced in a large number in vacuum arc discharges [124, 131–135]. By definition, these multiply charged ions are subject to a higher electrostatic or electromagnetic acceleration in the inter-electrode gap. As can be seen in figure 3.18, according to the population velocity and the occurrence time, these ions are created during the steady state arc regime and not during the spark [86, 124]. Their presence may be due to a sausage-type or pinch instabilities leading to local micro pinches that heat the plasma [136, 137]. These kind of instabilities usually need a high current to arise (i.e. > 1 kA), and could explain why the apparition of this fast population occurs

when the discharge current is almost at its maximum. Note that theoretical models allow pinch effects in the context of high current vacuum arcs [138].

4.6 Comparison and outcomes

The first observation that stands out when examining the velocities of copper and titanium ions is that their speed is significantly higher here than in the literature. Table 3.4 provides values from various references. It shows that, in average, titanium exhibits a velocity of 20 km/s, while copper is closer to 15 km/s. The ions probed in the present work, with the PJP, are approximately 1.5 to 2.5 times faster. Two important parameters should then be noted here. First, the discharge current of the PJP is about ten times higher than that of the experiments set up to study vacuum arcs in the given references. Yet, the correlation between the increase of discharge current and the increase of the ion velocity has already been observed for lower current vacuum arcs in ref. [85]. This leads us to the second parameter of interest: the growth rate of the discharge current $\dot{I}_d = \frac{dI_d}{dt}$.

Table 3.4: Values for the velocities of Ti and Cu ions generated by various vacuum arc experiments.

Reference	v [km/s] Ti	v [km/s] Cu	I_d [A]	$\dot{I}_d$ [A/s]
Yushkov et al. [127]	22	12	200	0
Byon et al. [128]	17	15	100	0
Hohenbild et al. [85]	20		200	2×10^6
Zhuang et al. [139]	18		80	5×10^6
This work	32	36	3000	9×10^8

The increase in the growth rate has been linked to the increase in the energy of the ejected ions. Particularly, when exceeding a threshold value of 3×10^8 A/s, the increase in the energy $\bar{\epsilon}_i$ of the ions seems to follow a power law $\bar{\epsilon}_i \propto (\dot{I}_d)^\alpha$ with $\alpha \approx 1.3$ [140]. Far below this value, the growth rate does not appear to play a predominant role in the evolution of the ion velocity. For the PJP, it is approximately 4×10^8 A/s in average during the rise time. Referring to figure 3.19, copper $\dot{I}_d$ is slightly higher than for titanium, rising up to respectively 9×10^8 A/s versus 8×10^8 A/s. Once again, these values are significantly higher than those in the cited references, also explaining the difference in velocity. An important point of this model is how the growth rate influences the ion energy. It suggests that the acceleration mechanism no longer relies on a classical electrostatic acceleration where $\bar{\epsilon}_i \propto Z$ (charge number), but rather on an energy transfer through electron-ion friction. In fast current rise regimes, electrons react much faster than ions. Since the electron-ion friction is proportional to the difference between electron and ion velocities, heavier, less mobile ions acquire more energy than lighter ions. This mechanism is therefore more mass-dependent than charge-dependent. It explains why here, copper ions, being heavier and subjected to a higher growth rate, acquire more energy than titanium ions. In the end, this mechanism shows a tendency towards equalizing the velocities of the different sorts of ions, for a given growth rate [140]. It should be noted that in table 3.4, the rate of rise values are provided

for indicative purposes when the ion velocity was measured by modulation or by perturbation/spike of the discharge current. For the others, the measurements were carried out during the steady-state period of the arc, operated over hundreds of μs .

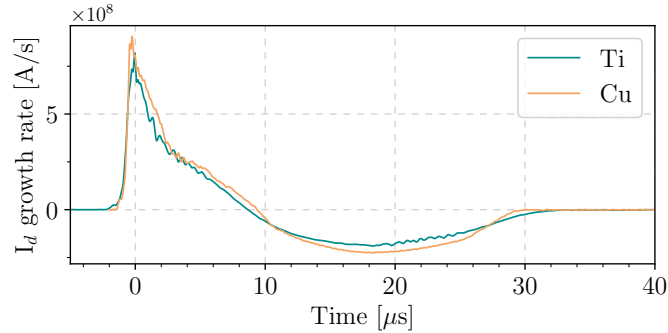


Figure 3.19: Evolution of the discharge current growth rate for Ti and Cu during a PJP discharge.

Another interesting observation is the significant dispersion of velocities within each population. On that specific point, there are so many voltage fluctuations and instabilities that no conclusive answer can be given. This becomes evident when looking at the variations in ion current traces. Although the discharge current is highly repeatable from one pulse to another, significant differences are observed in the current collected by the probes. The plume itself is highly non-homogeneous due to plasma instabilities, recombination phenomena, locally strong fields, and ion-electron coupling.

Concerning the ordering of the population depending on their velocity, it is noted that the first peak appearing in the waveform is the fastest in average, and the last one is the slowest. Assuming ions are created throughout the discharge, peaks occurring at the beginning of the discharge are faster than the ones appearing in the middle although the maximum in I_d is not yet reached. This ordering is observed in the overwhelming majority of the discharges. There is likely a mix of phenomena here to explain this ordering. The change in $\dot{I}$ is surely a key driver in this observation, but another phenomenon comes at play. As the discharge lasts, metal evaporation from macroparticles and vanishing cathode spots fill the inter-electrode gap with neutrals and reduce the ion mean charge [85, 123]. This naturally leads to a decrease in the mean ion velocity. Moreover, as the capacitor bank discharges, the voltage drops between the cathode and the anode [65]. The external electric field is then gradually reduced and ions experience less electrostatic acceleration from the anode, when the exit plane is passed. A few exceptions can however be found for some pulses. For example, in figure 3.12a the fastest ions are around 50 km/s and they correspond to the second population that appears during the discharge. The ion acceleration phenomenon is therefore complex, and it is difficult to identify a single mechanism that predominates in the acceleration of ions within the plume itself.

Finally, no significant spatial evolution of the ion speed is observed in the probed region. This indicates that the acceleration process takes place close to the cathode-anode region, which is consistent with previous studies [65, 85, 127–129, 141, 142]. Most references in the literature indicate that the ion acceleration near the spot seems to be mainly caused by the electron and ion pressure gradients, and electron-ion coupling.

Chapter IV

Fractal analysis of long exposure images

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1 Motivation

The vacuum arc is an intrinsically chaotic, complex, and highly non-stationary phenomenon. This complexity manifests both in the temporal and spatial domains. Among the many aspects that illustrate this, one may note: the inhomogeneity of the cathodic arc plasma, instabilities related to the high current, noise in the ion current within the plume, and the cathode spot process. The latter is particularly significant, as the spots are the very regions where the plasma is emitted. Numerous approaches have been attempted to characterize these extremely dense and energetic zones, the main directions of which were outlined in the first chapter of this thesis.

One way to explore highly complex phenomena is through fractal analysis. Fractals are omnipresent in science, across fields as diverse as biology, geology, and finance [143, 144]. Although the concept of fractality originates in mathematics, it is frequently encountered in physics. Fractals are self-similar objects, whose properties remain invariant across different scales. The most familiar examples include snowflakes or lightning branches in a stormy sky, whose structure appears infinitely intricate and composed of recurring patterns. Evidence of the fractal nature of vacuum arcs has been found in the noise of the ion current and in the traces left by cathode spots on the cathode surface [67].

Studying vacuum arc cathode spots through the lens of fractal analysis makes it possible to quantitatively assess their overall geometric complexity, even though they do not conform to classical or regular patterns. The aim of this chapter is to explore the insights that can be gained from fractal analysis on the cathode spot patterns during a complete discharge.

2 Fractal dimension

2.1 Image processing

Figure 4.1a shows a long exposure photograph of PJP discharge triggered with a titanium cathode, see chapter II for details about the setup. The physical boundaries of both the cathode and the anode are highlighted in figure 4.1b. The dendritic nature of the plasma emission sites throughout a PJP discharge can clearly be seen from there. The luminous light blue structures extend from the inner edge of the cathode to the outer edge. One can note that the dendrites are more sparse and less luminous near the outer edge of the cathode. The cathode resembles a hollow disk when seen from the front. The center of this disk corresponds to the insulator, where a reddish glow is observed as can be seen in the photograph of figure 4.1. In the latter, parts of the internal edge of the anode are illuminated, and the reddish glow at the bottom right part certainly corresponds to a so-called anode mode [65].

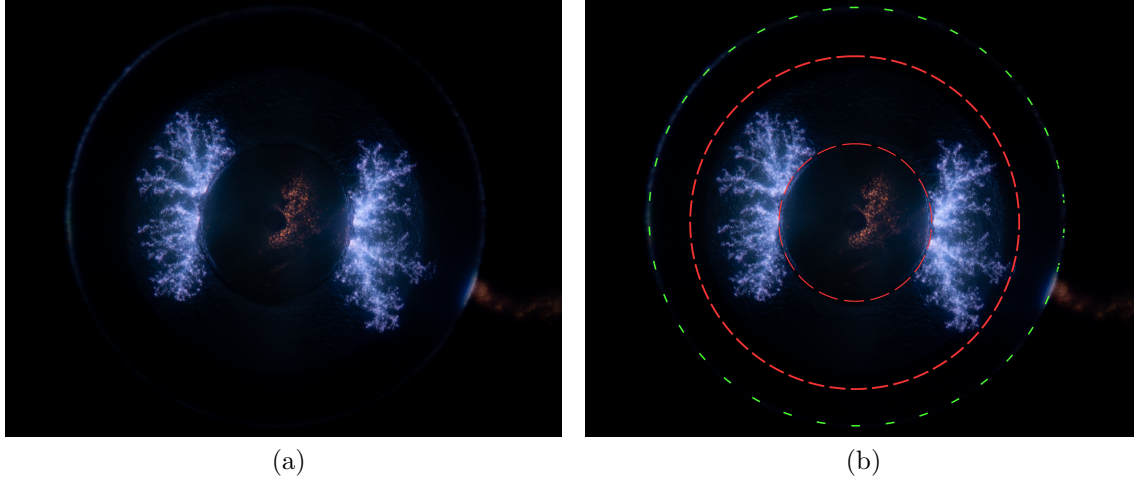


Figure 4.1: Raw photograph of a vacuum arc on a titanium cathode (left). Red densely-dashed and green loosely-dashed lines indicate cathode and anode boundaries respectively (right).

An image processing algorithm must be able to identify the structure of interest prior to further analysis. Here, the structure is the cathode spot traces, i.e. points with high light intensity. However, other bright zones appear in the pictures as previously discussed. It is then crucial to remove those parasitic elements before image analysis as they can perturb data treatment and lead to erroneous results and conclusions. This situation is exemplified in figure 4.2. First, for most VAT discharges anode modes are observed which leads to a glow in the anode region [65]. The anodic arc area is circled in yellow in figure 4.2. Second, a glow is visible on the center-mounted insulator due to a breakdown on its surface. The metal deposition that accumulates on the insulator during PJP operation is ionized at the early stages of the discharge, when the high voltage trigger is activated. This area, which is circled in red in figure 4.2, is not considered as part of the cathode spot area as ionized particles do not originate directly from the cathode body here [64]. Last, the cathode spots illuminate the outer edge of the central insulator, green rectangle and enlarged image in figure 4.2. This zone could be mistaken with the cathode spots by the algorithm and must be removed. The three described areas have been manually covered with black pixels by means of an image manipulation program. Note the inner edge of the anode, see figure 4.2, is also removed. Plasma jet direction is indicated to ease representation of the phenomena for the reader.

The light halo generated by the arc, see figure 4.2, must also be treated before image processing. This halo is the diffuse light seen around the cathode spots, between the dendritic branches. It mainly results from the intense light emission of the arc, but misalignment of the camera and optics also favors its appearance. During image processing the picture is transformed into a binary matrix, indicating whether the pixel belongs to the set of cathode spots or not. This halo being especially intense in the space between the branches of the structure, the fractal nature of the arcs is lost if the halo is kept. Keeping this halo would then distort the calculation of the fractal dimension.

To decrease the impact of the luminous halo and extract the cathode spot traces, the solution is to fine tune white balance and tonal curves of each photograph.

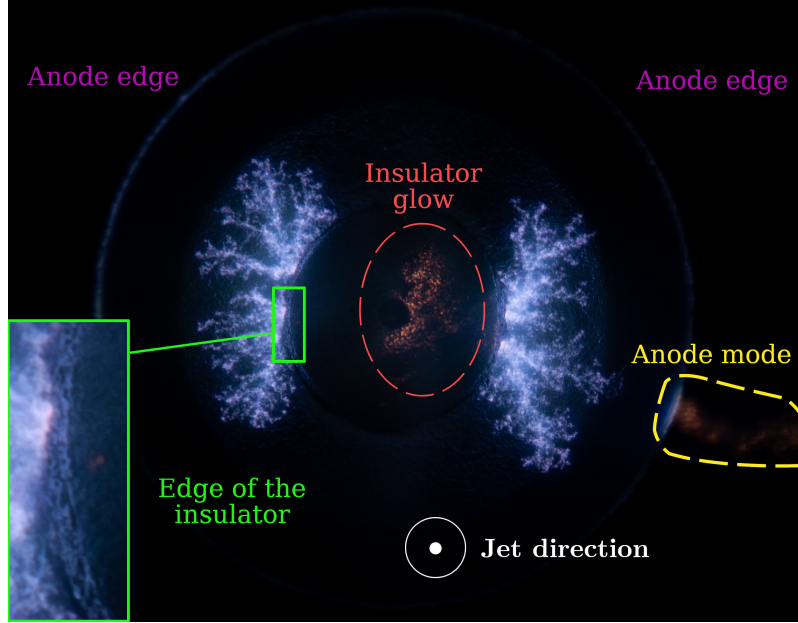


Figure 4.2: Zones of possible mistakes while algorithmically searching for cathode spots in a photograph of a vacuum arc. Image in figure 4.1 is used as a reference.

This operation permits to sharpen the color difference between the trace and the halo. The color components of the halo are strongly desaturated with regard to the ones of the spots, reducing its brightness. Afterwards conversion to gray scale is performed. The result is a gray-scale picture of the arc discharge in which the spot traces are sharper than in the original picture, without any additional numerical filter. Outcome of preprocessing is illustrated in figure 4.3 for the vacuum arc image presented in figure 4.1a.

Before determining the fractal dimension, the image must be turned into a binary matrix. In this study each pixel of the preprocessed images is associated with a value that varies between 0 and 1 according to a 8-bit gray scale. This numerical value is given by the normalized brightness of the pixel. A brightness threshold is then defined for each material: above this threshold the pixel is considered as part of the spot pattern and the value of 1 is assigned to the pixel. Below the threshold, the value 0 is assigned to the pixel as it does not belong to the pattern. The result is a binary matrix that is used for the calculation of the fractal dimension D of the spot structure. Threshold is given in table 4.1 for the different cathode materials. In order to assess the threshold, gray scale images are compared with binary images. The threshold is defined as the value that gives a binary image representative of the real image. When the threshold is low, the overall aspect of the image is changed. When it is high the image is smoother and the dendritic geometry tends to vanish. A precise value is hard to define for this threshold, fortunately it has a moderate impact on the fractal dimension. In the case of Ti, for instance, the change in D is only of 1% when considering the lowest and the highest value of the threshold, which is below the standard deviation for D as it will be seen in the next section.

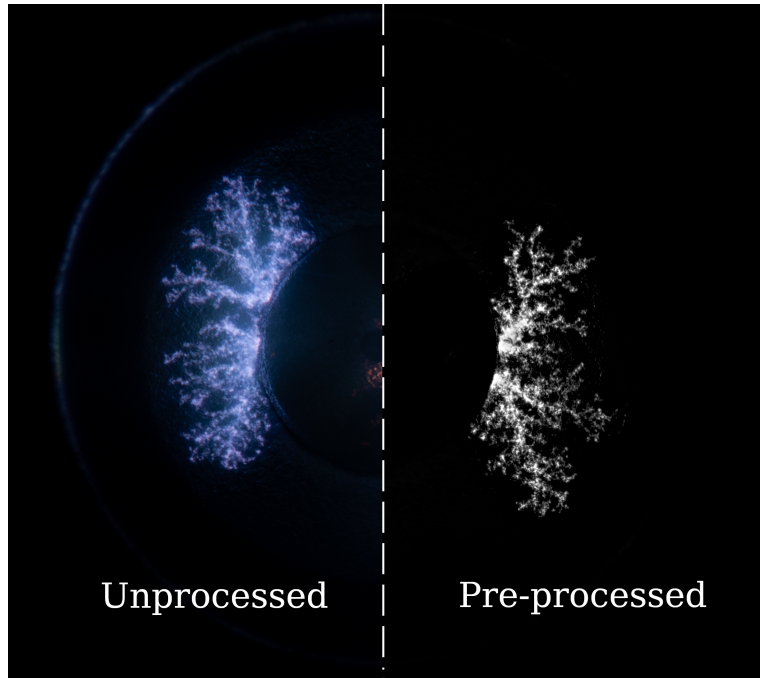


Figure 4.3: Vacuum arc image before (left) and after (right) preprocessing.

Material	Threshold
Ti	0.30
Al	0.25
Ni-Cr	0.40
Fe	0.35
Brass	0.48
Cu	0.40

Table 4.1: Threshold for different cathode materials.

2.2 Minkowski dimension

To recover the fractal dimension of the cathode spot luminous traces, the Minkowski-Bouligand method, also known as box counting method, has been used. According to this method a fractal S is covered with an evenly spaced grid and the number of squares required to cover S is counted. The dimension is calculated by evaluating how this number changes as the grid is made finer. The Minkowski-Bouligand dimension, or box-counting dimension, is defined as:

$$D_{\text{box}} = \dim_{\text{box}}(S) = \lim_{\epsilon \rightarrow 0} \frac{\log N(\epsilon)}{-\log \epsilon} \quad (4.1)$$

where $N(\epsilon)$ is the number of squares of side length ϵ required to cover the structure [145, 146]. The box counting process is interrupted here when ϵ equals the size of the camera pixel. The camera sensor contains here 6×10^6 pixels, so multiple iterations can still be made before reaching this limit. As the mesh is refined, N is calculated for each step, eventually the $\log(N)$ versus $-\log(\epsilon)$ curve is plotted and a line is fitted to the dataset. The slope of the linear fit gives the fractal dimension. Details about the computation methodology can be found in reference [147] for the inclined reader.

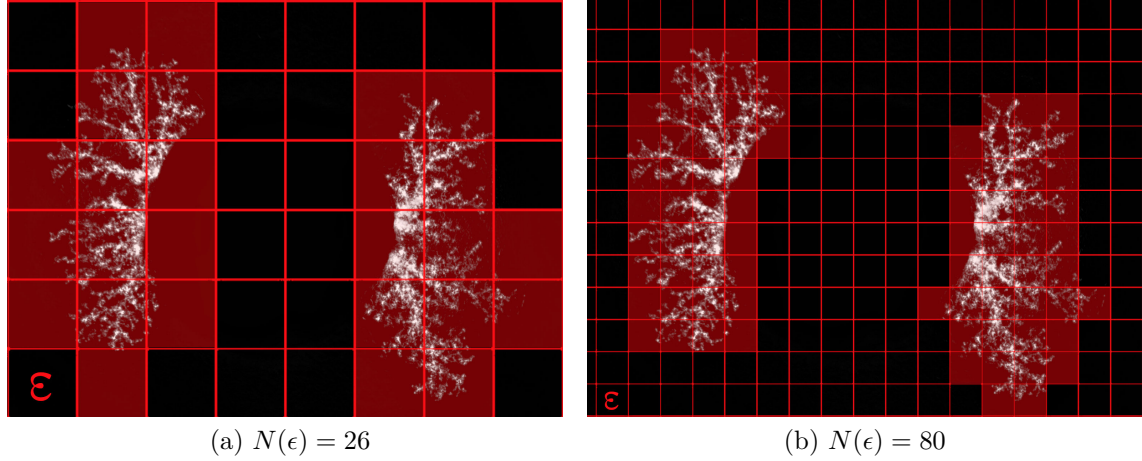


Figure 4.4: Iterations of the box-counting method on the example photograph.

Two iterations of the Minkowski method on the image in figure 4.1a are shown in figure 4.4. Figure 4.5 shows the curve obtained for that same photograph, along with the associated linear fit. As can be seen points are well aligned. The R^2 value is 0.996. On this example the fractal dimension is $D_{\text{box}} = 1.448$, a value close to the dimension of the Viscek snowflake and the quadratic Von Koch curve type 1.

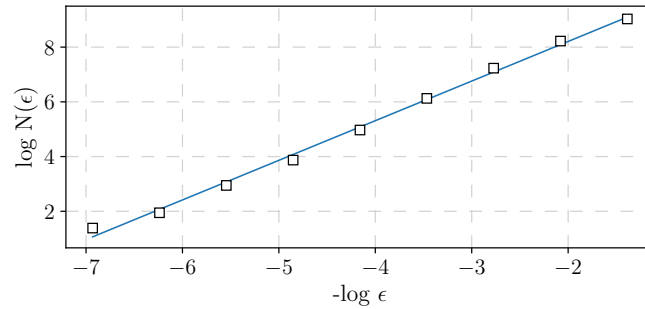


Figure 4.5: Plot for fractal dimension of vacuum arc image shown in figure 4.1a obtained with the box-counting method: $D = 1.448$.

The Minkowski dimension is only one possible way to characterize the complexity of a fractal. Notably, the Hausdorff dimension D_H and the similarity dimension D_{sim} are often mentioned in works dealing with fractals in physics [148–150]. When these different concepts are applied to the same fractal, the resulting dimension slightly varies. This is due to the fact that the mathematical definition of these dimensions differ, which in turn affects the results for a same measurement. The Hausdorff dimension requires a rigorous mathematical definition of the set to be characterized and is very difficult to implement in our case. The similarity dimension, on the other hand, assumes a strictly self-similar fractal. This would mean that the photograph as a whole is composed N perfect copies of itself, which is not the case here. The Minkowski dimension was therefore chosen here for its ease of implementation and its broad applicability.

2.3 Cathode material and fractal dimension

Figure 4.6 shows photographs of the vacuum arc cathode spots for the six different cathode materials. While for titanium the spots show large branches that separate into small dendrites, for copper and brass the sub-structures are much less developed and they form small and compact islands, especially for copper. Aluminum exhibits a pronounced luminous halo, which makes the determination of its fractal dimension somewhat more challenging compared to the other materials. The Cu-Zn alloy (brass) appears to share geometric characteristics with copper, which is the dominant element in that cathode composition, accounting for 60%. A common point between all images is that the spots seem to develop and propagate mainly in the radial direction with the external edge of the central insulator as origin. It should be noted that when more than one spot is active, which is the case here, the direction of new ignitions is influenced by the magnetic field of the discharge current paths (active spots) [68, 151].

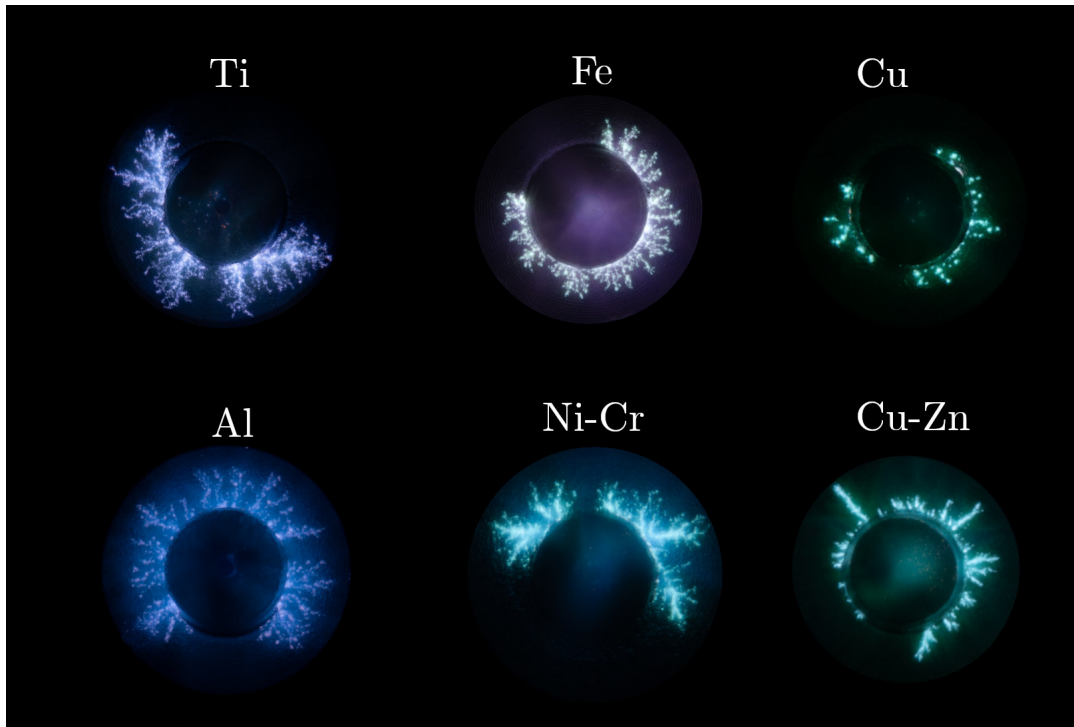


Figure 4.6: Raw photograph of cathodic arcs using different materials.

The mean fractal dimension determined with the box-counting algorithm for each cathode material along with the associated standard deviation are presented in table 4.2, sorted in descending order. As can be seen, the standard deviation remains low for all materials with a relatively constant magnitude. Moreover, except for Al and NiCr datasets, each element has a D_{box} value outside the $D_{\text{box}} \pm \sigma_D$ interval of the other elements. It means that there is a significant difference between the fractal structures of all studied materials. Titanium has the highest D_{box} whereas copper exhibits the lowest.

Material	D_{box}	σ_D
Ti	1.450	0.028
Al	1.329	0.038
Ni-Cr	1.326	0.034
Fe	1.288	0.019
Cu-Zn	1.198	0.032
Cu	1.145	0.021

Table 4.2: Mean fractal dimension and standard deviation for different cathode materials.

It is known, referenced and usually admitted that the cathode spot patterns depends on the surface condition, i.e. the degree of contamination by oxides [152–154] and the roughness [67, 155, 156]. In order to verify if the fractal dimension is impacted by the cathode surface properties under our arc condition, experiments have been performed with a brand new Ti cathode and a worn-out cathode exposed to more than 10^4 discharges. A set of 100 photographs has been collected and analyzed for the two cathodes. The fractal dimension is given in table 4.3. It results that there is no significant differences between the two values as they overlap when accounting for the standard deviation. Two explanations can be proposed then. Either the surface roughness and state of the two cathodes are comparable in spite of a very different exposure to the high current plasma discharge, or there is indeed no influence of the surface roughness on D_{box} . The second explanation seems more plausible given that the erosion of the “old” cathode was clearly noticeable, with a change in the geometry visible by the naked eye. The fractal dimension of the vacuum arc cathode spots pattern would therefore mostly depends on the material properties and contamination by oxides. There is unfortunately no way to quantify the degree of contamination by oxygen with the present experimental setup. It has however been ensured that the operating frequency and the background pressure were the same for all cathodes, respectively 1 Hz and 10^{-6} mbar. Moreover, the thruster is fired during about 2 minutes (~ 120 pulses) after installation in the vacuum chamber to clean the cathode and anode surfaces.

Cathode	D_{box}	σ_D	S [mm ²]	σ_S
Ti new	1.45	0.03	4.2	0.4
Ti old	1.42	0.03	4.6	0.5

Table 4.3: Fractal dimension and covered area for a fresh and a worn titanium cathode.

2.4 Total active spot surface

The cathode material has a strong impact on D_{box} as previously demonstrated. It is also obvious from the pictures that the area covered by the spots depends upon the material. From the binary matrix of an arc, one can easily determine the area covered by the spots. Scaling the picture allows to determine the exact size of one pixel. Then, the total area of cathode spots is given by the number of pixels

associated with value 1 in the binary matrix times the area that corresponds to one single pixel. Results are presented in table 4.4, with area sorted in descending order.

Material	S (mm ²)	σ_S	% of the cathode surface
Ti	4.2	0.4	11.1
Ni-Cr	3.2	0.3	8.5
Al	2.6	0.4	6.9
Fe	2.4	0.3	6.3
Brass	1.4	0.1	3.7
Cu	0.9	0.09	2.3

Table 4.4: Mean surface area covered by spots for different cathode materials.

The largest spot area is obtained with Ti whereas the smallest area is obtained with Cu. In fact the fractal dimension D_{box} and the surface area S are ordered in the same way keeping in mind that D_{box} is very similar for Al and Ni-Cr. Said differently this study reveals a strong link between the fractal dimension of the vacuum arc cathode spots and the covered area: a large D_{box} corresponds to a large S . In this experimental setup, unlike in a strict and formalized mathematical framework, the space containing the fractal is finite (limited by the sensor size), and the smallest possible value of ϵ corresponds to the pixel size. In that specific case, the fractal dimension characterizes then the space-filling capacity of a pattern, and the relation between D and S becomes visible.

Results are similar when comparing a fresh and a worn cathode: the covered area does not depend on the wear degree. Table 4.3 shows that the value of S is about 4.4 mm² in the case of a titanium cathode whatever the cathode surface state. Figure 4.6 shows as well an interesting characteristic of the arc discharge: there is always a sector of the cathode with no arc and spots. The sector location and size change from one pulse to the other, but empty sectors have been observed for all photographs, for the six tested materials. The existence of such sectors and their dynamics remain unexplained. They, however, participate in the fact that the area covered by the arcs is a small fraction of the total cathode area as illustrated in table 4.4.

2.5 Interpretation

The dendritic aspect of the traces left on cathode surfaces has been consistently observed either on metal plates [67] or on films [156]. Note that for studies described in references [56, 67] vacuum arcs were operating at 50 A, a current magnitude much below the one of the PJP thruster. Striking similarities were found between the cathode spots patterns and Lichtenberg figures, diffusion-limited aggregation structures [157] and Hastings-Levitov growth model [158, 159]. In our work, despite the fact that the discharge current I_d is almost 100 times higher, fractal patterns are still identified, especially when taking a close view to the dendrites. Similarities come from the fact that a spot cell can sustain a limited current density j [62]. When j becomes too high, another spot is initiated elsewhere and j decreases as the current flows through two different spot cells. Hence, the number of spots increases linearly with I_d [160]. On the other hand, cathode spots have a limited lifetime which is proportional to j^2 . At the end of its lifetime, the spot extinguishes and

an other one ignites elsewhere. This sequence of ignitions and extinctions spreads over the cathode surface in an random walk process [56, 161]. Note that the random walk in the case of vacuum arcs is not necessary a self avoiding walk, which means that a previously active spot may reignite later. Photographs taken in this thesis certainly contain spots that have been ignited several times. As spot and vacuum arc dynamics is governed by the current density, it is not surprising that high-current short duration arcs show structural similarities with low-current long duration arcs when the photograph exposure time is longer than the discharge duration in such a way all spots have been captured. The main difference between low- and high-current arcs is just the number of simultaneously active spots [68].

The size of a group of spots, generally speaking, is closely related to the pre-explosion specific action of the cathode material [58, 129]. This parameter h is written in wire explosion theory as :

$$h = \int_{t_0}^{t_{br}} j^2 dt \quad (4.2)$$

where t_0 is the time at which the energy is released in the circuit, t_{br} the instant the explosion occurs and j is the current density. This quantity represents the thermal toughness of the material heated with a current pulse [162, 163]. This parameter h has been linked to the ecton formation criterion in several studies [62, 164]. The ecton can be seen in this case as a potential candidate to the physical spatial cutoff of the fractal structure. It should be emphasized that in the present study, the spatial cutoff is the pixel size. It has been shown in [58] that h strongly depends on the cathode material, in such a way that it can be considered a function of the cathode material only, regardless of the energy input in the discharge or the cathode temperature. Unfortunately, consensual values of h are not available in the literature neither for pure metals nor for alloys. Therefore h could not be accounted for the fractal dimension analysis. However, it confirms that the structure of the plasma emission sites strongly depends on the cathode material for a similar geometry. It is therefore of interest to examine the physical properties of the different cathode materials. The objective is here twofold, namely: (i) to find which material properties influence the most the cathode spot structure during a plasma discharge and (ii) to reveal a possible relation between the fractal dimension of the vacuum arc cathode spots and cathode material characteristics.

3 Influence of material properties

To the best of my knowledge, there is currently no equation ruling the evolution of the fractal dimension with the cathode material properties. This work shows, however, that when applying the same voltage and releasing the same energy in the discharge for the same cathode geometry, the fractal dimension and the covered area are material-dependent. This idea is supported by the fact that the ordering in D_{box} and S follows the ordering of the mean diameter of macropots in [68], for titanium, aluminium and copper, despite different experimental conditions. Two different yet complementary approaches, namely Machine Learning and Heuristic method are proposed here to investigate the impact of the cathode material on the arc pattern.

The physical properties considered in these two approaches are the following: thermal conductivity λ [W/(m · K)], electrical conductivity σ [S/m], density ρ [kg/m³], vaporization energy E_{vap} [kJ/mol], cohesive energy, E_{coh} [kJ/mol], ionization energy E_{ion} [kJ/mol] and work function W_f [kJ/mol]. Values have been taken from the handbook of chemistry and physics [165] as well as from [166] and are presented in table 4.5. Exact values of energies for alloys could not be found from the literature. Therefore the weighted average with respect to the composition of the alloy was used. According to several works [167–171] such an approximation is reasonable.

Material	λ [W/(m · K)]	$\sigma \times 10^6$ [S/m]	$\rho \times 10^3$ [kg/m ³]	E_{coh} [kJ/mol]	E_{vap} [kJ/mol]	E_{ion} [kJ/mol]	W_f [kJ/mol]
Ti	22	2.34	4.51	468	421	659	418
Al	237	37.7	2.69	327	293	577	402
Ni-Cr	11	0.93	8.55	421*	365*	721*	488*
Fe	80	9.93	7.87	413	350	762	457
Brass	125	15.6	8.45	259*	231*	803*	419*
Cu	401	59.6	8.96	336	300	746	459

*Average between the properties of the materials composing the alloy

Table 4.5: Physical properties of cathode materials considered in this study.

3.1 Machine learning model

In order to determine a relation between the material properties and the fractal dimension D , an approach based on machine learning (ML) has first been considered as it allows to quantify each term contribution to the prediction function. Such insight is useful to capture the most contributing physical properties to the change in fractal dimension. The numerical model consists in a multivariate polynomial regression (MPR). A correlation between the fractal dimension and relevant material properties is sought in the form

$$D = \sum_k \sum_i \alpha_{i,k} x_i^k \quad \text{with} \quad \alpha_{i,k} \geq 0 \quad (4.3)$$

where D is the predicted fractal dimension, $\alpha_{i,k}$ are linear coefficients and x_i are physical constants relative to the cathode material, here extracted from table 4.5. Each x_i appears in the expression with exponent k . As for the polynomial order, since most scaling laws in physics feature exponents ± 0.5 , ± 1 and/or ± 2 , it is chosen to limit k to these values. No intercept is allowed in the model shown in equation 4.3, which means that a null fractal dimension is assumed when all physical properties are equal to zero. Physically, it means that no discharge can occur without a propagation medium.

The analysis consists in determining the value of each $\alpha_{i,k}$ that outputs the best fit to the experimentally observed fractal dimension. For this purpose, a ML program was developed in Python. Having selected 7 physical properties and 6 exponents, 42 coefficients are to be determined. In practical terms, the algorithm targets at minimizing the residual sum of squares between the experimentally observed fractal dimension and the one retrieved by the polynomial approximation. The dataset on which the analysis is performed comprises 6 materials and 411 pictures. This is

randomly split into a training set (85%) and a test set (15%). The training and test sets are depicted in figure 4.7 along with the fractal dimension predicted by the ML program. The quality of the model is quantitatively assessed on the test set through the MAE and R^2 metrics, which read 2.15% and 0.94, respectively. These parameters outline that the model does satisfactorily predict the experimental findings.

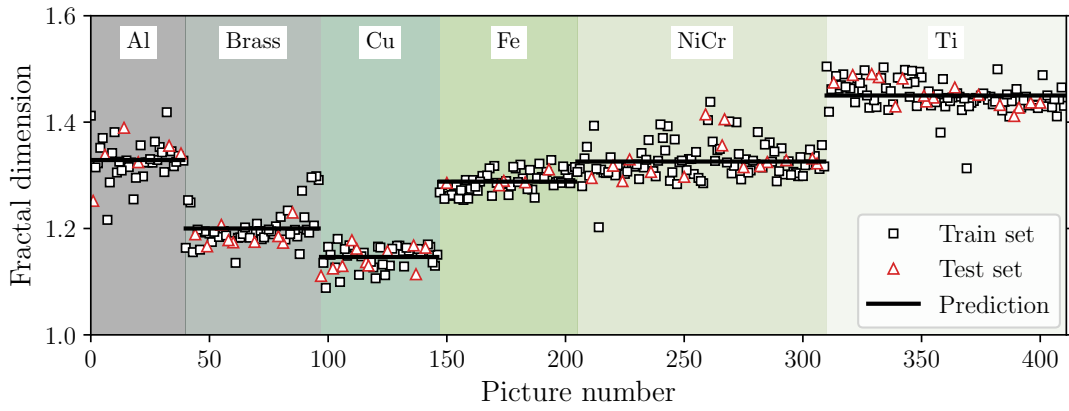


Figure 4.7: Scatter plot of train set, test set and predicted value of fractal dimension.

Figure 4.8 shows the normalized values of every $\alpha_{i,k}x_i^k$ for the six tested materials, that means their respective contribution to the fractal dimension prediction function. In all cases, the reciprocal of the work function term contribute more than 55% to the fractal dimension. The sum of the squared ionization energy and squared cohesive energy accounts for more than 35%, with varying proportions. The remaining quantities visible at the bottom of Figure 4.8 appear with variable weights depending on how the dataset is randomly divided into train and test sets. This effect possibly results from the relatively limited amount of data available. Therefore the precise influence of ρ , λ and σ is difficult to quantify with this approach, however it certainly remains secondary. Note that the contribution of the vaporization energy is null in each case.

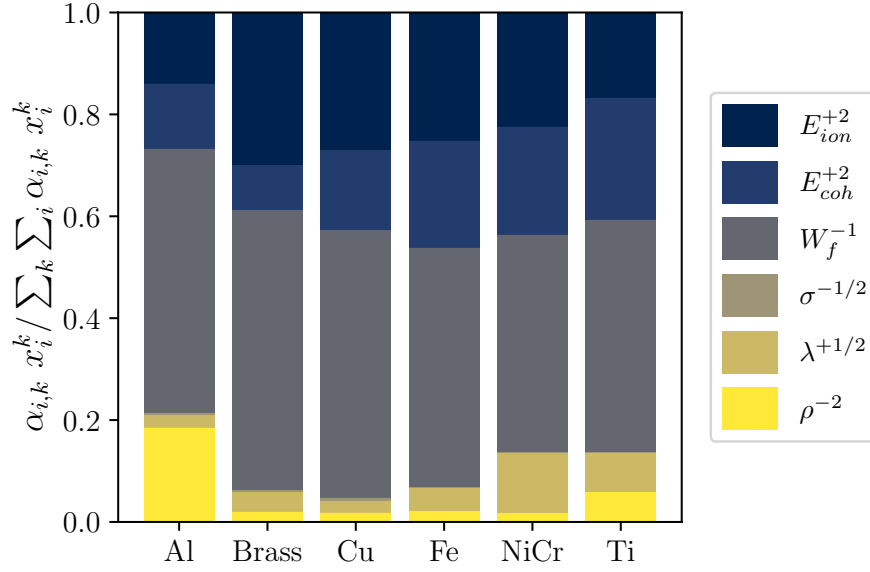


Figure 4.8: Normalized cumulative bar plot of the absolute value of each $\alpha_{i,k} x_i^k$. Omitted units are consistent with Table 4.5.

3.2 Heuristic analysis

As a means to complete the ML model and examine whether a product relation between the material properties can also explain the change of fractal dimension D , a heuristic analysis has been implemented. It relies here on an expression in the form

$$D \propto \prod_i x_i^k \quad (4.4)$$

where x_i is a physical parameter relative to the cathode material and k are usual exponents in physical laws, in a similar fashion to the approach discussed beforehand. Values of k are limited once again to $k = 0, \pm 0.5, \pm 1, \pm 2$. As this approach relies on an expression in a product form, a risk of overfitting the data can arise. One should then limit the number of x_i 's. In that respect, since the ML model brings out the main contributing properties, the method proposed here only considers parameters with a contribution above 10% in the ML model i.e. E_{coh} , E_{ion} and W_f . Note however that the heuristic model considers a product of the various physical properties while the ML model is based on a sum. All the 343 possible combinations have been tested in an iterative way for the six materials using a custom program in Python. For each combination a linear regression analysis is performed that includes the six materials. Unlike with the previous model, the intercept of the linear regression is not forced to be null. The expression giving both the lowest mean absolute error (MAE) and the highest R^2 is considered the most probable and the most appropriate relation of proportionality between the physical parameters of the material and the fractal dimension. Following this procedure, the best suited relation between D and the material properties is:

$$D \propto \frac{E_{coh}^2}{W_f E_{ion}^2} \quad (4.5)$$

The evolution of the fractal dimension D with this combination of parameters is displayed in figure 4.9. With the relation given by equation 4.5, the linear regression yields a mean absolute error of 0.025 and a R^2 equal to 0.865. It is quite clear from this figure that the copper is responsible for the decrease in R^2 . The linear regression without taking Cu into account has an R^2 of 0.987. This may be due to the accuracy of the box counting method with the copper dataset. The copper spots covering the smallest surface with small filaments, the box counting algorithm is less accurate as the size of the spot pattern is close to the pixel size.

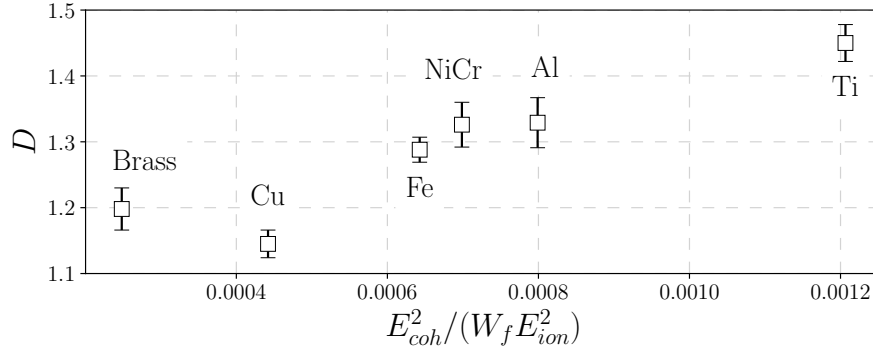


Figure 4.9: Evolution of the fractal dimension according to equation 4.5.

Relation	a	b	R^2	MAE
$\frac{E_{coh}^2}{W_f E_{ion}^2}$	305.7	1.08	0.865	0.025
$\frac{E_{coh}}{W_f \sqrt{E_{ion}}}$	13.4	0.86	0.841	0.030
$\frac{E_{coh}}{W_f^2 \sqrt{E_{ion}}}$	5096.8	0.91	0.813	0.030
$\frac{E_{coh}}{W_f^2 E_{ion}}$	111107.8	0.98	0.771	0.039
$\frac{\sqrt{E_{coh}}}{W_f^2}$	5616.3	0.72	0.623	0.049

Table 4.6: Most appropriate relations of proportionality between the physical parameters of the material and the fractal dimension.

The five relations having the most linear behavior, as well as the coefficients of the regression and their respective R^2 and MAE, are given and ranked in table 4.6. This table indicates that the cohesive energy increases the fractal dimension while the work function and the ionization energy decreases D . Studies have shown that

the noise amplitude in the vacuum arc burning voltage scales linearly with the cohesive energy of the cathode material [66, 172]. According to the theory of colored noise [173, 174], this noise in burning voltage could possibly indicate a Brownian motion of the active cathode spots, often approximated with random walk models. A link could therefore be expected between the spot distribution over the cathode surface and the cohesive energy of the cathode material. On the other hand, the ionization energy and the work function counteract the cohesive energy in the relation, playing a crucial role in reducing the fractal dimension magnitude.

Chapter V

Cathode spots dynamics by high-speed imaging

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1 Observations

1.1 Protocol

In a similar manner as for long exposure photographs presented in the previous chapter, a series of about 100 pulses is performed prior to measurements. These pulses are used to remove the oxide layer formed on the cathode, which modifies the behavior of the vacuum arc [152–154]. To ensure the best possible level of detail, focus is achieved by setting the camera’s exposure time to 10 ms, its maximum. This increases the image brightness when no arc is triggered, thus facilitating manual focusing. A calibration image is taken after focusing, and used as a reference for automatic cathode edge detection as it will be explained later. The acquisition period is set to 2.2 μ s. With this setting, the maximum exposure time of the camera is 1.9 μ s. The 0.3 μ s “dead time” between acquisition period and exposure time is necessary for each frame to be recorded and for the sensor pixels to be reset. Therefore, there is losses of information during 0.3 μ s between each frame, which is the smallest time period caused by hardware limitations. The camera is triggered by the PJP trigger signal. The associated discharge current is recorded on the oscilloscope. A series of 150 associations of videos and discharge current were recorded for titanium, iron and aluminium cathodes. Only 50 videos and current traces were recorded with the copper cathode due to thruster malfunction. Examples of a typical videos are shown in the following section.

Due to the geometry of the PJP H2020, arcs can form on the side of the cathode or anodic modes can occur. Although such phenomena are fairly rare, these luminous areas must not be confused with cathode spots, as this would distort the result when calculating the mean surface covered by the spots and spot propagation speed. To avoid this kind of issue, only the central part of the image is retained for further analysis. Parts of the image beyond the cathode boundaries are automatically removed using a computer vision library (OpenCV). For a pixel to be identified as a cathode spot, its brightness must exceed a given threshold. This threshold is empirically adapted to the datasets and optical equipment. This value was found manually, by matching the patterns of detected pixels to the patterns observed with the naked eye. Values of threshold for the four different materials are given in table 5.1 for the inclined reader. Setting the threshold too low results in false positives, i.e. pixels detected when there are no active spots. Conversely, setting it too high makes the algorithm too selective and some active spots are not detected. A manual fine tune of this threshold is important because the spots produce a lot of light and illuminate the cathode surface. The spots are a primary source of light and the cathode itself is a secondary source, not an active plasma emission site. Reflected light should not be counted as a cathode spot, to reduce inaccuracies on the results. Detail on the problematic of false positives and the way they may appear is provided in chapter IV.

Table 5.1: Threshold values for a pixel to be counted as an active spot.

Material	Ti	Al	Fe	Cu
Pixel intensity	240	200	240	220

1.2 Videos

Titanium

On the whole, see figure. 5.1, the titanium spots tend to cluster near the center at the beginning of the discharge. This was to be expected, as the high voltage trigger tip is located at the center of the cathode central insulator. The breakdown is therefore initiated where the electric field is the strongest: i.e close to the trigger tip. As explained in the previous chapters, when the trigger is switched off the arc migrates toward the main anode through the existing metal vapor. The latter is circular in shape, and its diameter is larger than the outer diameter of the cathode. The electric field $\mathbf{E}$ is therefore radial in the picture, and *should* thus lead to a purely radial propagation of the cathode spots. However, this is not what is observed. Although the propagation is partly radial, one can see that some spots move to the left or to the right of the picture, develop tangentially, or even remain active along the edge of the central insulator. Some spots initiated at frame $t = 2.2 \mu\text{s}$ are still active at frame $t = 22.0 \mu\text{s}$, close to the central insulator. It should also be noted that in this figure, the spots propagate in the top half section of the picture. The preferred side is in fact determined by the breakdown initiation area, as the spots successively ignite close to the already existing ones [64, 175]. The propagation area changes then from one pulse to another.

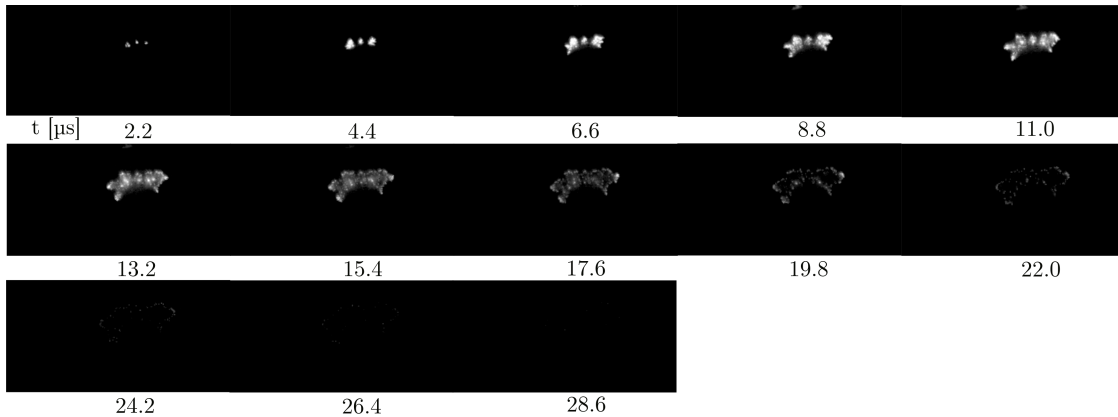


Figure 5.1: Spatial evolution of the Ti cathode spots captured over 13 video frames

Aluminium

When the PJP is equipped with an aluminium cathode, it is observed that the spots are much more diffuse and show a less intense brightness. They propagate quickly, cover a large area of the cathode, and leave very little inactive surface as can be seen in figure 5.2. This behavior had not been observed in the long-exposure photographs as these diffuse spots could not be distinguished from the luminous halo present between the branches. Very bright and dense anode modes are visible in this video. Moreover, it can be seen that some cathodic spots are initiated on the side of the cathode starting from frame $t = 11.0 \mu\text{s}$. Overall, the behavior of aluminium spots is chaotic and difficult to describe.

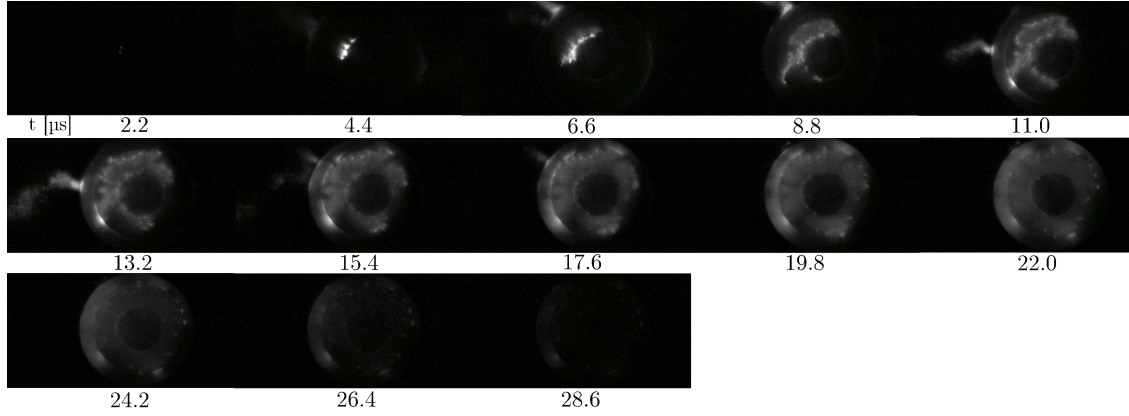


Figure 5.2: Spatial evolution of the Al cathode spots captured over 13 video frames

Iron

In contrast to the two previous materials, iron exhibits a more regular and predictable behavior. As can be seen in figure 5.3, the iron spots first appear all around the edge of the insulator and then begin to develop purely radially. They slowly move away from the center and gradually fade away. It takes 15 frames, or $33\text{ }\mu\text{s}$, to fully capture the discharge, whereas less than $29\text{ }\mu\text{s}$ were sufficient to capture the titanium and aluminum discharges. The spots are also quite grouped and bright. Here, numerous anodic modes are present and can be observed from frames $6.6\text{ }\mu\text{s}$ to $22\text{ }\mu\text{s}$.

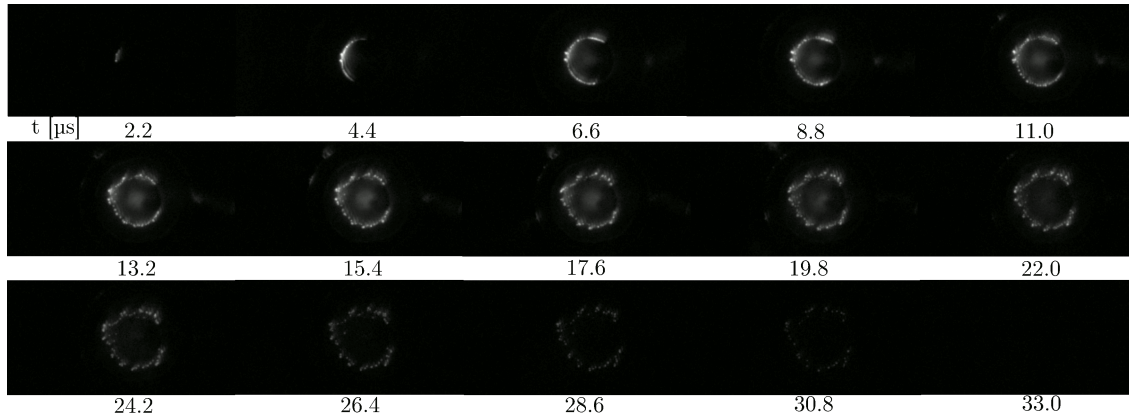


Figure 5.3: Spatial evolution of the Fe cathode spots captured over 15 video frames

Copper

The behavior of copper cathode spots is quite similar to that of the iron spots, see figure 5.4. They are initiated at the periphery of the central insulator, and form small islands that barely propagate during the discharge. Copper cathode spots also exhibit a relatively long activity time and only extinguish after $33\text{ }\mu\text{s}$. Hence, copper spots videos require 15 frames to fully capture the discharge.

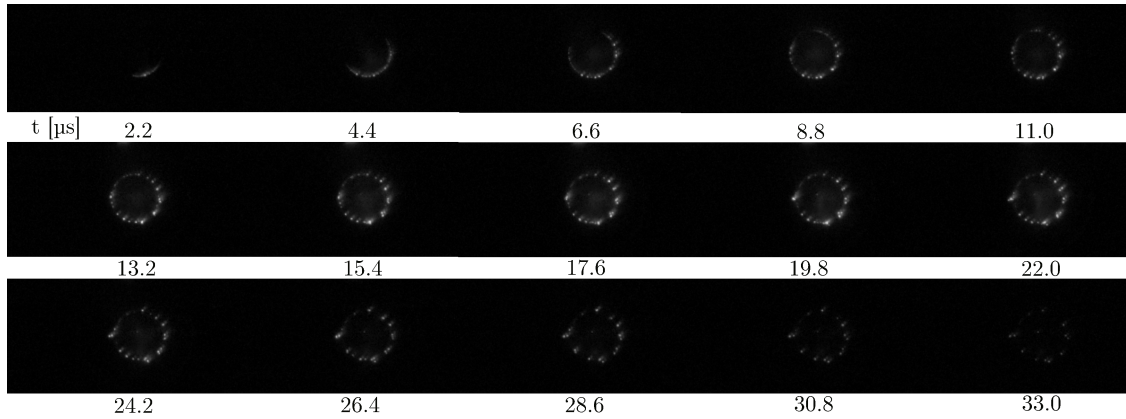


Figure 5.4: Spatial evolution of the Cu cathode spots captured over 15 video frames

2 Instantaneous spot surface

In order to avoid overloading the present qualitative analysis, only titanium and iron will be compared in the text of the following sections. This choice was made because titanium and aluminum both exhibit a chaotic behavior, whereas iron and copper are somewhat more predictable. Nevertheless, comparative values for all four materials will be provided in tables. For every cathode material, the instantaneous surface covered by the spots shows a significant evolution during the discharge. As spots propagate towards the external edge, they cover an increasingly larger surface. Figure 5.5 presents a comparison between the discharge current and the evolution of the covered area as a function of time, for the videos in figure 5.1 and figure 5.3. For both cathode materials here, the maximum surface is reached when the discharge current reaches its maximum, then it decreases and falls back to zero. One can see that the I_d and surface curves exhibit a high degree of similarity, which is observed consistently across all recorded videos. In particular, one can see from figure 5.5b that the PJP slightly malfunctioned at the start of the discharge but that the evolution of the surface still followed the discharge current waveform. The $2.2\mu\text{s}$ offset between the two curves in figure 5.5 originates in the time taken by the camera to acquire an image (exposure time).

The correlation between the discharge current $I_d(t)$ and the surface covered in time $S(t)$ has been quantified by determining the Spearman rank-order correlation coefficient r_s . This coefficient ranges from -1 to +1: +1 indicating an exact monotonic correlation, -1 an inverse monotonic correlation, and 0 no correlation. Its calculation is relatively simple and well implemented in python libraries as it is widely used in different research areas [176, 177]. In the present examples, $r_s = 0.95$ for Ti and $r_s = 0.894$ for Fe which means a strong correlation between $I_d(t)$ and $S(t)$ on these videos as the naked eye could tell. The averaged values of spot surface per frame, maximum instantaneous surface achieved during a pulse and r_s for the whole dataset are shown in table 5.2. Of course the term “instantaneous” is, strictly speaking, not entirely accurate here as each frame actually represents $2\mu\text{s}$ of the discharge. It can be seen that for each material the Spearman coefficient is very high, except for copper. Once again, this is due to the operational difficulties of the PJP with the copper cathode: contamination of the insulators, short circuits, and thermal expansion of the cathode. The copper discharge current is then often very noisy, and show bumps, which impacts the computation of r_s . The fact that copper spots

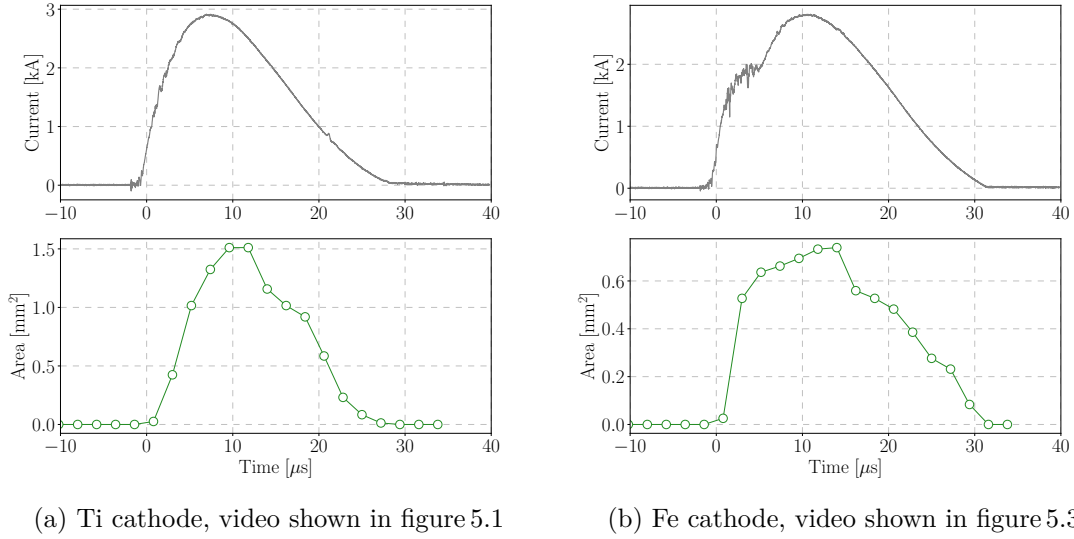


Figure 5.5: Comparison between the discharge current (top) and the evolution of the spot surface (bottom) for the Ti and Fe example videos.

are small and about the size of a pixel could also play a role in that observation. For all other materials, high values of r_s indicate that on a large sample, the evolution of the surface covered correlates with the evolution of the discharge current. With the PJP, the discharge current waveform is smooth and highly reproducible. One can therefore remark that despite an apparent random motion observed at smaller scales [68, 155, 175], the instantaneous spot surface as an absolute value shows a quantifiable evolution. This evolution being correlated with the discharge current.

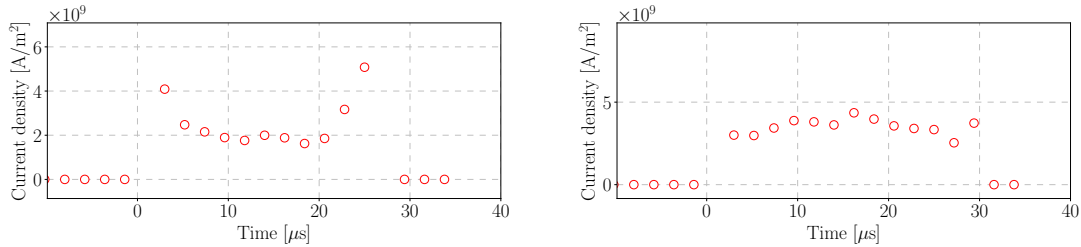
Table 5.2: Average values of surface covered, maximum surface and spearman coefficient for the 4 materials. Each dataset represents 150 pulses, except for copper with 50 pulses.

Material	Ti	Al	Fe	Cu
S [mm ²]	0.54	0.73	0.48	0.29
max S [mm ²]	1.12	1.42	0.85	0.51
r_s	0.95	0.72	0.84	0.48

3 Current density

Once the evolution of the active spot surface has been determined, it becomes possible to calculate the evolution of the current density at the spots during the discharge. Figure 5.6 shows the evolution of current density as a function of time for the titanium and iron example videos. This figure is obtained by dividing $I_d(t)$ by $S(t)$. For the titanium cathode, one can note 2 periods with high current density j at the beginning and the end of the discharge. Indeed, due to the spots characterization method, uncertainty on the surface computation is large for these two instants. 1-At the start, the spots are tightly grouped regardless of the material. This leads to an

overestimation of j because several spots are present per pixel. 2-At the end of the discharge, the spots are scattered and not very bright. The optical arrangement of neutral density filters prevents these spots from being seen clearly, as their brightness falls below the threshold. This results in an underestimation of the surface area and, consequently, an overestimation of j . This can be seen around 0 and 28 μs in figure 5.6a, and around 0 μs in figure 5.6b.



(a) Titanium cathode, video shown in figure 5.1

(b) Iron cathode, video shown in figure 5.3

Figure 5.6: Evolution of the current density during the discharge, for the two example videos.

If one disregards these 2 peaks, j remains stable around $2 \times 10^9 \text{ A/m}^2$ for titanium and around $4 \times 10^9 \text{ A/m}^2$ for iron. The correlation between $I_d(t)$ and $S(t)$ has already been discussed in section 2, it is then natural that current density should be relatively stable as current and surface evolve together. This clearly illustrates, when looking at the problem from a different perspective, that each spot can sustain only a limited current density. When this limit is reached, additional spots are simultaneously initiated. This implies that, in high-current vacuum arcs, the higher the discharge current, the larger the total active spot surface becomes. Here the average current density can then be calculated by taking the average current and dividing it by the average spot surface covered during the pulse. Results are given in table 5.3. It can be observed that the four materials have a fairly similar average current density. References in the literature [68, 178] give values in the range of $10^9 - 10^{10} \text{ A/m}^2$ according to the “large spot” model. This model predates the fractal spot model, developed in the early 2000s to better describe the cathode spot behavior [67, 120]. Here, because of the image resolution, the fractal model cannot be directly applied to the observations. Current density is then underestimated due to overestimation of the spot surface. Note that the calculated j for copper and iron might be slightly more overestimated than that of titanium and aluminium, due to the sizes of their spots which are closer to the size of the camera pixel. In addition, direct observation of the cathode spots by optical means inherently overestimate their surface [179, 180]. The dense, rapidly expanding plasma from the spots can show a brightness almost equal to the one of the spot, within a certain radius. More recent studies suggest that the spot current density would be closer to 10^{11} A/m^2 , based on the ecton model [62] and advanced techniques such as pulsed laser absorption photography [181] or pulsed laser interferometry [182].

Ultimately, the fractal model gives another perspective on the quest to determine the exact current density in cathode spots. They are self-similar and thus appear, when observed with increasingly precise and modern techniques, to be composed of ever-smaller fragments. The question of the real spot current density is then still

Table 5.3: Average current density for the four cathode materials.

Material	Ti	Al	Fe	Cu
Discharge current [A]	1.5×10^3	1.8×10^3	1.6×10^3	1.0×10^3
Spot current density [A/m ²]	2.7×10^9	2.4×10^9	3.4×10^9	3.5×10^9

open and, in the end, is rather abstract.

4 Spot clustering

Despite this difficulty in estimating the true size of a spot, values of interest can be estimated from groups of spots, or clusters, which are clearly visible in the pictures. In fact, the spots captured by the camera consist of numerous smaller spots, each of them on the scale of tens of μm . Siemroth et al. [68] propose to classify spots into 3 categories, depending on their size. Ranging from the smallest to the largest, first are the microspots, each approximately tens of μm in size. Then are the macrospots, observable through a camera. Ultimately, group spots are clusters of macrospots. The group spots can vary in size. Each macrospot can sustain a limited current density, which depends on the cathode material. When the maximum current density is reached, another spot is initiated and may or may not cluster with the previous one [65, 120]. In high-current vacuum arcs, a large number of macrospots are simultaneously active [62, 175]. It is therefore interesting to look at the spot clustering in the context of a PJP discharge.

4.1 Depth-first search

To automatically identify structures of clustered active spots on each image, a depth-first search (DFS) algorithm has been implemented in the data analysis process. This type of algorithm is used for exploring tree data structures [183] and generally finds applications in computer science or artificial intelligence related topics. The DFS initiates at a root node and explores as far as possible along each branch before backtracking [184]. The functioning principle of a DFS algorithm is shown in figure 5.7. The numbers indicate the order in which the algorithm explores the nodes. It starts on the left and explores as far as possible before backtracking on the last partially explored node. Once the branch has been fully visited, it goes back on the root node and does the same on every branch. In our case, the algorithm starts at a pixel and explores neighboring pixels with a brightness greater or equal to the imposed threshold as defined in section 1.1. When a darker pixel is encountered, the algorithm returns to the previous pixel and explores its neighbors again. Doing so, the algorithm recursively explores spatially linked bright pixels: clustered spots structures. Subsequently, each group of spots is assigned a unique identifier, facilitating the mapping and counting of these clusters on a given picture. Note that the DFS implemented here does not allow to track a given cluster from one frame to the other.

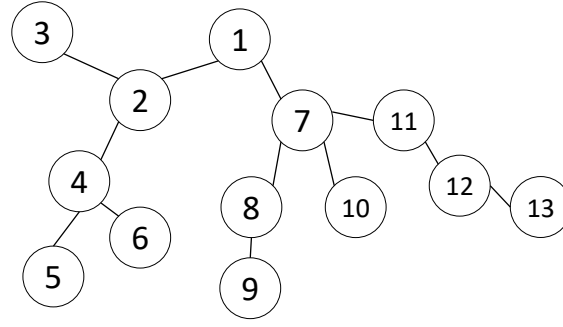


Figure 5.7: Functioning of a DFS algorithm. The numbers indicate the order in which the nodes are visited.

4.2 Groups of spots

Figure 5.8 and figure 5.9 show the group spots as identified by the DFS algorithm on 3 different images, on the videos exemplified in figure 5.1 and figure 5.3 respectively. The different groups are highlighted in different colors. In most frames, the algorithm detects more group spots than there are colors in the palette: different clusters can therefore be highlighted in a similar color in the figures. It is noticeable that at the beginning of the discharge the spots are clustered in large groups. As the discharge continues, they split and move further apart. The spots are therefore smaller and more numerous. The average size and average number of simultaneously active group spots during a PJP discharge is given in table 5.4 for the four cathode materials, as well as the average maximum size of the group spots. Time of occurrence indicates when the group spots are the largest during the discharge. σ_N and $\max(N)$ indicate respectively the standard deviation of the number of simultaneous group spots, and the maximum number of simultaneous group spots.

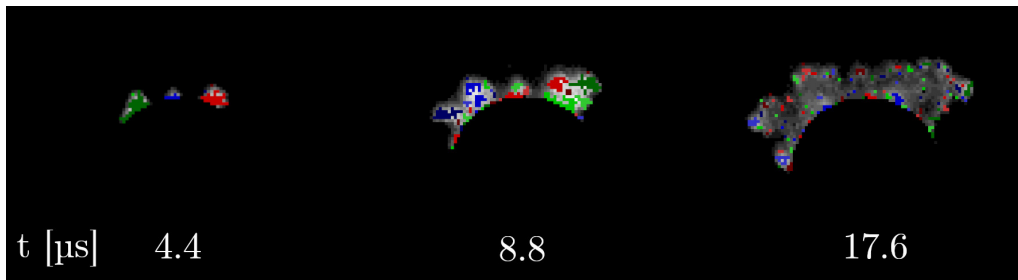


Figure 5.8: Group spot recognition with the DFS algorithm. Ti cathode.

Spot clusters present a surface on the order of 10^{-2} mm^2 on average, whatever the cathode material. However, what significantly varies between materials is the average maximum cluster size. Aluminum generally exhibits the largest instantaneous cluster sizes, reaching nearly $6 \times 10^{-1} \text{ mm}^2$ on average, while iron shows the smallest average increase in cluster size during the discharge. The largest cluster observed across all materials was detected for titanium and is presented in figure 5.10. One can yet remark here that the cluster size is surely overestimated due both to

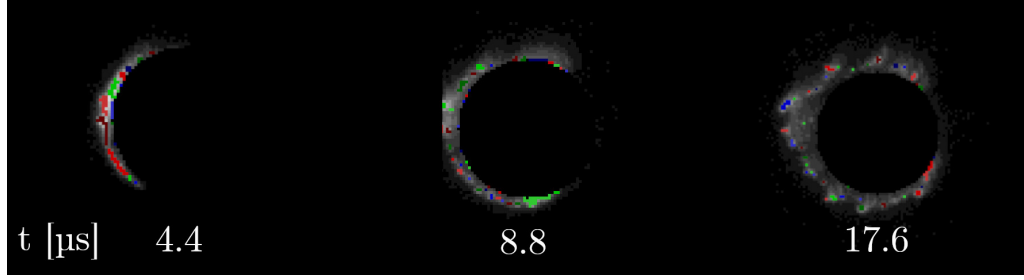


Figure 5.9: Group spot recognition with the DFS algorithm. Fe cathode.

Table 5.4: Average spot properties for the four materials.

Parameter	Ti	Al	Fe	Cu
Size [mm ²]	1.82×10^{-2}	1.72×10^{-2}	1.34×10^{-2}	1.39×10^{-2}
Maximum size [mm ²]	3.29×10^{-1}	5.68×10^{-1}	2.26×10^{-2}	6.64×10^{-2}
Time of occurrence [μs]	4.9	4.3	2.2	16.9
Number of groups	37	26	35	18
σ_N	25	22	18	9
$\max(N)$	100	106	83	36

the resolution of the camera and the overestimation of the spot surface as explained in the previous section. In reality, there might be several different clusters on this large one, but the separation between them is not visible here. The value of this maximum size should therefore be taken with caution.

On an other aspect, the average number of simultaneous groups per discharge provides an interesting insight on a previously described phenomenon. It was observed that iron exhibits a behavior similar to that of copper, but with a larger total active spot surface. It is clear that the difference here lies in the number of active clusters: 35 versus 18, despite a similar average size. Ultimately, by multiplying the average cluster size by their number, one can retrieve the same ordering of active spot surface as previously found. This ordering, however, is not maintained in the case of aluminum due to its diffuse spots whose clusters are difficult to identify.



Figure 5.10: Largest spot cluster detected by the DFS algorithm, highlighted in brown. Picture taken 6.6 μs after discharge ignition.

Another observation is that the larger spots form at the very beginning of the discharge, not when the maximum in I_d is reached. There is then no apparent relation between the maximum active spot surface during the discharge and the maximum clustering of the spots. It is yet known that the apparent spot motion on the cathode surface follows a special type of random walk called self avoiding

walk [67]. This means that an extinguished spot has little chance of being active again, at least in a short period of time. This implies that large clusters of spots tend to split without ever reforming. It was also observed in ref. [185] that there is a size effect: small spots disintegrate faster than large ones. Spots therefore separate more and more quickly as the discharge progresses, which can be clearly observed here for titanium and aluminium: spot clusters start from the center and move outwards while disintegrating in the course of the discharge.

5 Propagation velocity

5.1 Tracking algorithm

TracTrac [186] is an open-source particle tracking velocimetry (PTV) algorithm implemented by J. Heyman. Developed as an alternative to particle image velocimetry (PIV) algorithms, it can simultaneously track thousands of points on videos and estimate, among other parameters, their trajectory and velocity. The special feature of TracTrac is that it can track particles separately regardless of the surrounding moving particles, as it was originally developed for tracking particles in granular flows. This feature enables, in our case, the tracking of emergent spots throughout the discharge, and is of particular interest when it comes to the tracking of separate spot clusters. Moreover, the weight of the motion prediction model can be adjusted in TracTrac thus enabling the tracking of quasi-random motion. In the present study, this weight is set very low. TracTrac can be used with a graphical interface using Matlab, but all the features are written as well in Python scripts. In this work, the python version is used so that it can be integrated with other processing algorithms. The models used in TracTrac, its code structure, performance and robustness are detailed in J. Heyman's original work [186].

5.2 Titanium

Once the average size of a cluster has been determined, the input parameters of the tracking algorithm can be refined. This makes tracking more precise by forcing it to only track objects of a given size. Result of the tracking for the video shown in figure 5.1 is shown in figure 5.11. The background picture is a sum of all the frames composing the video, making it analogous to a long-exposure photograph. Since the spots develop only on part of the cathode, the image has been cropped to better visualize the trajectories. The displayed lines are the trajectories of the different group spots tracked on this video. Trajectories are colorized according to their respective average speed (norm of the velocity vector), the crosses symbolize the exact location where the spots have been identified. As seen previously, trajectories can be tangential or radial, with no apparent correlation between the propagation direction and the speed. As an example, two orange trajectories with speed of about 80 m/s are observed on the left of the figure despite that they are respectively almost purely radial and tangential. A closed loop trajectory is observed at coordinates [150,45], showing the robustness of the algorithm when it comes to identify quasi-stochastic movement. However, most clusters appear to propagate in relatively straight lines. In particular, tracking of a fast spot moving at 119 m/s is observed on the left of the figure. This spot is identified over 9 consecutive images and show

no abrupt change of direction throughout its course. Several possible explanations for this phenomenon are discussed at the end of this section.

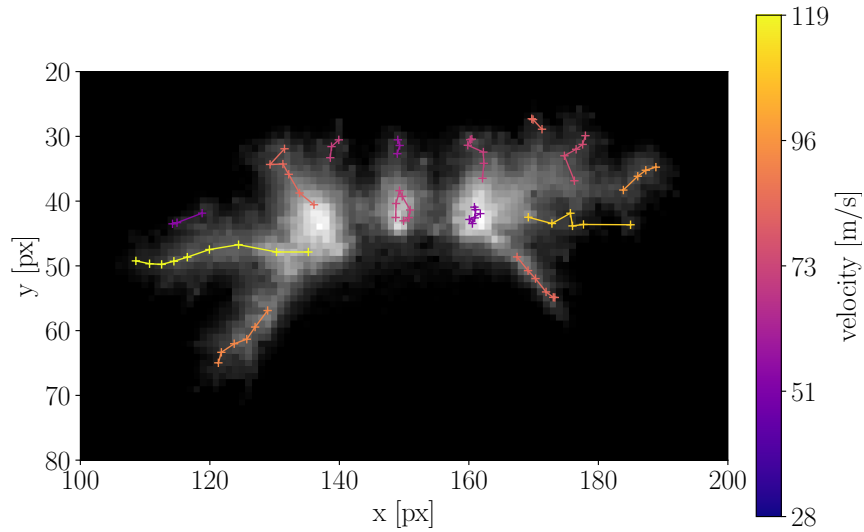


Figure 5.11: Tracking of the Ti spots. Data taken from video shown in figure 5.1.

In this example, the spots apparent velocities ranges from 28 m/s to 119 m/s, which is fairly representative of the total data set. Figure 5.12 shows a histogram of tracked spots velocities over the whole dataset. Out of a set of 150 videos, 2655 spots were successfully tracked. The average speed is close to 60 m/s, with a standard deviation of 32.6 m/s which is rather high.

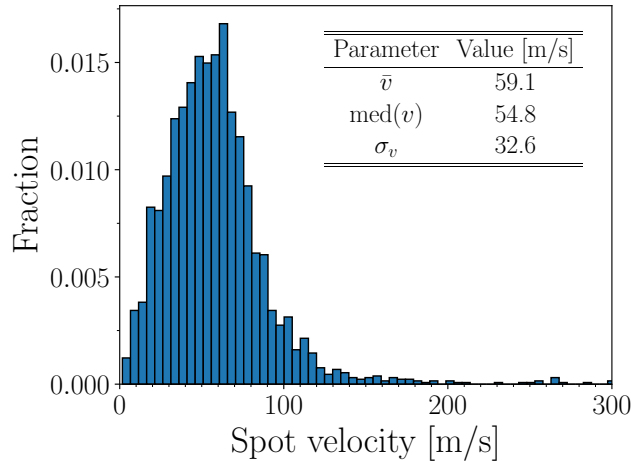


Figure 5.12: Velocity distribution of all tracked Ti spots. Average and median velocities as well as the standard deviation are provided in the table.

5.3 Aluminium

For aluminium, results of the tracking on the example video are shown in figure 5.13. The technical difficulties encountered during the automatic analysis of this cathodic material are immediately observed. The spots are small and dim, which results in

the fact that not all of them are tracked, or only for a limited number of successive frames. In other words, the trajectories are shorter but more numerous. Despite applying a mask to exclude anodic modes, a number of spots are still tracked along the edge of the cathode, as can be seen at coordinates [110, 30]. Those tracked spots appear to move mainly tangentially, because once they reach the edge, they propagate in the z direction towards the plate in which the cathode is screwed. This slow and tangential tracked motion is just the projection of this movement along the z axis on the plane of the picture.

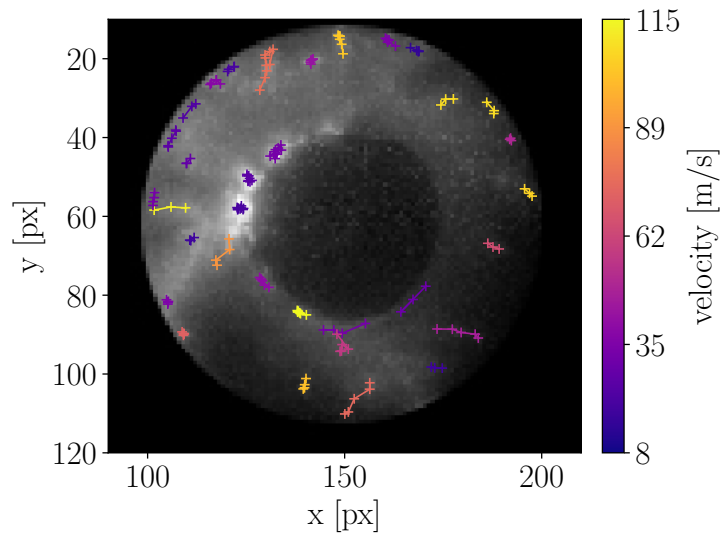


Figure 5.13: Tracking of the Al spots. Data taken from video shown in figure 5.2.

Overall, more than 6000 spots are tracked over the whole dataset, which is way more than for titanium. Repartition of the tracked velocities is shown in figure 5.14, as well as statistical values of interest. According to that figure, aluminium is the cathodic material with the largest difference between the mean and the median spot velocity. This observation and the shape of the distribution clearly indicates the presence of a dominant type of spot with moderate speed, along with a smaller number of faster, more rare spots that form the tail of the distribution. This tail is likely even more pronounced in reality, as the algorithm was unable to track all of the dimmer spots.

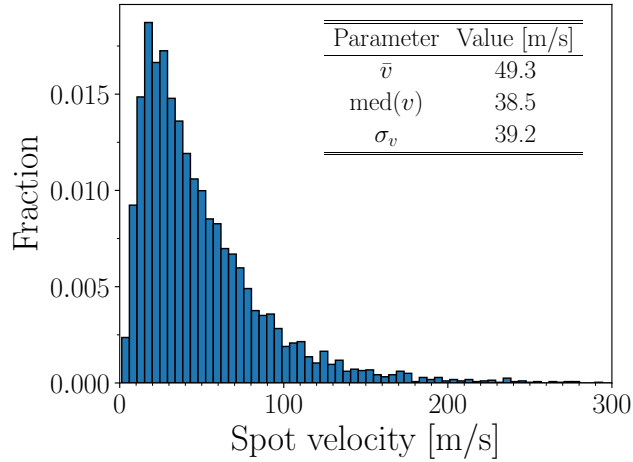


Figure 5.14: Velocity distribution of all tracked Al spots. Average and median velocities as well as the standard deviation are provided in the table.

5.4 Iron

The tracking results for the iron example video are shown in figure 5.15. Here, the spots propagate in an almost purely radial direction, in contrast to titanium and aluminum. No tangential or loop-shaped trajectories are observed. In this video, nearly all spots move at approximately the same speed, around 40 m/s. It is also noticeable that the spots are tracked over many consecutive frames, forming long and straight trajectories. In total, across the entire iron dataset, 3610 spots were tracked. The velocity distribution is shown in figure 5.16. The average speed is 37.5 m/s, and once again, the distribution is not Gaussian. A secondary population of slower spots is observed, in addition to the main group centered around 40 m/s.

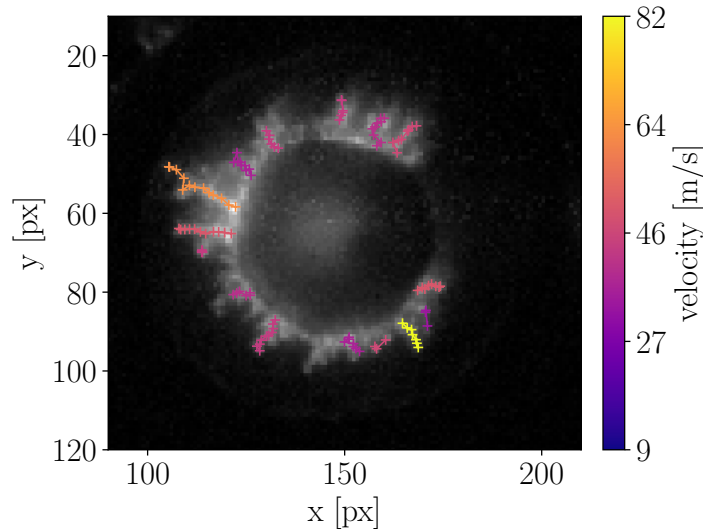


Figure 5.15: Tracking of the Fe spots. Data taken from video shown in figure 5.3.

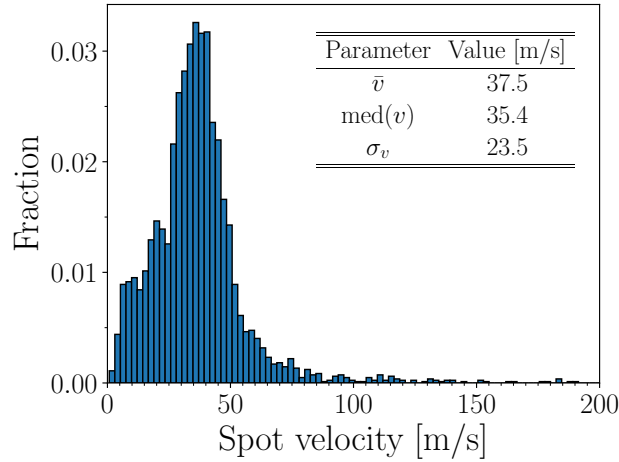


Figure 5.16: Velocity distribution of all tracked Fe spots. Average and median velocities as well as the standard deviation are provided in the table.

5.5 Copper

For copper, see figure 5.17, the spots are tracked over many successive frames but low velocities are observed. The spots appear almost static in the videos although a radial propagation can be identified, in a similar manner as the observations made with the iron cathode. In this example, the average speed of the tracked spots is 19 m/s, which is representative of the entire dataset. It comprises 853 trajectories over 50 videos. Figure 5.18 shows a relatively narrow velocity distribution for copper spots, with an average speed of 19.3 m/s. A few faster spots are observed, but their occurrence appears to be more of a statistical exception than a recurring behavior. In figure 5.18, a spot was also tracked on the tip of the central insulator, illustrating the concept of a false positive.

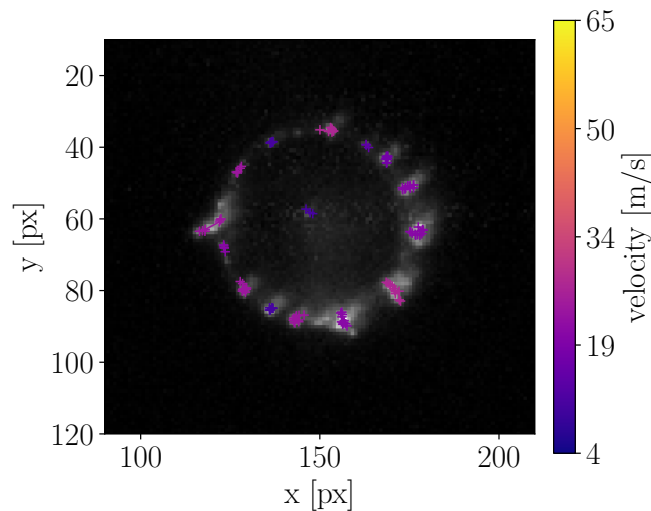


Figure 5.17: Tracking of the Cu spots. Data taken from video shown in figure 5.4.

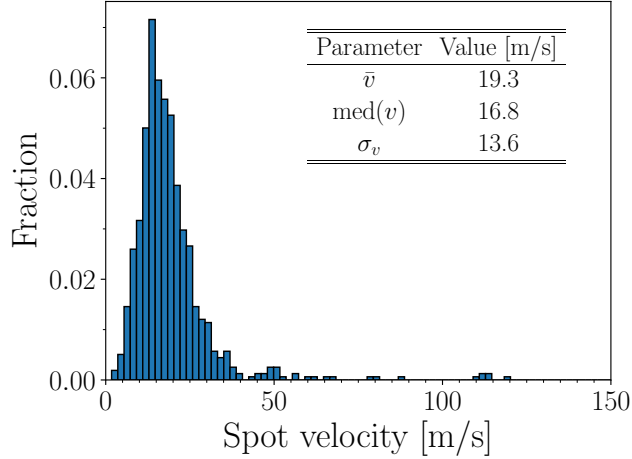


Figure 5.18: Velocity distribution of all tracked Cu spots. Average and median velocities as well as the standard deviation are provided in the table.

6 Comparative overview and discussion

Table 5.5 summarizes the average spot velocities for the four cathode materials. It can be observed that the ordering of velocities also follows the ordering of the total surface areas covered, as detailed in the previous chapter. This indicates two key points: First, and most evidently, the total spot surface covered during a discharge is correlated with the spot propagation speed. Since the PJP is pulsed over a well-defined time interval, $30\ \mu\text{s}$, it logically follows that the fastest propagating spots are able to cover the largest area within that time frame. This would not have been true if the spots were frequently propagating backwards along their path. This leads to the second major observation from this study: spots propagate predominantly in a forward direction. In the titanium example video, loop-shaped trajectories are visible, but such behavior appears to be statistically marginal. The assumption that spots propagate following a self-avoiding walk [67] is therefore statistically valid, although this description should be nuanced. The self-avoiding walk behavior holds over relatively short time intervals, as looped trajectories have been observed over spans of approximately $13\ \mu\text{s}$. Similarly, when observing groups of spots, the apparent randomness of the trajectories seems to diminish. While the propagation directions do not align with the electric field $\mathbf{E}$ in the case of titanium or aluminium, the trajectories remain relatively straight and not fully stochastic. This randomness disappears entirely for iron and copper, where spot propagation is radial and linear along $\mathbf{E}$. This could indicate a macro effect of spot clustering, meaning that spots tend to lose their stochastic behavior when they cluster and show a preferential propagation direction. It may also be linked to the camera low spatial and temporal resolution compared to the spot propagation phenomenon, making it impossible to see small variations in the trajectory. In a similar manner to the self-avoiding walk, it is possible that the random walk solely occurs at smaller spatial scales, beyond the resolution of the high-speed camera.

Regarding another aspect, for the four cathode materials, spot propagation speeds presented here are higher than what is usually found in the literature [56, 187, 188]. Still, the PJP operates at a higher discharge current than the experi-

Table 5.5: Spot tracking results for the four materials.

Parameter	Ti	Al	Fe	Cu
number of trajectories	2655	6007	3610	853
mean velocity $\bar{v}$ [m/s]	59.1	49.3	37.5	19.3
standard deviation σ_v [m/s]	32.6	39.2	23.5	13.6

mental arrangements used in these references. Specifically, the discharge current rate of rise $\dot{I}_d$ is notably higher in our experiment. Since the evolution of active spot surface increases linearly with current, and spots cannot ignite again shortly after being active, the propagation speed of cathode spots is naturally linked to $\dot{I}_d$. Similar conclusions have been drawn by I. Beilis in [187], with low current vacuum arcs under the influence of an external magnetic field.

According to this combination of simultaneous spot ignition and no backpropagation, the propagation speed of the spots may be related to a value called the splitting frequency, for which a model was proposed by Harris et al.[185]. According to this model, the initiation of new spots would not depend solely on the thermal parameters of the cathode surface, but also on the interactions between the arc plasma and the cathode surface. The expression for the spot splitting frequency ν being :

$$\nu = 0.071 \left(\frac{J_s}{J_{is}} \right) \frac{3\pi V_p \Lambda \sigma_{ei} v_{crit}}{2r_s I_s} \left[1 + \left(\frac{r_s}{R_s} \right)^2 \frac{2\epsilon_0 V_p v_{crit}}{J_{is} \Lambda^2} \right] \quad (5.1)$$

with J_s is the current density at the spot surface, J_{is} the ion current density at the spot surface, V_p the plasma potential in the ionisation zone, Λ the or height of the ionization zone over the spot, σ_{ei} the electrical conductivity of the ionization zone plasma, I_s the current at the spot, r_s the spot radius, R_s the distance between the center of the two spots and v_{crit} the speed of ions leaving the ionization zone towards the cathode. This equation is not strictly applicable in our case, as many parameters cannot be determined. However, where the model becomes interesting here is in highlighting the strong dependence of the splitting frequency on the spot size. According to equation 6, it can be seen that larger spots tend to exhibit lower splitting frequencies and consequently, as previously discussed, lower propagation velocities. The atomic mass of the cathode material is also linked to this splitting frequency, through both v_{crit} and Λ . Assuming that the expression for the critical ion velocity mentioned in the reference is in the form:

$$v_{crit} \approx \sqrt{\frac{k_B T_e}{m_i}}, \quad (5.2)$$

one can deduce in equation 6 that a heavier atom will make the splitting frequency decrease. It can therefore be concluded that a cathode material with a higher atomic mass should present a lower spot propagation velocity.

Comparing the ordering of atomic masses and spot propagation velocities, the ordering appears to be preserved except in the case of aluminum. However, it should be kept in mind that aluminium exhibits many very diffuse and faint spots that are not tracked in the present study. By the naked eye, these untracked spots seem to show a high propagation velocity. As mentioned in previous sections, the tail of the velocity distribution for aluminum is likely more significant than what is measured,

which would result in an increased average propagation velocity. In conclusion, although multiple factors are involved, atomic mass undoubtedly plays a role in the spot propagation velocity. One must yet consider that the mentioned model does not provide an explanation for the tangential propagation of the spots, nor for the wide dispersion of velocities observed for a given material. The underlying mechanisms of cathode spot propagation remain then a subject of ongoing debate.

Chapter VI

Performance: Thrust measurements

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1 Torsional thrust balance dynamics

The architecture of the balance developed in this study is described in chapter II. The dynamics of torsional thrust stands, when not subjected to any external force, are similar to those of a damped oscillating pendulum. When the PJP fires, the balance dynamics correspond to those of a forced damped oscillating pendulum. The purpose of this chapter is twofold: to detail the thrust acquisition method, which combines a mathematical model with experimental measurements, and to analyze the PJP thrust measurements with different cathode materials.

Since the thrust of a single PJP pulse induces a very low signal, even with the present balance and sensor, the chosen approach is to perform a resonance based measurement. The principle is as follows: the natural frequency of the balance is first determined; the PJP is subsequently pulsed at this frequency, in phase with the rising edge of the balance oscillation. This way, each pulse constructively amplifies the oscillation of the balance, and the difference in amplitude between successive peaks allows us to determine the energy injected into the oscillating system, from which the thrust can be derived. The displacement of the balance being then maximized, so is the signal-to-noise ratio and the measurement accuracy.

1.1 Damped pendulum

Mathematical model

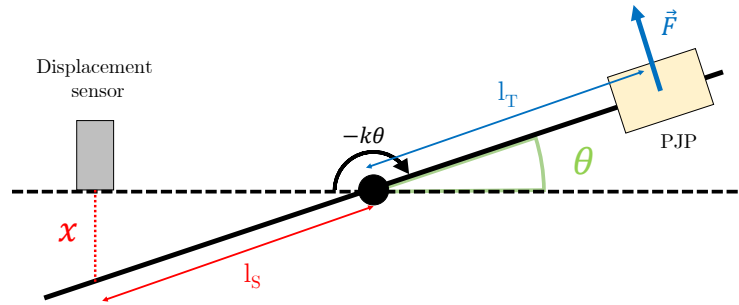


Figure 6.1: Simplified diagram of the torsional thrust stand, as seen from the top.

A simplified diagram of the balance is shown in figure 6.1. The thrust F is assumed to be applied at the center of gravity of the thruster, and the barycenter of the balance is aligned with its axis of rotation. This condition is ensured by a counterweight system, which is not depicted on the present figure. Let I the moment of inertia $I = ml_T^2$ where m is the mass of the balance with the PJP mounted on it and l_T the arm length of application of the force. With k the stiffness of the flexural pivot and γ the damping coefficient, the differential equation ruling the dynamics of the balance is:

$$\ddot{\theta}(t) + \frac{\gamma}{I}\dot{\theta}(t) + \frac{k}{I}\theta(t) = 0 \quad (6.1)$$

Where the variable $\theta(t)$ is the angle between the arm and the equilibrium position of the thrust stand, as a function of time. Let the damping ratio $2\lambda = \gamma/I$ and natural frequency $\omega_0^2 = k/I$. Equation 6.1 becomes

$$\ddot{\theta}(t) + 2\lambda\dot{\theta} + \omega_0^2\theta(t) = 0 \quad (6.2)$$

Such second-order homogeneous differential equations are solved by first addressing the so-called characteristic equation, which takes the form:

$$r^2 + 2\lambda r + \omega_0^2 = 0 \quad (6.3)$$

Here, the form of the solutions depend on the sign of the discriminant $\Delta = 4\lambda^2 - 4\omega_0^2$. Given the characteristics of the balance, see next section, we know that $\omega_0 \gg \lambda$. The oscillation regime where $\Delta < 0$ is called the underdamped regime. In that case the general solution to the differential equation takes the form:

$$\theta(t) = e^{-\lambda t}(A \cos(\omega t) + B \sin(\omega t)) \quad (6.4)$$

with $\omega = \sqrt{\omega_0^2 - \lambda^2}$ the pseudo-frequency and where the constants A and B are given by the initial conditions of the dynamic system, here:

$$A = \theta_0 \quad \text{and} \quad B = \frac{\dot{\theta}_0 + \lambda\theta_0}{\omega} \quad (6.5)$$

where $\theta_0 = \theta(0)$ is the angle at $t = 0$ and $\dot{\theta}_0 = \dot{\theta}(0)$ is the initial angular velocity. In the model presented here and for the remainder of this chapter, the equations describe the evolution of the angle θ through the moment of inertia. Experimentally, however, what is measured is a displacement from equilibrium position at the tip of the balance arm. All θ values are therefore converted into displacement values using the relation:

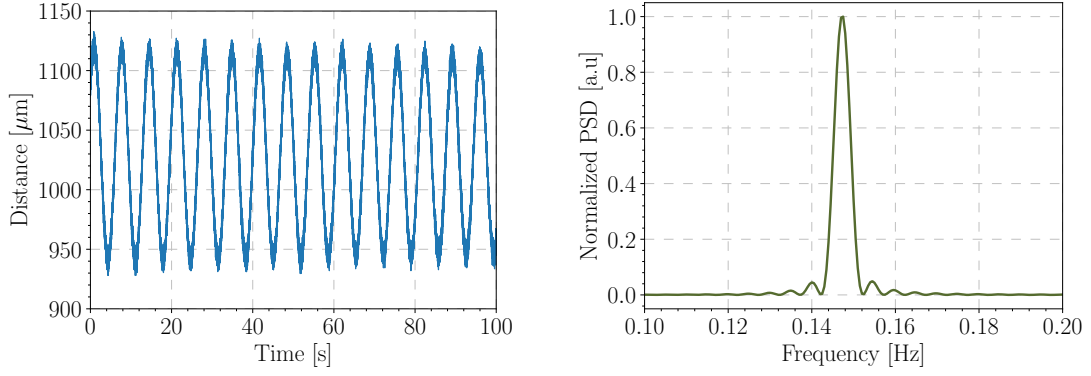
$$x(t) = l_S \sin \theta(t) \quad (6.6)$$

with l_S the length between the axis of rotation and the displacement sensor.

Experimental characterization

To determine the natural frequency of the balance, a certain number of PJP pulses are first triggered to make the balance oscillate. The oscillations are then recorded with the sensor for a few seconds. The natural frequency of the balance can be determined by performing a Fourier analysis on the oscillations. An example is shown in figure 6.2, where the natural frequency of the balance is 147.5 mHz. By definition, the natural frequency should remain unchanged when switching cathode materials. Since all cathodes have nearly the same mass, differing only by a few μg , the proportion of mass variation is negligible given the inertia of the balance with the thruster mounted on. However, to ensure measurement accuracy, the natural frequency determination is repeated after each cathode change because small variations might still occur from one measurement to the other. When the cathode is changed, the thruster is handled, and the equilibrium position slightly shifts due to the stiffness of the electrical connections. The natural frequency may then, in practice, change by approximately ± 0.5 mHz.

Once the natural frequency is retrieved, the only remaining unknown variable in equation 6.5 is the damping ratio λ . The equation is turned into a function which is then fitted to the experimental data by means of a Levenberg-Marquardt algorithm. In the example of figure 6.2a, $\lambda = 0.0016$ and this value is kept for analysis of the iron dataset.



(a) Oscillation induced by one PJP pulse on the balance. Iron cathode. (b) Periodogram associated to the signal. Zoomed-in performed around the natural frequency (noise removed).

Figure 6.2: Characterization of the natural frequency of the balance, here 147.5 mHz. Iron cathode.

1.2 Forced movement and resonance

Mathematical model

In the presence of an external exciting force applied to the pendulum, the equation becomes:

$$\ddot{\theta}(t) + 2\lambda\dot{\theta} + \omega_0\theta(t) = \frac{l_T}{I}F(t) \quad (6.7)$$

The general solution of this equation is expressed as the sum of the homogeneous solution $\theta_h(t)$ and a particular solution $\theta_p(t)$ to the non-homogeneous equation, in other words:

$$\theta(t) = \theta_h(t) + \theta_p(t) \quad (6.8)$$

where θ_h is the solution of the homogeneous equation as written in equation 6.4:

$$\theta_h(t) = e^{-\lambda t} \left[\theta_0 \cos(\omega t) + \frac{\dot{\theta}_0 + \lambda\theta_0}{\omega} \sin(\omega t) \right] \quad (6.9)$$

Using the Green's function method [189], one can retrieve the particular solution for the forced underdamped harmonic oscillator:

$$\theta_p(t) = \frac{l_T}{I\omega} \int_0^t F(\tau) e^{-\lambda(t-\tau)} \sin[\omega(t-\tau)] d\tau \quad (6.10)$$

In the case of the PJP, the thrust $F(t)$ generated by the thruster can be described as a force of amplitude F_0 applied over a very short duration τ_p :

$$F(t) = \begin{cases} 0 & t < 0 \\ F_0 & 0 \leq t \leq \tau_p \\ 0 & t > \tau_p \end{cases} \quad (6.11)$$

A square function is used here as an approximation for the thrust profile. In reality, the PJP discharges are better represented by a Weibull-like function, but using it would significantly complicate the calculations. Since the pulse duration τ_p is very short compared to the oscillation period of the balance, this approximation is not problematic in this context. Solving the general equation 6.8 with this expression, while considering the system initially at rest with $\theta_0 = 0$ and $\dot{\theta}_0 = 0$ yields the dynamics of the balance for a single PJP pulse. However, in practical terms, the balance is never truly at rest. It always oscillates slightly, as it is set into motion during the pumping of the vacuum chamber, when the air volume is evacuated through the pumps. Vibrations from the pumps and human activity in the room also contribute to the oscillations of the balance. In addition, experimental observations showed that a single PJP pulse is not powerful enough for its induced displacement to be distinguished from the natural motion of the balance in its environment. This is why, as mentioned at the beginning of this chapter, impulse bit measurements were performed with the resonance method. Consequently, the mathematical model must be adapted, particularly in the way the thrust force is defined. In resonance mode, the thrust is applied periodically, synchronized with the natural frequency of the balance, and can be written as follows:

$$F(t) = \begin{cases} F_0 & (N-1)T \leq t \leq (N-1)T + \tau_p \quad (1) \\ 0 & (N-1)T + \tau_p < t \leq NT \quad (2) \end{cases} \quad (6.12)$$

With $N = 1, 2, \dots, \infty$ being the number of the period. The analytical development of this model was detailed by Zhang et al. in reference [190]. We will focus here on the expression of $\theta_{\text{peak}}(N)$, which is the peak value of θ at period N . This value is expressed as follows:

$$\theta_{\text{peak}}(t) = \frac{F_0 l_T \tau_p}{I \omega_0} e^{-\lambda(t_{\text{peak}}(N) - \tau_p)} \sum_{n=1}^N e^{(n-1)\lambda T} \quad (6.13)$$

Knowing that the phase is $\phi = \arctan(\omega/\lambda)$, and assuming that τ_p is negligible compared to the oscillation period T , the time of occurrence t_{peak} of θ_{peak} can be written as:

$$t_{\text{peak}}(N) = \frac{\phi}{\omega} + (N-1)T \quad (6.14)$$

And the equation 6.13 can be simplified as a function of the impulse bit, recalling that $I_{\text{bit}} = F_0 \tau_p$:

$$\theta_{\text{peak}}(N) \approx I_{\text{bit}} \frac{l_T}{I \omega_0} e^{-\lambda \phi / \omega} \frac{1 - e^{-N \lambda T}}{1 - e^{-\lambda T}} \quad (6.15)$$

Implementation

Once the natural frequency of the balance has been determined, the PJP is pulsed at this exact frequency, synchronized with the rising edge of the oscillation. In principle, the resonance pulsation is given by $\omega_r = \sqrt{\omega_0^2 - \lambda^2}$ but since here $\lambda \ll \omega_0$ we can approximate that $\omega_r \approx \omega_0$. As in the damped oscillations model, the task is to identify a function that can be fitted to the experimental signal. Because the presence of a sum of discretized terms is not optimal for implementation in Python,

we set $N = t/T$. Then, we know that the expression for θ_{peak} represents the peak amplitude at each period. This expression, coupled with a sinusoidal function of pulsation ω and phase ϕ , can be considered a particular solution of the differential equation governing the evolution of $\theta(t)$. We therefore define:

$$\theta(t) = \theta_h(t) + \theta_{\text{peak}}(t) \sin(\omega t + \phi) \quad (6.16)$$

as the function to be fitted. In this expression, the only value that is not yet known at this stage of the analysis is I_{bit} . Hence, the model is fitted to the dataset with the impulse bit let as a variable. Figure 6.3 shows the result of the fitting on an acquisition performed with an iron cathode. It can be seen that the model follows accurately the oscillations of the balance over the entire duration of the acquisition. A zoomed-in view is presented in figure 6.3b, where a peak can be observed in the raw data. This peak corresponds to the electromagnetic perturbation associated with the PJP discharge. It is seen that this peak is synchronized with the rising edge of the oscillation, and that its duration is negligible compared to the balance oscillation period.

Another method, which we will refer to as the increment method, makes it possible to determine the impulse bit at each period. Indeed, if one sets:

$$\Delta\theta_{\text{peak}}(N) = \theta_{\text{peak}}(N) - \theta_{\text{peak}}(N - 1) \quad (6.17)$$

then, from equation 6.15 comes

$$\Delta\theta_{\text{peak}}(N) = I_{\text{bit}} \frac{l_T}{I\omega_0} e^{-\lambda\phi/\omega} e^{-(N-1)\lambda T} \quad (6.18)$$

and in the same way as for the fitting method, I_{bit} remains the only unknown here. By measuring the amplitude difference between two periods, I_{bit} can be directly obtained, since all other values are either known or have been experimentally determined beforehand. The advantage of this method is that it permits to retrieve statistics of the I_{bit} values over a complete thrust acquisition dataset.

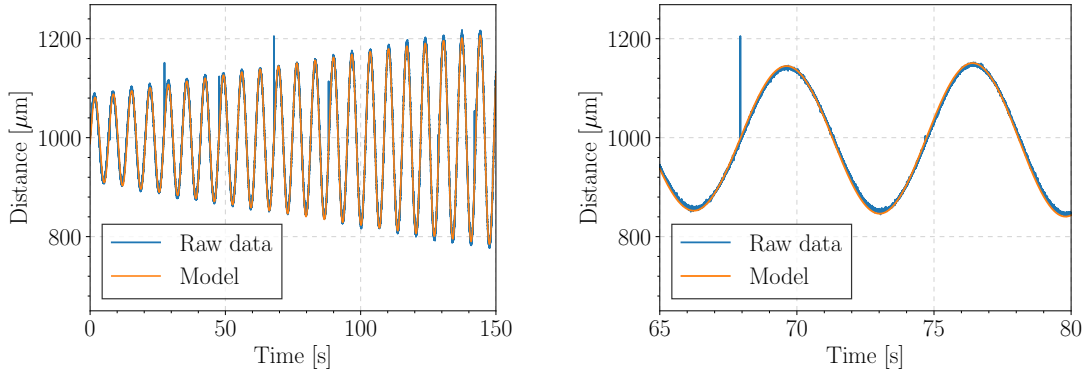
Results of the analysis on the data from figure 6.3a are presented in table 6.1. Note that the value of impulse bit obtained with the increments method is the average value over the whole acquisition.

Table 6.1: Results of the model fitting to the Fe-001 dataset.

Parameter	Value
λ	0.0016
θ_0 [rad]	-8×10^{-5}
$\dot{\theta}_0$ [rad/s]	16.2×10^{-5}
I_{bit} (fitting method) [$\mu\text{N s}$]	27.5
I_{bit} (increment method) [$\mu\text{N s}$]	24.5

2 Average method

Another method consists in operating the PJP at a frequency higher than the natural frequency of the balance. The equilibrium position of the balance will then shift,



(a) Resonance thrust acquisition. Iron cathode. (b) Zoomed-in view of two oscillation periods.

Figure 6.3: Oscillations of the balance, with the PJP pulsed at resonance frequency. The fitted forced oscillations model is shown in orange. Iron cathode.

and after calibration of the balance, the thrust exerted by the thruster can be determined [191]. This method is simpler to implement from a theoretical standpoint but requires calibrating the balance with the application of known forces. The advantage of this method is that it directly provides the thrust produced by the thruster for a given operating frequency.

Calibration

Example of a calibration trace is shown in figure 6.4. The acquisition displays seven measurement points corresponding to the application of different forces by the voice coil. Because the damping is very low, the oscillations are only weakly attenuated. What should therefore be looked at during the calibration process is the balance deflection from its original equilibrium position, when no force is applied. The different force–displacement pairs are reported in table 6.2.

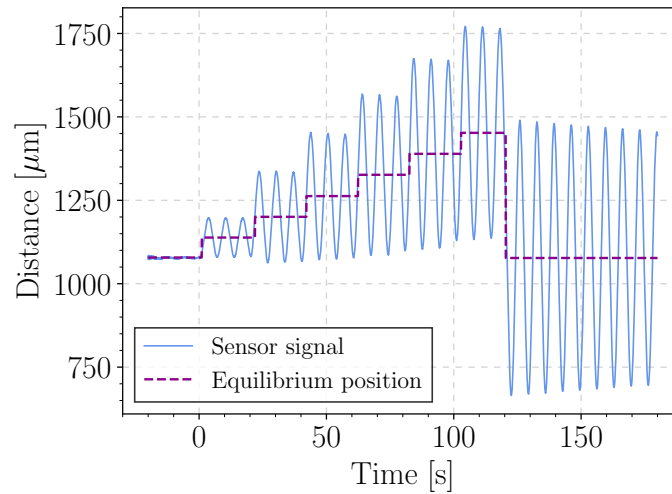


Figure 6.4: Sensor signal recovered during calibration of the balance. The shift in equilibrium position is shown in purple.

Since the calibration force is applied at the voice coil, it is necessary to determine the equivalent force at the thruster. To do so, we apply Varignon's theorem:

$$F_{cal}l_{VC} = F_{eq}l_T \quad (6.19)$$

with F_{cal} the applied calibration force through the voice-coil, l_{VC} the length of torque arm between the voice coil and the flexural pivot, l_T the length of arm to the thruster and F_{eq} the equivalent force. To determine the sensitivity of the balance, a linear regression of the form $y = ax + b$ is performed on the displacement-vs- F_{eq} curve. The coefficient a represents then the sensitivity of the balance, expressed in m/N, and the intercept b is forced here to be 0. The calibration curve corresponding to figure 6.4 is shown in figure 6.5. The calculated sensitivity of the balance is there $254 \mu\text{m}/\text{mN}$.

Table 6.2: Calibration points of the thrust stand.

Applied force at l_{VC} [mN]	Equivalent force at l_T [mN]	Displacement [μm]
0	0	0
0.376	0.291	61.756
0.754	0.582	123.293
1.131	0.872	184.987
1.208	1.163	248.074
1.885	1.454	311.456
2.262	1.744	375.118

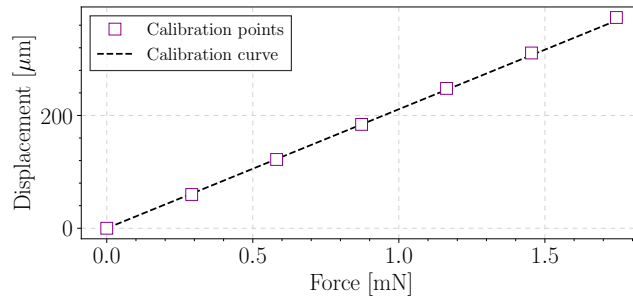


Figure 6.5: Calibration curve of the thrust balance.

Aquisition

Once the calibration has been performed, the PJP is pulsed at a given frequency $\omega_{PJP} > \omega_0$. In doing so, the balance oscillates around a new equilibrium point, from which the thrust of the PJP can be determined. Figure 6.6 shows acquisition examples where the PJP is operated at 1 Hz for 20 s, then stopped for 20 s, and pulsed again at 1 Hz for 20 s. A clear deflection of the equilibrium position can be observed, as highlighted in purple. The titanium example clearly illustrates the differences in the balance natural oscillation amplitudes regarding the datasets. In this case, the measurement was performed shortly after pumping the air out of the chamber, the oscillations therefore exhibit a large amplitude. Nevertheless, the

shift in the equilibrium position due to the PJP firing is clearly visible. The first deflection during this acquisition indicates a thrust level of $73\text{ }\mu\text{N}$ whereas the second one reads $99\text{ }\mu\text{N}$. The budget error of the balance is around 5%, see appendix for more details. For copper, see figure 6.6b, the balance natural oscillation is smaller in amplitude as the period between the pumping and the acquisition was longer. The first deflection in that dataset reads $43\text{ }\mu\text{N}$ and the second $46\text{ }\mu\text{N}$. A significant difference in thrust can be observed between the two materials, as well as between measurements of the same material. It has already been reported in chapter III that the ion ejection velocity is not constant from one PJP pulse to another, sometimes exhibiting a large statistical spread. It has also been observed in chapter IV that the number of cathode spots (eg surface covered) varies from one pulse to another. Following these observations, it appears reasonable that the thrust would also vary from one discharge to the next.

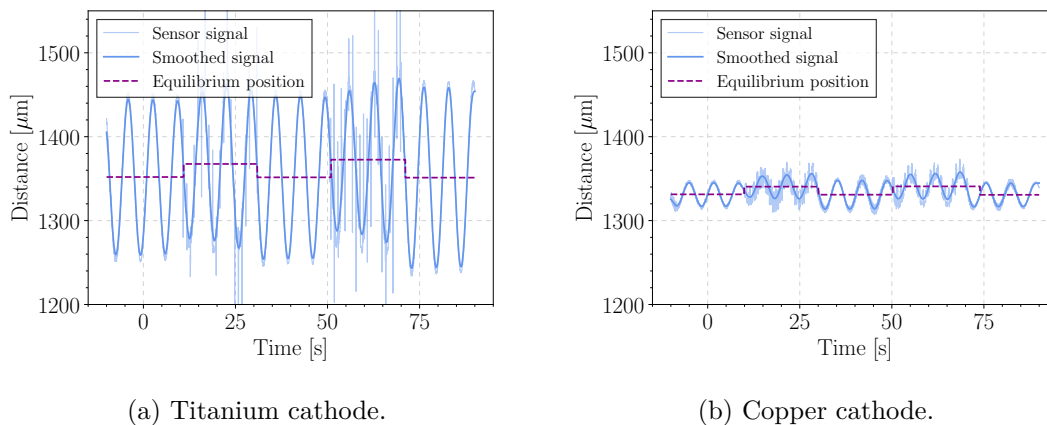


Figure 6.6: Example of thrust measurements with the two cathode materials. PJP pulsed at 1 Hz.

3 Results

3.1 General observations

The average thrust values are given in table 6.3. The values are presented in order of decreasing average thrust. The first observation is that the ordering is preserved with respect to the trends already noted for total active spot surface, see chapter IV. The trend indicates a clear link between the spot surface and the thrust. This interesting result needs a dedicated discussion, which is the subject of the next subsection.

Regarding the resonance method, the values of impulse bit obtained according to the two computation methods are presented in table 6.4. It can be observed that the I_{bit} values, although similar in magnitude, slightly differ depending on the computation method. For some materials, such as Al and NiCr, the difference between the increment and the fitting method can be significant. This naturally raises the question of which approach is more reliable. Since both methods result from the same theoretical model, the difference arises in their application to the experimental data. The fitting method relies on a global analysis of the system

Table 6.3: Thrust values according to the average deflection method.

Material	Mean thrust (1 Hz) [μN]	standard deviation [μN]
Ti	71.7	19.2
Al	50.8	10.3
NiCr	50.7	4.4
Fe	42.4	2.8
Cu	38.6	6.9

and is sensitive to errors in the initial conditions. Likewise, the search for a global behavior necessarily comes at the expense of some accuracy at the local level. In particular, it may happen that the PJP fails to pulse even though the trigger signal is sent. As already discussed in the previous chapters, this behavior occurs due to pollution of the insulators with deposition from the cathode material. In such cases, the balance enters a free damped regime, and the oscillation no longer exhibits a peak increment but show instead a decay. The model will then fit this segment, but the result is unrealistic and introduces an error in the output values. This is where the increment method proves to be especially useful. If the amplitude increment is negative from one period to the next, it indicates that the PJP did not pulse, and this point is excluded from the calculation of the average impulse bit. The increment method is therefore more robust to variations and provides additional insight into the distribution of impulse bit values.

Table 6.4: Impulse bit values according to the two resonance methods.

Material	I_{bit} (increments) [$\mu\text{N s}$]	I_{bit} (fitting) [$\mu\text{N s}$]	Difference
Ti	44.1	42.4	4%
NiCr	29.1	32.5	10 %
Fe	21.6	20.1	7 %
Cu	15.4	15.5	-
Al	13.0	16.5	21%

The only peculiarity observed is that, for the resonance measurements, aluminium shows large discrepancies between the average thrust and the resonance-based values of impulse bit. This difference is so significant that it would place aluminum last in the impulse bit ordering, if the increments method is taken as reference. When examining the raw data, without applying the forced oscillation model, it is found that the amplitude increments at each period are indeed the lowest in the case of aluminium. This is quite surprising, given that in the average thrust measurements, aluminium ranks among the materials with relatively high thrust levels, see appendix. At this stage, the most likely explanation is a mechanical issue with the balance during the resonance measurement acquisition.

Regarding the amplitude increments at each oscillation period, for all materials it is observed that the amplitude increments do not strictly follow the model from one pulse to the next. This indicates that the thrust is not exactly the same from one pulse to another. Hints of such behavior had already been noted in ion velocity measurements as well as in the fractal dimension of cathode spots, see previous chapters. Each PJP discharge is different and appears to erode a variable portion of the cathode material. If the eroded mass Δm per pulse varies from one discharge

to another, it naturally follows that the different pulses will produce different thrust levels. Similarly, in the thrust measurements obtained using the averaging method, the retrieved thrust values vary slightly from one firing period to another, as indicated by the standard deviations given in table 6.3. This figure supports the idea that the thrust level differs from one discharge to the other, with relatively limited control over the exact thrust level produced for a given time period. The distributions of the I_{bit} obtained for different resonance acquisition datasets are shown in figure 6.7. Box plots are drawn to ease visualisation, and the median value is highlighted in red.

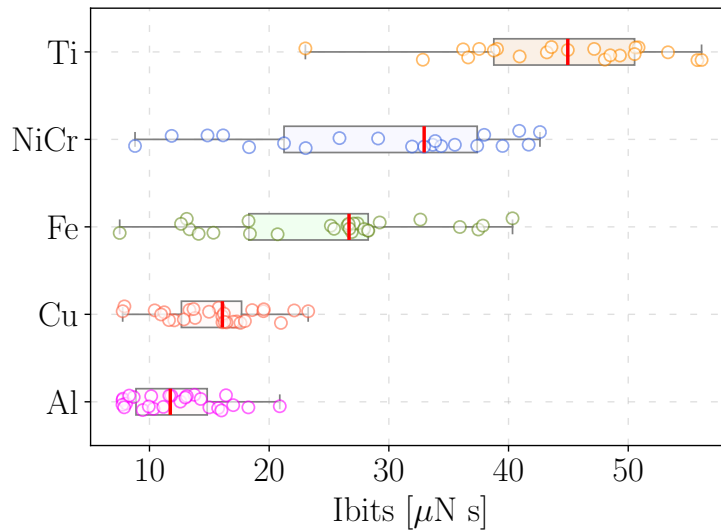


Figure 6.7: Distributions of I_{bit} on a representative dataset for each cathode material. The median value is shown in red, and a box plot is drawn to ease visualization.

It is noted that some materials such as NiCr and Fe exhibit very large standard deviations. From an engineering perspective, this implies that the choice of cathode material can directly influence the precision of spacecraft maneuvers. For higher thrust, Ti is preferable due to its larger average impulse bit, while for greater precision, Cu would be the better choice given its smaller dispersion in impulse bit values.

Finally, it should be noted that the average thrust depends on the operating frequency of the thruster [20]. This dependence arises from the fact that at higher operating frequencies, the mass flow rate is naturally greater. The same reasoning applies to operation under different duty cycles. It has also been observed that for different operating frequencies, the mass eroded per pulse varies, increasing with the frequency [30]. Here, the choice has been made to operate the PJP at 1 Hz to easily compare the mean thrust values and the impulse bit values in $\mu\text{N s}$. The maximum operating frequency of the PJP is 2 Hz, and no significant evolution of thrust was observed with the present balance. Operating the PJP at lower frequencies would have brought it closer to the natural frequency of the balance and would have biased the calculation of the average thrust.

Influence of the computation method

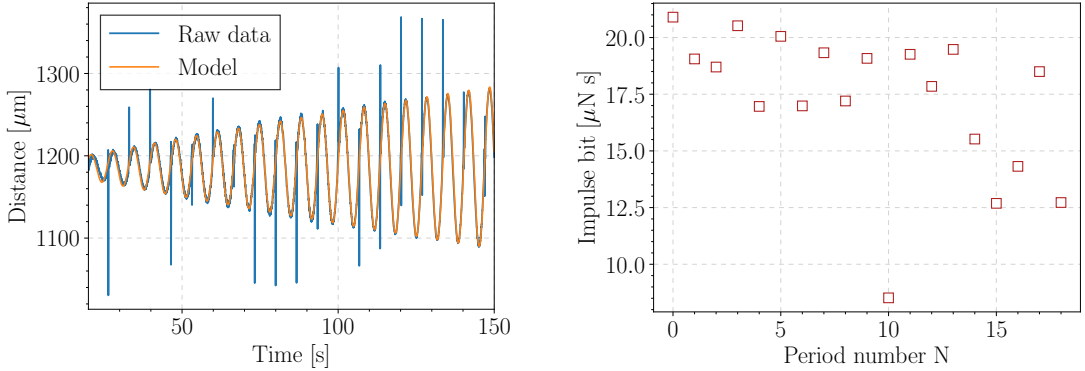
Although the ordering of the impulse bits is preserved, except for aluminum, a factor ranging from 1.3 to 2 is observed between the impulse bit values and the thrust values at 1 Hz. However, these quantities should theoretically be identical, given the definition of the impulse bit:

$$I_{\text{bit}} = m_{\text{bit}} v_e = \dot{m} \tau_p v_e \quad (6.20)$$

One can see the impulse bit is the impulse produced by a single pulse. For a thruster operating at a frequency $f_{\text{thruster}} \geq 1\text{Hz}$, the impulse bit is simply :

$$I_{\text{bit}} = \frac{T}{f_{\text{thruster}}} \quad (6.21)$$

which is why the values of impulse bit and thrust at 1 Hz should be the same. The issue most likely arises from the implementation of the stiffness constant in the model, and more specifically from discrepancies between its theoretical value and the real one. When plotting the evolution of the impulse bit over the course of an acquisition, see figure 6.8, it can be observed that the I_{bit} values tend to decrease after several periods. The example is taken from the aluminium dataset, as the low standard deviation of the impulse bits with this cathode material makes the decreasing trend easier to visualize.



(a) Resonance thrust acquisition. Al cathode. (b) Impulse bits retrieved as function of the period number.

Figure 6.8: Aluminium cathode resonance acquisition, and associated impulse bits retrieved with the increments method.

In a torsion spring, the restoring force increases with θ . Forced damped mechanical oscillators driven at resonance frequency eventually reach a steady-state regime in which this restoring force balances the excitation force. Hence, it is likely that in this case, the force exerted by the thruster is too low to assume that the resonance acquisitions remain in a purely amplifying transient regime. If the simplifications in the model are not valid, the thrust is probably underestimated since the contribution of the restoring force could also be underestimated. In other words, the steady-state regime is reached too quickly to assume that the amplitude increments purely increase according to the magnification factor

$$\alpha(N) = \frac{1 - e^{-N\lambda T}}{1 - e^{-\lambda T}} \quad (6.22)$$

and they should instead be considered as decreasing. In doing so, the impulse bit values would be expected to remain approximately constant throughout the acquisition.

3.2 Influence of the cathode material

The thrust and impulse bit values, regardless of the method used, follow the same ordering as the fractal dimension and the total surface area covered by the spots during a discharge. The only material for which a notable discrepancy exists between the three methods is aluminum, as previously mentioned. Figure 6.9 shows the evolution of thrust as a function of the fractal dimension, and figure 6.10 the evolution of thrust as a function of the total surface area covered by the spots. The bars represent the standard deviations of the different values. It can be seen that the trend is very clear, despite the large dispersions.

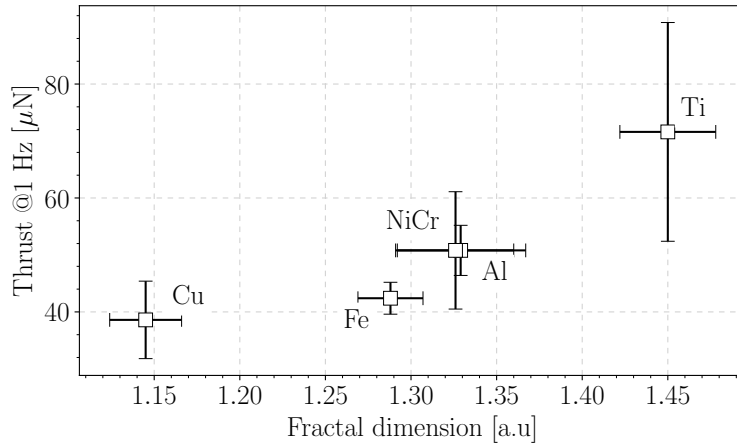


Figure 6.9: Thrust as a function of fractal dimension of the cathode spots pattern. The error bars represent the standard deviations. Values of fractal dimension taken from chapter IV

A theoretical model of the thrust generated by the vacuum arc, based on momentum conservation, was proposed by Polk et al.[192]:

$$F = \frac{m_i f_i I_d v_i}{e \bar{Z}} C_t(\bar{L}, \bar{r}_a) \quad (6.23)$$

where m_i is the ion mass, f_i the ion current fraction with regards to the discharge current $I_i = f_i I_d$, v_i the ion velocity, $\bar{Z}$ the ion mean charge number and $C_t(\bar{L}, \bar{r}_a)$ a thrust correction factor that accounts for plume divergence, electrodes geometry and cathode material deposition on the thruster. It can be seen here that the force depends on measurable quantities such as m_i , I_d et v_i , as well as on quantities that are somewhat more difficult to determine experimentally such as $\bar{Z}$ or f_i . The coefficient C_t can be calculated using the expression given in Polk's original work. What should be emphasized here is the strong intrinsic influence of the cathode

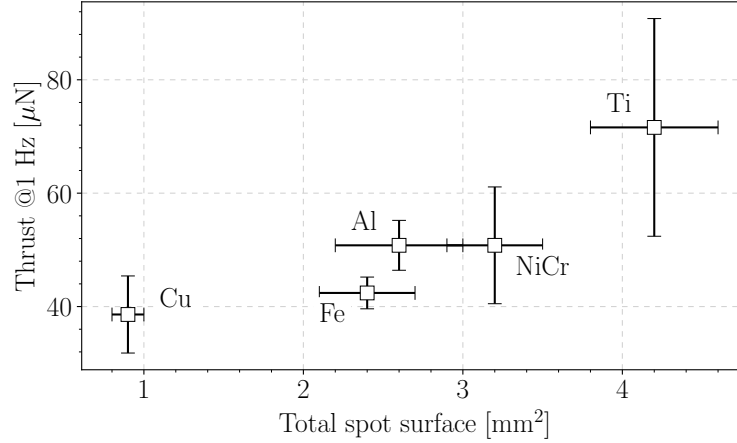


Figure 6.10: Thrust as a function of the total spot surface covered during the discharge. The error bars represent the standard deviations. Values of surface taken from chapter IV.

material on the thrust. This reference predicts the thrust ordering listed in table 6.5, considering a low current VAT operating under identical discharge conditions for each material:

Table 6.5: Predicted performance of VATs according to the cathode material [192].

Material	Predicted thrust [mN]
Cr	3.27
Cu	2.73
Ti	2.70
Ni	2.51
Fe	2.50
Al	1.6

This ordering is not the same as the one observed in the case of the PJP, and the large difference in thrust level observed between the two materials is not predicted. However, Polk notes that for certain materials such as Ti, the model overpredicts the performance by 36–50%. Moreover, this model is based on data from arc with a discharge current of about 100 A and pulse duration of 250 μ s. In comparison, the PJP operates with a discharge current nearly 30 times higher and a pulse duration almost 10 times shorter. This significantly longer pulse duration and the higher operating frequency are most likely the reasons for the much higher thrust level predicted by the model compared to the observations made with the PJP. On the other hand, parameters such as f_i and $\bar{Z}$ are very likely to differ in the case of the PJP [65, 120] due to the higher discharge current and higher current growth rate. It was also noted in a previous chapter that the evolution of v_i scales in a non linear manner with the discharge current growth rate [140]. Similarly copper was higher than that of titanium in the case of the PJP [193], contrary to the data used to develop the model. This highlights the strong influence of discharge conditions on the model outcomes.

Finally, and most importantly, this model is based on the operation of a single

spot. It does not account for the multitude of spots that a vacuum arc can generate, nor for the differences in the number of simultaneously active spots between materials. However, it can clearly be seen here that the thrust evolves in the same manner as the total surface area covered by the spots, which depends mainly on their propagation velocity. When plotting the spot velocity as a function of thrust, a very clear trend can be observed despite the large standard deviations, see figure 6.11. Hence, a clear link can be established between the cathode spot dynamics and the thrust produced by a vacuum arc thruster. This also indicates that the macroscopic dynamics of cathode spot groups are more influential in the thrust level achieved than the microscopic phenomena observed at the individual spot level.

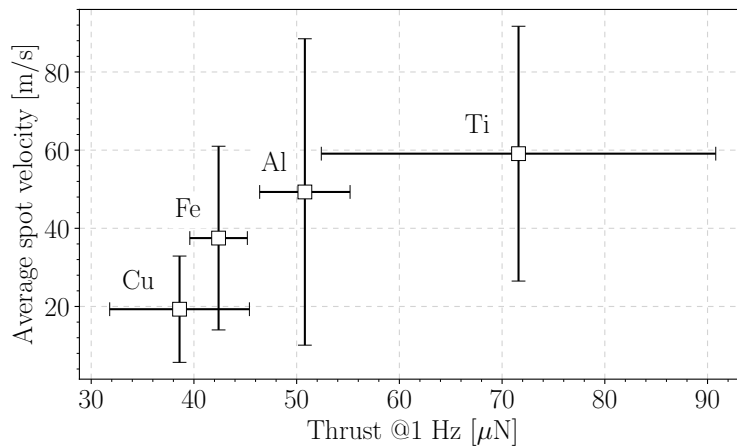


Figure 6.11: Average spot velocity as a function of the average thrust at 1 Hz. The error bars represent the standard deviations. Values of fractal dimension taken from chapter V.

3.3 Comparison with other VATs performance

The PJP is a rather unique VAT in the literature, due to its high operating current and very short pulse duration. It is therefore difficult to directly compare its impulse bit and thrust values with the other VATs found in the literature. A first version of the PJP, dissipating 2 J per pulse, yielded impulse bit values of about 30 $\mu\text{N s}$ and a thrust of 57 μN at 2 Hz [20]. In this reference, neither the cathode material nor the pulse duration is specified, making the exact comparison rather complex. However, the same order of magnitude in thrust level has been measured in COMAT, for the PJP H2020 equipped with a Ti cathode and operating at 2 Hz [194].

Comparative values reported in the literature for different VATs are given in table 6.6. It can be seen that the performances are generally lower, except for the reference by Lun et al. The main difficulty when attempting to compare with our experimental data is that, in most cases, the operating frequencies are not specified. In the given example of Lun et al. [195], it is stated that the thruster was pulsed at frequencies of up to 20 Hz, but without specifying which frequency was set for the thrust measurements. A relevant reference for comparison is the work of Zhang et al. [190], where an impulse bit value of 50 $\mu\text{N s}$ is reported for a VAT releasing

3 J in the discharge through a capacitor bank. However, neither the peak discharge current nor the electrode voltage is specified.

It is commonly accepted that the instantaneous thrust increases with the discharge current [26]. More specifically, some references state that approximately 0.2 mN of force can be extracted per ampere of discharge current [196–198]. The impulse bit could then be approximated by:

$$I_{\text{bit}} = F_0 \tau_p \approx I_d \tau_p 0.2 \times 10^{-3} \quad (6.24)$$

which would read an impulse bit of around 25 $\mu\text{N s}$ for the PJP. This value would be consistent with the impulse bit values obtained by the resonance method, but once again, it is slightly lower than the thrust values measured at 1 Hz. One possible explanation for the discrepancies between theoretical and experimental values for the PJP once again lies in the discharge current ramp, reaching up to almost 10^9 A/s . Such a steep growth rate results in a strong electron–ion friction, leading to high ion kinetic energies at the thruster exhaust, see chapter III. The ion ejection velocity is notably higher in the case of the PJP, and this undoubtedly impacts the evolution of thrust as a function of the discharge current.

Table 6.6: Different VAT performance and characteristics.

Reference	Ibit	Thrust	Peak I_d	Voltage	Pulse duration
This work	15-40 $\mu\text{N s}$	38-71 μN	3.5 kA	250 V	35 μs
Kuhn et al. [19]	-	5-15 μN	20 A	25 V	200 μs
Lun et al. [195]	-	20-50 mN	150-250 A	-	400 ns
Kronhaus et al. [199]	0.3-1.5 $\mu\text{N s}$	5-22 μN	40 A	27-49 V	250-450 μs
Kronhaus et al. [31]	0.15 $\mu\text{N s}$	4.4 μN	17 A	28 V	200 μs

Chapter VII

Hybrid MC-PIC simulations

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1 Numerical simulations of vacuum arcs

The multi-scale and multi-physics nature of the vacuum arc presents a significant challenge for numerical simulation. A wide variety of modeling techniques has been developed to date, each of them representing a different trade-off between physical fidelity and computational cost. The selection of a particular method depends on the specific phenomena being investigated, and understanding these trade-offs is essential to contextualize the hybrid approach at the core of this chapter.

1.1 Fluid approach

Fluid models treat the plasma as a single or multi-fluid continuous medium. These models solve a set of coupled conservation equations for mass, momentum and energy, along with Maxwell equations. This allows the description of the evolution of macroscopic plasma properties like density, velocity, and temperature [200, 201]. The principal advantage of the fluid approach is its computational efficiency. By averaging over microscopic particle kinetics, fluid simulations can model large plasma volumes over extended timescales, up to to milliseconds [202, 203]. This makes them well-suited for investigating global, macroscopic phenomena. For instance, fluid models have been successfully used to study the interaction of the plasma plume with applied magnetic fields in magnetic nozzles, the plasma plume expansion dynamics, the distribution of current within the arc column [204–206].

In the context of vacuum arcs, a 1D stationary two-fluid model was developed by Karimi et al. to describe the supersonic acceleration of the dense plasma jet from a cathode spot [207]. Langlois et al. developed a 2D two-fluid numerical model to describe the diffuse vacuum arc, including the potential influence of an axial magnetic field (AMF) [208]. A key feature of their work is the implementation of energy balance equations for both ions and electrons separately. This model also accounts for different ion flow regimes, based on the current density. At low current densities the flow is considered supersonic, then it becomes subsonic at high current densities. This two-fluid approach is shared by other research teams. Wang et al. presented a 2D axisymmetric magnetohydrodynamic (MHD) model to specifically analyze the influence of different AMF distributions on vacuum arc characteristics [209]. Their model was an improvement upon earlier work by Schade and Shmelev [201], as it incorporated an ion viscosity coefficient and kinetic terms into the momentum and energy equations. Their approach also implemented the Biot-Savart law to calculate the self-induced magnetic field. More recently, Zhao et al. extended this framework to a 3D MHD model to simulate the entire burning process of a AC high-current vacuum arc [210]. This work provides insight into the transient evolution of plasma parameters, correlating the plasma pressure, temperature, and current density distributions with the changing instantaneous arc current.

It should be noted that the computational efficiency of the fluid models comes at the cost of physical fidelity. Most fluid models are built on the assumption that the plasma is collisional and quasi neutral. The latter assumption breaks down severely in the near-cathode region of a vacuum arc. By treating the plasma as a fluid, those models inherently average the electron and ion velocity distribution functions. In particular, according to the theory formulated by Beilis, the near cathode region can be divided into distinct sub-regions, each of them exhibiting specific electron

and ion properties [65]. As an example, fluid models can not resolve the high energy tail of the EEDF that is responsible for stepwise ionization. Moreover, due to the core assumptions of those models, they can not accurately model kinetic effects such as sheath formation. Consequently, while fluid models can provide an accurate description of the bulk plasma flow, their physical relevance comes short when it comes to investigate the fundamental mechanisms at play for generation of multiply charged ions. It can be emphasized that, to date, the majority of fluid models aiming at describing vacuum arcs have primarily focused on the interaction between the inter-electrode plasma and an external magnetic field, with particular attention to axial magnetic fields.

1.2 Kinetic approach: Particle-in-Cell

At the opposite end of the physical relevance spectrum are fully kinetic models. The most notable approach is the particle-in-cell (PIC) method. In a full PIC simulation, both ions and electrons are treated as particles. The trajectories of millions of these particles are integrated in time, based on the Lorentz force, where the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are calculated self-consistently on a grid. The values of both $\mathbf{E}$ and $\mathbf{B}$ are inferred from the charge and current densities produced by the particles themselves. Collisional processes, which are critical in the dense near-cathode plasma, are usually incorporated through stochastic Monte Carlo Collision (MCC) or Direct Simulation Monte Carlo (DSMC) methods[211–213].

In the context of vacuum arcs, numerous PIC models have been developed, focusing on different aspects of arc physics from those addressed by fluid models. In particular, a significant research effort has been focused on modeling the initial breakdown of the arc. One can primarily cite the different works of H. Timko, that developped a one-dimensional PIC model to simulate the plasma generation from a field emitter tip, under electrical breakdown conditions [214]. That model assumed an external electric field and an initial field enhancement factor as starting points. This work was aimed at identifying the general requirements for plasma formation, formulating them in terms of a critical initial local electric field. This initial value of $\mathbf{E}$ was experimentally found to be 10 GV/m for copper. This 1D work was later extended by the same team, presenting a new 2D PIC tool named ARCPIC [215]. A key advancement of this 2D model is its ability to simulate not only the plasma formation but also the subsequent propagation of the cathode spot across the cathode surface. This model bridged the gap between the nano-scale mechanisms of field emission and the macroscopic phenomena of cathode spot.

More on the 2D approach, Wang et al. later developed a 2D PIC/MCC model to investigate plasma formation during the vacuum breakdown stage [216]. A distinguishing feature of this model is its inclusion of multiple ion species. It is able to track electrons, neutrals, and copper ions with charge states going from $Z = 1$ to $Z = 4$. The team modeled the plasma as expanding hemispherically from the emitter, and identified a characteristic structure including a plasma sheath, a quasi-neutral region and an acceleration layer. These simulations confirmed the presence of a potential hump near the cathode, which is responsible for accelerating ions to the high velocities observed in experiments.

It is also worth mentioning here the 2D PIC/MCC model of Lüsken et al., aimed at simulating the vacuum arc thruster from the company Neumann Space [217]. As

already noted in the introduction chapter, the Neumann Space thruster and the PJP are quite similar in terms of discharge conditions. The primary goal of this model was an analysis of the plasma transport and plume dynamics. It employs a simplified model for the arc trigger, injecting fast electrons to simulate the arc breakdown. This approach successfully identified a critical plasma instability arising from the interaction between rapid electrons and the slower bulk plasma. This instability was found to create a significant charge separation, generating powerful internal electric fields up to 10 kV/m. This strong $\mathbf{E}$ is found here to be the cause of the acceleration of plasma particles, and so directly contributes to the thrust of the thruster.

Overall, the strength of the PIC approach is its very good physical accuracy. It provides a relevant description of the plasma, fully resolving the velocity distribution functions of all species. It is capable of capturing a wide range of kinetic phenomena without any assumptions of thermal equilibrium, including the structure of plasma sheaths, non-Maxwellian EEDF and energy-dependent collision processes. This high fidelity, however, comes at the cost of computational constraints. To ensure numerical stability and accuracy, PIC simulations must resolve the smallest temporal and spatial scales in the plasma, i.e. the electron plasma period $1/\omega_{pe}$ and the Debye length λ_D , respectively. For the dense plasmas found in vacuum arcs, these scales are on the order of ps and nm. Simulating the entire inter-electrode gap of a VAT for several μ s, for instance, becomes prohibitive in terms of computational cost. The application of full PIC models is thus restricted to studies in highly localized plasma regions or scaled-down geometries.

1.3 Hybrid approaches

The hybrid models combine fluid and kinetic descriptions for different species, or sometimes regions within the plasma[218, 219]. These different descriptions aim at achieving an optimal balance between physical fidelity and computational feasibility. A common and effective hybrid strategy is to treat the heavy species, i.e. ions and neutral atoms, kinetically as particles while describing the electrons as a fluid [220, 221]. Detail on this specific approach is provided in the next section, as it is the approach retained here to model the PJP plasma.

The primary constraints on a full PIC simulation comes from the need to resolve the fast dynamics of the electrons. By treating electrons as a fluid, the simulation time steps are no longer limited by the electron plasma frequency, and can then be increased by several orders of magnitude. At the same time, the model keeps a fully kinetic description of the ions, which is essential for accurately capturing their non-equilibrium velocity distributions.

Hybrid Monte Carlo-PIC model

The foundation of the simulation model used in this thesis is the one-dimensional (1D) hybrid model proposed by Sarrailh et al. [73] for studying the post-arc phase in vacuum circuit breakers. The work of Sarrailh aimed to understand the risks of re-ignition in an inter-electrode gap filled with a dense metallic vapor and a residual plasma. The model is called “hybrid” because the different particle populations are treated in a different way. The electrons populations are separated in 2 groups: low-energy bulk electrons that are treated as a fluid and high energy electrons, originating from secondary emission at the cathode and accelerated in the sheath,

that are treated using MCC. This MCC method was added as an improvement of a previously developed model [222] to better calculate the ionization rate during the sheath expansion. The heavy species are treated kinetically using a PIC approach. Collisions between these heavy species are managed by a DSMC method. This initial model demonstrated its ability to accurately describe the cathode sheath dynamics and the conditions leading to a Townsend-type breakdown in copper vapor.

In the frame of the H2020 PJP european project, this model has been adapted to better reflect the plasma conditions in the PJP thruster. Although the core of the hybrid approach (Maxwell-Boltzmann/PIC/DSMC/MCC) has been kept due to its relevance for describing a dense metallic plasma, several major modifications have been implemented. First, the focus is no longer made on the re-ignition of a discharge, but on the description of its steady-state operation and the mechanisms by which it is sustained. Second, the input data have been changed to correspond to the main cathode material under study, namely titanium. This includes collision cross-sections, surface emission properties, and evaporation characteristics. Lastly, the ionization model has been enhanced to include stepwise ionization processes. Unlike direct ionization from ground state, this mechanism is essential for explaining the formation of highly charged ions, such as Ti^+ , Ti^{2+} , Ti^{3+} and so on. This modelization is of particular interest here, as it is a fundamental characteristic of vacuum arcs [123, 161, 223] and a key factor for their application in propulsion.

2 Hybrid Monte Carlo-PIC model

2.1 Core approach

The model has the particularity of accounting for two distinct electron populations, which differ in their energy levels and the way they are modeled. Bulk electrons are treated as a fluid in equilibrium with the electric potential. Their energy is assumed to be low enough, so that they are trapped by the ambipolar electric field. The density of bulk electrons, referred to as population 1, is described by the Maxwell-Boltzmann relation:

$$n_e = n_{e,ref} \exp \left[\frac{eV}{k_b T_e} \right] \quad (7.1)$$

with V the electric potential, $n_{e,ref}$ the reference electron density computed at the anode where $V = 0$, e the elementary charge, k_b the Boltzmann constant and T_e the electron temperature.

The second population of electrons is considered sufficiently energetic to ionize neutral atoms. This population is treated using a Monte Carlo procedure. In essence, these highly energetic electrons originate from the cathode via thermionic emission and are accelerated as they cross the sheath region. A key simplification in the model is that the density of this energetic population is assumed to be several orders of magnitude lower than the bulk electron density, and thus their contribution to the overall space charge in Poisson's equation is neglected. Their primary function within the simulation is to act as the source term for the creation of new electron-ion pairs through ionization. The relationship between the two electron populations and the reference electron density is obtained through the integration of the continuity equation between the two electrodes.

$$\frac{\partial p n_{e,ref}}{\partial t} + \frac{1}{4} n_{e,ref} v_{th,e} \left(\exp \left[\frac{eV_c}{k_b T_e} \right] + 1 \right) = S_{21} \quad (7.2)$$

On the right hand side, S_{21} represents the rate at which energetic electrons from population 2 lose enough energy to become part of the bulk electrons population. S_{21} is essentially a source term for the electron population 1. On the left-hand side of equation 7.2, p is the space integration in the inter-electrode gap termed d and $v_{th,e}$ is the thermal velocity for Maxwellian electrons, namely:

$$p = \int_0^d \exp \left[\frac{eV}{k_B T_e} \right] dx \quad \text{and} \quad V_{th,e} = \sqrt{\frac{8k_B T_e}{\pi m_e}} \quad (7.3)$$

with m_e the electron mass. For the remaining of this chapter, T_e is considered constant at 1 eV in accordance with commonly used data in the litterature, which allows to calculate $n_{e,ref}$ and n_e through equation 7.2 and equation 7.1 respectively. The second population is described using a particle approach. The neutrals and ions are also treated with a PIC approach, the electric potential being solved here through the 1D Poisson's equation:

$$\frac{\partial^2 V}{\partial x^2} = -\frac{e}{\epsilon_0} (Z_i n_i - n_e) \quad (7.4)$$

where ϵ_0 is the permittivity of free space, Z_i the ion charge state and n_i the ion density. Note that both the ions and hot electrons are considered non-magnetized. The Newton's law are applied on the charged particles, and integrated using a leapfrog scheme [211], meaning that the position of the particle x is computed at integer time steps whereas velocity v is computed at half time steps. A schematic of the leapfrog scheme is shown in figure 7.1, where T^n represents the n th time step of the iteration. The update cycle can be summarized as:

1. The velocity is updated from $t - \Delta t/2$ to $t + \Delta t/2$ using the acceleration a computed at the integer time t

$$v(t + \Delta t/2) = v(t - \Delta t/2) + a(t) \Delta t \quad (7.5)$$

2. The position is updated using the calculated velocity

$$x(t + \Delta t) = x(t) + v(t + \Delta t/2) \Delta t \quad (7.6)$$

The leapfrog scheme is a popular numerical algorithm due to its stability, computational efficiency, and the fact that it conserves the system energy over long periods.

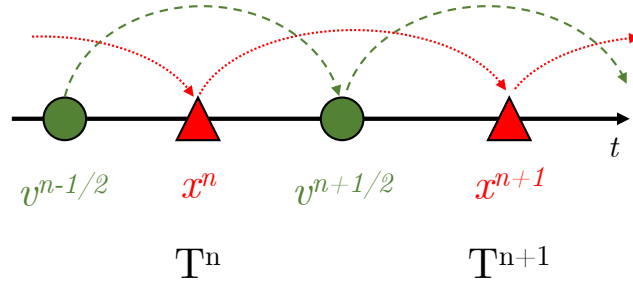
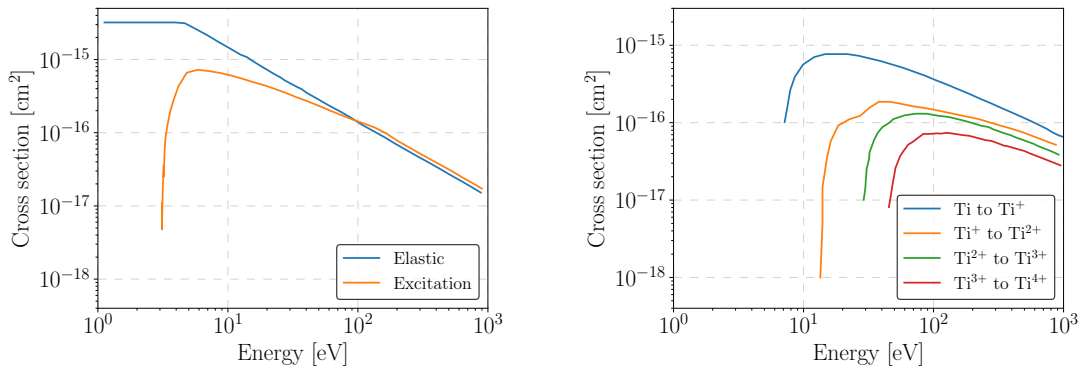


Figure 7.1: Working principle of the leapfrog algorithm, where v is the particle velocity and x its position.

2.2 Collisions

Electron-heavy species collisions, handled by a Monte Carlo procedure, include stepwise ionizations, elastic, and excitation processes. The cross sections for elastic and excitation are sourced from the Quantemol database, while stepwise ionization data comes from reference [224]. Direct ionization from ground states was not included in the model due to a lack of data. The Direct Simulation Monte Carlo (DSMC) is specifically used for neutral-neutral and ion-neutral collisions. For neutral-neutral interactions, constant variable hard sphere (VHS) cross sections are used [225]. For ion-neutral collisions, only charge exchange processes are considered as it is the dominant process. Due to a lack of specific data for titanium, the cross sections for these ion-neutral collisions were taken from data for Cu-Cu⁺ collisions. The different collision processes and the methods by which they are implemented are summarized in table 7.1. Note that the model only allows generations of ions up to 4⁺ due to lack of data for the ionization cross section. This limitation is further discussed in section 4.2.



(a) Titanium elastic and excitation cross sections.

(b) Titanium stepwise ionization cross sections.

Figure 7.2: Electron-neutral and electron-ion cross sections considered in the model. Reproduced from L. Garrigues with permission.

Table 7.1: Summary of the different collision processes implemented.

Collision Type	Reactant	Product	Numerical method
Stepwise ionization	$e_{\text{kinetic}}^- + \text{Ti}^{n+}$	$e_{\text{bulk}}^- + e_{\text{bulk}}^- + \text{Ti}^{(n+1)+}$	MCC
Electron excitation	$e_{\text{kinetic}}^- + \text{Ti}$	$e_{\text{kinetic}}^- + \text{Ti}^*$	MCC
Electron elastic	$e_{\text{kinetic}}^- + \text{Ti}$	$e_{\text{kinetic}}^- + \text{Ti}$	MCC
Charge exchange (CEX)	$\text{Ti}_{\text{fast}}^+ + \text{Ti}_{\text{slow}}$	$\text{Ti}_{\text{fast}} + \text{Ti}_{\text{slow}}^+$	DSMC
Neutral elastic	$\text{Ti} + \text{Ti}$	$\text{Ti} + \text{Ti}$	DSMC

2.3 Role of the cathode

The primary source of electrons is modeled by field-enhanced thermionic emission. Electrons are emitted from the cathode surface through thermionic emission. The strong electric field that is present in the plasma sheath, just above the cathode surface, makes it easier to extract electrons from the cathode body hence amplifying their emission. This phenomenon, known as the Schottky effect, is ruled by the Richardson-Dushman law:

$$J_{em} = AT_c^2 \exp \left[-\frac{e\phi_{eff}}{k_B T_c} \right] \quad (7.7)$$

where $A = 6 \times 10^5 \text{ A m}^{-2} \text{ K}^{-2}$ is a constant fitted from experimental results [120] and T_c the cathode temperature in K. The effective work function ϕ_{eff} is here the cathode material nominal work function W_f reduced by the field at the cathode:

$$\phi_{eff} = W_f - \sqrt{\frac{eE_c}{4\pi\epsilon_0}} \quad (7.8)$$

where E_c is the electric field in front of the cathode. Note that E_c has to be lower than 10^9 V/m for equation 7.8 to be valid, which is the case here, as the model does not consider local field enhancement due to micro-protrusions. $W_f = 4.33 \text{ V}$ here for titanium. This formulation creates a crucial amplification loop: a denser plasma creates a stronger electric field, which enhances electron emission, itself leading to more ionization and denser plasma.

The neutral atoms present in the simulation domain originate from evaporation of the cathode surface. Note that sublimation is not considered here. This phenomena is described as a flux of which the magnitude reads:

$$\Gamma_{a,evap} = \frac{P_s}{\sqrt{2\pi M_a k_B T_c}} \quad (7.9)$$

where P_s is the saturated vapor pressure and $M_a = 8 \times 10^{-26} \text{ kg}$ is the mass of the evaporated atoms. P_s is calculated the Clausius-Clapeyron relation, which directly depends here on the properties of titanium, such as the latent heat of evaporation $L = 421 \text{ kJ mol}^{-1}$ and the boiling temperature $T_{boil} = 3560 \text{ K}$, so that:

$$P_s = P_{atm} \exp \left[-\frac{L}{R} \left(\frac{1}{T_c} - \frac{1}{T_{boil}} \right) \right] \quad (7.10)$$

with R the gas constant, equal to $8.314 \text{ J K}^{-1} \text{ mol}^{-1}$.

Lastly, the model accounts for the emission of secondary electrons (SEE) due to the impact of energetic ions and fast neutrals on the cathode surface. While

included for physical completeness, the contribution from this mechanism to the total emitted electron current is found to be negligible. Calculations testing different SEE for reasonable variations show that it contributes to less than 1% of the total electron emission current density at the surface [135].

3 Results

3.1 Input parameters

The simulations were carried out with different cathode voltages and discharge durations. To reproduce the PJP conditions as closely as possible, the “baseline” conditions used in the present simulations are the ones detailed in table 7.2 if not stated otherwise. In particular, the cathode voltage is identical to the one applied by the PJP. A discharge duration of $15\text{ }\mu\text{s}$ was chosen in order to limit computational time. This choice was supported by the conclusion that, at least at these timescales, the discharge duration had only a minor influence on the evolution of the simulated plasma properties. An example of $30\text{ }\mu\text{s}$ discharge duration simulation output is shown in the appendix. To provide an order of magnitude and a basis for comparison, a $15\text{ }\mu\text{s}$ discharge simulation typically requires between 24 and 48 hours to complete, whereas a $30\text{ }\mu\text{s}$ simulation takes approximately 72 hours. The ramp of cathode voltage used in the simulations is shown in figure 7.3.

Table 7.2: Input parameters.

Parameter	Value
Cathode temperature	3200 K
Discharge duration	$15\text{ }\mu\text{s}$
Electrode gap	8 mm
Cathode voltage	-250 V
Anode voltage	0 V
Cathode material	titanium
Electron temperature	1 eV

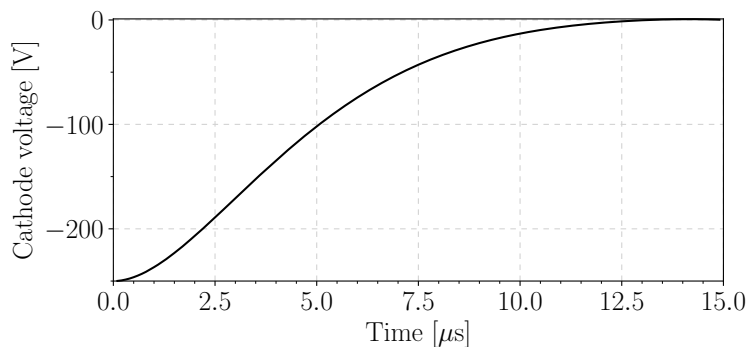


Figure 7.3: Ramp of cathode voltage.

3.2 Output

Figure 7.4 shows the time and space profiles of the ion density n_i , neutral density n_a and ion mean charge $\bar{Z}$. It can be seen that a significant ionization of the neutral gas occurs up to 4 μs . The peak in ion density is reached at the very beginning of the discharge, reading $5.9 \times 10^{23} \text{ m}^{-3}$. Those peak values are comparable to those of neutral density, showing $7 \times 10^{23} \text{ m}^{-3}$. This maximum ion density is reached in front of the cathode, just after the sheath, at the location where electrons reach their maximum in energy [135]. Beyond approximately 4 mm from the cathode, the electron energy decreases, thereby increasing the ionization mean free path. It can be seen as well that the ion mean charge is at the highest between 0 μs and 2 μs . Between 4 μs and 6 μs , a small fraction of these higher charge states is present, but the electron kinetic energy is insufficient to populate even higher states. After 8 μs , electron emission is significantly reduced, leading to less intense ionization of neutral atoms. Consequently, the plasma density drops to the order of 10^{21} m^{-3} or below. Finally, after 8 μs , the atom density increases in the inter-electrode region, reaching peak values as a result of cathode evaporation.

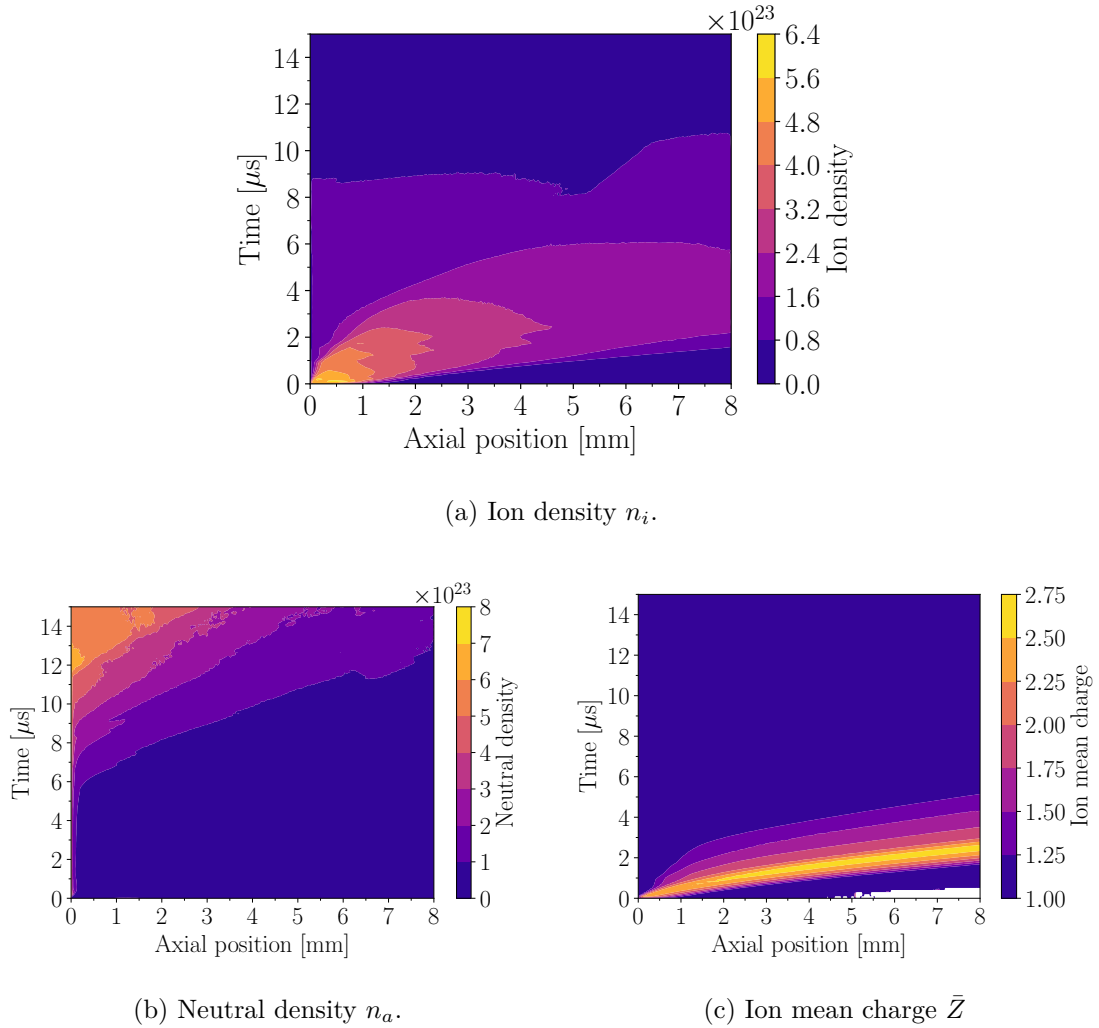


Figure 7.4: Time and space profiles of the ion density, neutral density and ion mean charge.

3.3 Influence of the simulation parameters

Cathode voltage

A significant influence of the cathode voltage on the simulation results can be observed. Figure 7.5 shows the output of the simulations with a cathode voltage of $V_c = -850$ V. The ion density peak is still occurring at the very beginning of the discharge, and is still located close to the cathode, but $9.23 \times 10^{23} \text{ m}^{-3}$ a ion density is reached. Once again, the maximum ion density and the maximum neutral density are of the same order of magnitude. The most visible difference compared to the simulation at $V_c = -250$ V lies in the ion mean charge. Instead of forming a distinct peak over approximately $2 \mu\text{s}$ that propagates relatively uniformly across the inter-electrode gap, the multiply charged ions are filling much more the simulation domain. It can be observed that the ion mean charge reaches a value of 4 between $1 \mu\text{s}$ and $4 \mu\text{s}$, close to the anode. As a reminder, 4^+ is the maximum charge state that the model can generate through stepwise ionization. This indicates a saturation effect in the calculated ion charge. This effect is further discussed in section 4.2. The presence of multiply charged ions is also observed up to $10 \mu\text{s}$ into the discharge, showing that stepwise ionization processes remain much more persistent throughout the discharge. Finally, the distribution of multiply charged ions extends across the entire inter-electrode region, with maximum charge states populations in the region closer to the anode.

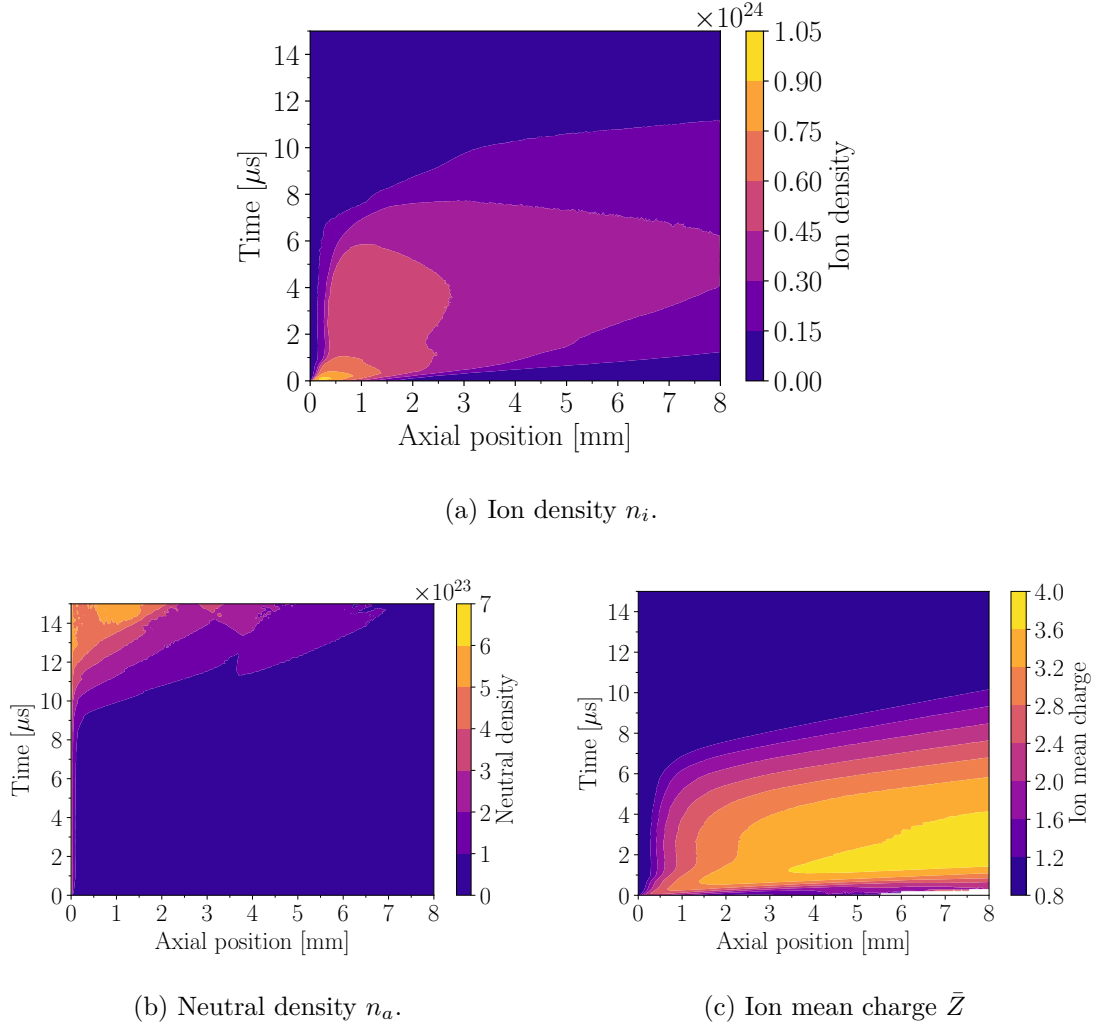


Figure 7.5: Time and space profiles of the ion density, neutral density and ion mean charge for a cathode voltage $V_c = -850$ V.

Electron temperature

Figure 7.6 shows a simulation ran with the electron temperature fixed at $T_e = 3$ eV. The ion density peaks at $4 \times 10^{23} \text{ m}^{-3}$. The overall distribution appears to follow the same trend as in the $T_e = 1$ eV, with a peak occurring at the beginning of the discharge and a gradual decrease over time and space. The difference being that the decrease in ion density is slower in the case of $T_e = 3$ eV, meaning that the medium remains ionized for a longer time period. A direct consequence of this is the lower average neutral density observed. It can be observed that the mean ion charge is significantly higher than in the 1 eV case, reaching up to $\bar{Z} = 3.6$ near the anode. This behavior was expected, as the electron temperature plays a major role in the ionization of the medium.

Cathode temperature

The cathode temperature T_c is a key parameter for the evolution of the arc properties throughout a discharge. A comparative study has been made by Garrigues et al. for T_c ranging between a 3200 K and a 3500 K, revealing a non-linear evolution [135].

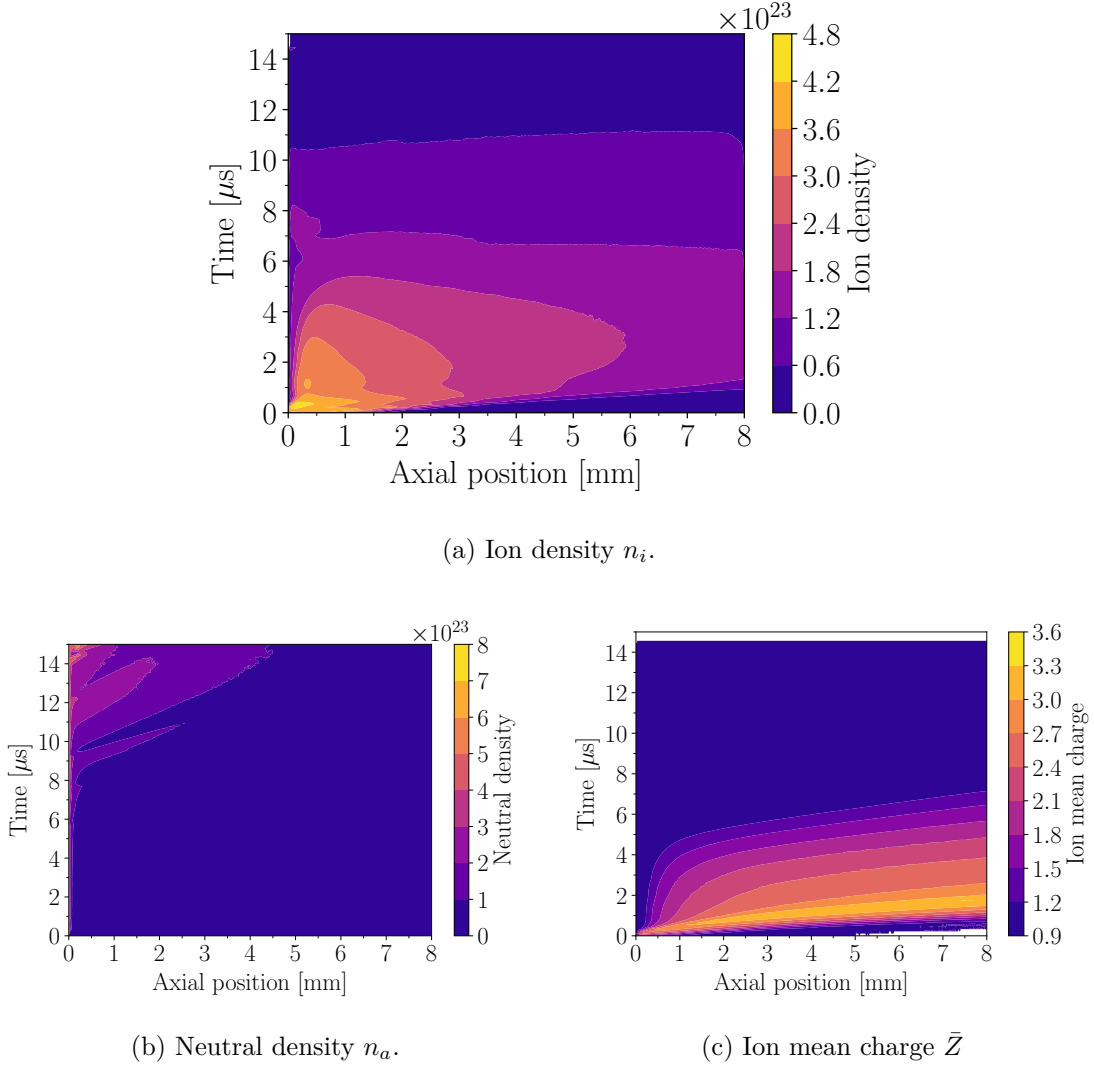


Figure 7.6: Time and space profiles of the ion density, neutral density and ion mean charge for a fixed $T_e = 3 \text{ eV}$.

Increasing T_c from 3200 K to 3500 K increases the evaporated neutral current density by nearly a factor of four, from 2.7×10^7 to 10^8 A/m^2 . Simultaneously, thermionic electron emission is enhanced. This is a coupled effect: the direct temperature increase creates a denser plasma, which in turn reduces the thickness of sheath and intensifies the cathodic electric field, as explained previously. The effects of this amplification loop can directly be seen on the resulting plasma. The simulation for $T_c = 3500 \text{ K}$ produced a maximum averaged plasma density approximately one order of magnitude higher than the 3200 K case.

This significantly denser plasma environment also implies more stepwise ionization. Consequently, the peak average ion charge state $\bar{Z}$ increased from $\bar{Z} \approx 1.2$ at 3200 K to $\bar{Z} > 2.2$ at 3500 K. Note that the time of occurrence of this peak remained unchanged in both simulations, occurring at approximately 4 μs .

4 Discussion

While the hybrid Monte Carlo-PIC model provides a robust framework for investigating the generation of multiply charged ions in VATs, a detailed analysis reveals both its strengths and areas where further refinement might be needed.

4.1 Strengths of the model

The model's most significant advantage is the compromise it makes between physical accuracy and computational cost. By treating electrons as a fluid while retaining a kinetic description for ions and neutrals, it successfully overcomes computational constraints of full-PIC simulations [135, 222]. This allows for the simulation of the entire 1D electrode gap over several microseconds, i.e the typical time scale of a VAT pulse. Simultaneously, it preserves the crucial kinetic nature of the heavy species, which is lost in purely fluid-based models.

Also, the explicit inclusion of stepwise ionisation makes this model rather unique, and innovative in explaining the formation of multiply-charged ions. This inclusion allows for parametric investigations into how different factors, such as cathode temperature, applied voltage or discharge duration can influence the ion charge state distribution.

The model self-consistently resolves the structure of the plasma and the critical cathode sheath by coupling the kinetic particle sources, the fluid electron response, and the solution of Poisson's equation. The sheath region is of particular importance in a vacuum arc, as the large potential drop in this thin layer simultaneously accelerates emitted electrons towards the bulk plasma and accelerates ions back to the cathode. This has the effect of, respectively, increase the ionization in the bulk and contribute largely to surface heating. The ability to capture this tightly coupled feedback loop between the plasma state and the cathode boundary conditions is a crucial element of the physical relevance of this model.

4.2 Limitations

Recombination process

It has been identified by L. Garrigues [135] that, for an electron temperature of 1 eV, electron-ion three body recombination becomes the dominant recombination process when plasma density exceeds $3 \times 10^{19} \text{ m}^{-3}$ [226]. This type of recombination occurs when two electrons and an ion collide, resulting in one electron being captured by the ion while the other carries away the excess energy and momentum. This process can be summarized as $e^- + e^- + A^{Z+} \rightarrow A^{(Z-1)+} + e^-$. The simulations here show densities up to approximately 10^{24} m^{-3} , thus largely surpassing the $3 \times 10^{19} \text{ m}^{-3}$ threshold. D. Wu et al. propose a direct method for implementing three body recombination as the inverse process of electron impact ionization [227]. The foundation of this approach is the detailed balance equation of species with ionization charge state i and $i + 1$:

$$\frac{\partial n_i}{\partial t} = \nu_{i+1}^{\text{rec}} n_{i+1} - \nu_i^{\text{ci}} n_i \quad (7.11)$$

where n_i is the ion density, ν_{i+1}^{rec} is the three-body recombination rate and $\nu_i^{\text{ci}}n_i$ is the impact (or collision) ionization rate. At recombination-ionization equilibrium, $\frac{\partial n_i}{\partial t} = 0$. Equation 7.11 then becomes

$$\nu_{i+1}^{\text{rec}}n_{i+1} = \nu_i^{\text{ci}}n_i \quad (7.12)$$

This equilibrium is well described by the Saha-Boltzmann equation [228]:

$$\frac{n_en_{i+1}}{n_i} = \frac{g_eg_{i+1}}{g_e} \left(\frac{2\pi m_e k_B T_e}{h^2} \right)^{3/2} \exp \left[-\frac{P_i}{k_B T_e} \right] \quad (7.13)$$

where g_e and g_i are the statistical weights, $h = 6.626 \times 10^{-34}$ J s the Planck constant and P_i is the ionization potential from i to $i + 1$ charge state. Including expression 7.12 in equation 7.13 gives an expression of the recombination rate in the form

$$\nu_{i+1}^{\text{rec}} = n_e \nu_i^{\text{ci}} \frac{g_i}{g_e g_{i+1}} \lambda_e^3 \exp \left[\frac{P_i}{k_B T_e} \right] \quad (7.14)$$

where $\lambda_e = \sqrt{h^2/2\pi m_e k_B T_e}$ is the thermal electronic de Broglie wavelength.

In their Monte Carlo simulations of dense aluminum plasmas, Wu et al. demonstrated that neglecting the recombination process leads to a significant overestimation of the final average ionization degree. They argue that models based solely on collisional ionization are incomplete because they lack the inverse process required to establish a dynamic equilibrium. Following this conclusion, the hybrid MC-PIC model reveals a potential weakness on that point. The model simulates the intense ionization occurring in the dense vapor near the cathode but does not account for the intense recombination that must also be occurring in the same region.

Ionization potential depression

If one takes Wu's work once again, a second enhancement for the model could address the environment in which the ionization occurs. The hybrid MC-PIC model uses ionization cross-sections based on data for isolated atoms. However, at the high densities calculated, the plasma is strongly coupled and the electric fields from surrounding free electrons and ions screen the nucleus. This screening effect lowers the energy required to ionize an atom. This effect is called Ionization Potential Depression (IPD), and has been introduced by H.R Griem in 1962 [229] and further studied by J. Stewart and K. Pyatt [230]. According to this theory, the lowering of ionization potential is described by

$$\Delta P = \frac{3(Z+1)e^2}{2R_i} \left(\left[1 + \left(\frac{\lambda_d}{R_i} \right)^3 \right]^{2/3} - \left(\frac{\lambda_d}{R_i} \right)^2 \right) \quad (7.15)$$

where λ_d is the Debye length, and R_i is ion-sphere radius defined by $4\pi n_i R_i^3 = 1$. In the hybrid MC-PIC model, the ionization potentials are currently static values. Considering $n_i = 10^{23} \text{ m}^{-3}$ and $T_e = 1 \text{ eV}$, this ΔP ranges from 0.12 eV if $Z = 1$ to almost 0.3 eV when $Z = 4$. By implementing the IDP, those static values would be replaced with a dynamic ionization potential, such as

$$P_{i,eff} = P_i - \Delta P \quad (7.16)$$

which would be updated at every time step, based on the local plasma properties calculated by the PIC and fluid components. This effect would certainly enhance the stepwise ionization mechanism.

Model simplifications

The modelization in a single spatial dimension fundamentally limits the scope of the physics the MC-PIC model can capture. In particular, some plasma behaviors are known to occur in the radial or azimuthal directions of the plasma column, and could not be modeled in a purely 1D model. The most important example of this is the self-magnetic field pinching of the vacuum arc. Experimental investigations by Yushkov et al. on high-current, short-pulse vacuum arcs have shown the existence of very high charge states, such as Bi^{17+} [133]. Their analysis links this high ionization to the constriction of the plasma column by its own azimuthal magnetic field, at a distance of several millimeters from the cathode. This pinching effect largely increases the local plasma density and temperature, enhancing the possibility of stepwise ionization that would not likely occur in other plasma conditions. This effect being a purely 3D effect, the generation of highly, multiply charged ions could therefore be inherently limited within a 1D model.

The assumption of a constant bulk electron temperature is a major simplification in the model. In a real plasma electrons are heated by Joule heating, and subsequently cooled down through inelastic collisions (ionization and excitation) and adiabatic expansion [53, 66]. This value is therefore expected to vary over the course of a discharge, as observed in chapter III. Since both ionization and recombination rates are highly sensitive to T_e , this simplification could have a significant impact on the model outputs, and still needs to be investigated.

Another significant simplification of the model lies in the omission of the presence of micro-protrusions on the cathode surface. These protrusions can cause a local enhancement of the electric field, leading to values at the tips of protrusions that can be an order of magnitude greater than the macroscopic field calculated by the model [231]. Since the calculations of field-enhanced thermionic emission is based here on the macroscopic field, it does not account for the electron emission that can occur at these sites. By omitting this emission mechanism, the model may underestimate an important source of electrons [61, 65]. On the other hand, for very high emitted current density, the forming cloud of emitted electrons creates a significant space charge in the vacuum above the emitting surface. This space charge can screen the applied field and thus causes a negative feedback that reduces the emission [231]. Impact of the presence of micro-protrusions might then be investigated.

A last simplification in the model concerns the role of the anode. Here, the anode is treated as a simple, passive boundary that collects current. However, detailed 3D hybrid simulations by Cao et al. have shown that the bombardment of the anode by energetic ions from the cathode can lead to significant anode sputtering [232]. The sputtered neutral atoms can then be ionized by the plasma, creating a secondary population of low energy ions traveling towards the cathode. This neglected process could influence the plasma potential profile and the overall momentum balance of the system.

Ultimately, the lack of data concerning the ionization cross sections lead to saturation of the ion charge in the simulations. It was seen that the maximum charge state allowed in the model is 4^+ , a limit that was reached even in the 250 V simulation case. This last point supports the idea that the stepwise ionization mechanism is likely underestimated. On the other hand, since recombination is not taken into account, it would be necessary to examine which process would be dominant, in order to identify a clear trend.

Chapter VIII

Conclusion

This thesis investigates the plasma and plume properties of a high-current, short-duration vacuum arc thruster (VAT) using various experimental methods. The objective of this research is to quantify and better understand the factors that influence the performance of such thrusters.

To this end, three main experimental approaches are presented in this work: the use of electrostatic probes in the plasma plume, imaging of the cathode surface, and performance quantification using a thrust balance. Each of these approaches is complementary, and correlations have been established between the results obtained from the different methods. The main findings from each approach, as well as the connections drawn between them, are summarized below.

1 Summary

The first part of this thesis provides an overview of the fundamental concepts commonly used to describe vacuum arcs. It also presents a brief overview of the main application domains of vacuum arcs. The state of the art is particularly focused on space propulsion, which forms the framework of the present work. The Plasma Jet Pack (PJP) thruster, developed by the company COMAT, is introduced there as the thruster under study. It is characterized by its ability to generate a high discharge current of approximately 3 kA, over a short pulse duration of 35 μ s. These discharge conditions make the PJP rather unique among all other existing vacuum arc thrusters. The technical details of the PJP, as well as the different experimental apparatus used to investigate its plasma are also presented.

Time-resolved Langmuir probe measurements reveal the temporal evolution of plasma density, electron temperature, and plasma potential in the plume of the PJP equipped with a titanium cathode. The measurements show that the plasma density generally follows the shape of the discharge current, reaching values up to 10^{18} m^{-3} . The temporal evolution of the plasma potential and electron temperature highlights the presence of two distinct arc regimes: the spark regime, characterized by a high electron temperature, and the steady-state arc regime with a high density. A brief comparison of different Langmuir probe analysis models indicates that the Orbital Motion Limited (OML) model is the most suitable for analyzing dense and pulsed plasma data, despite being the least theoretically complete. The two other models considered here, namely Allen-Boyd-Reynolds (ABR) and Bernstein-Rabinowitz-Laframboise (BRL), are found to be more sensitive to measurement noise, leading

to errors even though they include sheath modeling. A linear set of electrodes was used to measure the ion exhaust velocity, yielding values around 35 km/s for the PJP with a titanium cathode and about 40 km/s with a copper cathode. Although initially surprising, as copper atoms are heavier than titanium atoms, this result demonstrates the influence of the current growth rate on the ion exhaust velocity. Specifically, above 10^8 A/s, electron-ion friction effects are reported to occur, resulting in significant kinetic energy transfer from electrons to ions. The observation of very fast ions, with velocities up to 160 km/s, also suggests the presence of pinch-type instabilities in the PJP plasma.

Long exposure photographs of the cathode surface taken during firing reveal a fractal structure of the cathode spots patterns. The dimension of these structures is determined by the Minkowski-Bouligand method. It is found that the fractal dimension varies considerably as a function of the cathode material. Differences cannot be attributed to the typical dispersion resulting from several separated discharges, or to surface conditions. For the same energy released in the discharge, D spreads from 1.14 ± 0.02 for copper to 1.45 ± 0.03 for titanium. The total surface covered by the spots during a discharge shows the same variation and the same ordering. Two methods have been proposed to investigate the role of the cathode material physical properties in relation to both the fractal dimension and the total surface. A machine learning approach as well as a heuristic analysis show that the work function, the ionization energy and the cohesive energy of the material play an important role in the fractal pattern geometry.

High-speed imaging reveals the formation process of these fractal patterns. Although the direction of spots propagation is known to be random, the values for total surface covered, or emissive area, are reproducible with a fairly low standard deviation. Combined with the fact that the trajectories are ultimately quite straight, one can remark a duality of behavior. The literature demonstrates that the apparent movement of a spot cell is random, but the present experiments reveal that some parameters in the vacuum arc produced by the PJP can still be quantified with the proper use of high-speed imaging. The emission surface evolves linearly with the discharge current, and the current density at the spots therefore remains constant at around 10^9 A/m², for all investigated materials. This behavior is verified over the entire data sets, and all investigated cathode materials. It is observed that the spot propagation velocity depends on the cathode material, ranging from about 19 m/s for copper to over 59 m/s for titanium. In fact, it becomes clear that the fractal dimension is correlated with the spot propagation speed on the cathode surface. Over the 35 μ s duration of the discharge, the materials exhibiting the fastest spot motion also display the highest fractal dimension. On another level, maximum spot clustering at the start of discharge combined with the central position of the triggering system means that the cathode is not eroded homogeneously across its entire radius. More importantly, the images reveal that the arc develops asymmetrically around the central insulator. This asymmetry leads to an undesired thrust vectoring, which was also observed in the electrostatic probe measurements. This may therefore result in a technical limitation of the PJP, linked to weakly optimized cathode utilization and a lack of precision when it comes to single pulse maneuvers for the spacecraft.

Finally, thrust measurements confirm at the macroscopic level the observations made at the microscopic scale, namely:

- The PJP produces a different thrust level from one pulse to another.
- The cathode materials that display larger fractal dimensions, and consequently higher spot propagation speeds, are found to generate higher thrust levels.

This indicates that the thrust produced by VATs is primarily determined by the total surface covered by the spots during a discharge or, equivalently, by their fractal dimension rather than by the atomic mass of the cathode material. At least, for the cathode sizes and discharge current levels considered here, this parameter appears to dominate over the others. The total spot surface and fractal dimension are governed by the velocity at which the spots propagate across the surface, and is therefore likely related to the materials work function, ionization energy, and cohesive energy. The latter is systematically used by André Anders to establish correlations between the various plasma parameters of vacuum arcs. Thus, cohesive energy seems to emerge not merely as a parameter, but as the hidden thread binding the fractal tapestry of vacuum arc plasmas.

2 Perspective

Although six materials were investigated in this work, it would be of interest to measure the fractal dimension, spot velocity, ion velocity, and thrust level produced with other cathode materials. In particular, it would be valuable to perform such measurements with non-metallic materials, such as graphite, to determine whether the thrust remains correlated with the fractal dimension. It could also be of interest to study the influence of the cathode material in greater detail, including aspects such as its crystalline structure. The precise impact of the discharge current waveform, its growth rate, and the pulse duration on thruster performance is also a key topic in the context of propulsion. Establishing a clear and robust relationship between these different parameters would make it possible to optimize VAT design, by developing cathode geometries and compositions that are precisely matched to the desired discharge conditions. Such an approach would enable the design of vacuum arc thrusters optimized in terms of both thrust and specific impulse, tailored to mission requirements.

Another research direction, of a more theoretical nature, would be the detailed modelling the motion of the cathode spots. It has been observed that their propagation pattern follows a random walk behavior, and forms patterns similar to those generated by diffusion-limited aggregation (DLA) or Hastings–Levitov algorithms. These algorithms can be characterized by a diffusion constant, which shapes the final form of the patterns. Determining this diffusion constant as a function of the cathode material and linking it to the material intrinsic properties would be particularly valuable. If an evolution law can be established, it would become possible to identify the best suited materials for propulsion applications. An accurate model, whether purely numerical/theoretical or semi-empirical, would open the way to more optimized cathode designs. One could develop alloys composed of different metals, with optimized proportions, in order to achieve both high thrust and low impulse-bit dispersion.

Finally, the primary limiting factor in the use of VATs is the contamination of insulating surfaces by sputtering of the cathode material. As observed in this work,

repeated discharges lead to the formation of a conductive layer on the insulator surfaces, which induces inevitable electrical short circuits within the thruster. One of the solutions proposed to date is the application of an external magnetic field on the arc, forcing it to propagate in the $-\mathbf{j} \times \mathbf{B}$ direction and thereby homogenizing cathode erosion. The application of a magnetic field also appears to affect the way in which the eroded material is redeposited, as well as the microstructure of the deposited layer itself [233–235]. This represents a key research direction for increasing the lifetime, and therefore the total impulse, of VATs. Still, the interaction between the arc and an external magnetic field remains a topic in its own right, with many open questions and a considerable potential for future research.

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Appendix A

List of publications

1 Peer-reviewed journal papers

1. E. Michaux, S. Mazouffre and A. Blanchet. Time Evolution of Plasma Parameters in the Jet of a Low-Power Vacuum Arc Thruster. *Journal of Electric Propulsion*, 1(1), 7, 2022.
DOI : 10.5555/3495724.3496760
2. E. Michaux, A.E. Vinci, S. Mazouffre. Fractal dimension of cathode spots in a high-current vacuum arc thruster. *Vacuum*, 215, 112286, 2023.
DOI : 10.1016/j.vacuum.2023.112286
3. E. Michaux, S. Mazouffre. Time-of-flight measurements in the jet of a high-current vacuum arc thruster. *Aerospace*, 10(12), 1011, 2023.
DOI : 10.3390/aerospace10121011
4. E. Michaux, J. Julien, S. Mazouffre. High-speed imaging of titanium cathode spots in a high-current vacuum arc thruster. *Journal of Physics D: Applied Physics*, 58(13), 135206, 2025.
DOI : 10.1088/1361-6463/adafb9

2 Conference papers

1. E. Michaux, S. Mazouffre and R. Fritzsche. Evolution in time and space of ion and electron properties in the jet of a low-power Vacuum Arc Thruster. *8th Space Propulsion Conference, Estoril, Portugal*, 2022. SP2022-430.
2. E. Michaux, S. Mazouffre and R. Fritzsche. Spatial and temporal evolution of ion and electron parameters in the plasma jet of a 30 W VAT. *37th International Electric Propulsion Conference, Cambridge, USA*, 2022. IEPC-2022-557.
3. E. Michaux, J. Julien and S. Mazouffre. Cathode spots dynamics in a high-current Vacuum Arc Thruster. *9th Space Propulsion Conference, Glasgow, Scotland*, 2022. SP2024-251.

4. E. Michaux and S. Mazouffre. High-speed imaging of cathode spots in a Vacuum Arc Thruster. *38th International Electric Propulsion Conference, Toulouse, France*, 2024. IEPC-2024-291.
5. E. Michaux and S. Mazouffre. Cathode spots dynamics in a high-current Vacuum Arc Thruster: Aluminium vs Iron. *AIAA SciTech 2025 Forum, Orlando, USA*, 2025. AIAA 2025-2386.
DOI : 10.2514/6.2025-2386
6. E. Michaux and S. Mazouffre. Influence of cathode spots behavior on vacuum arc thruster performance. *39th International Electric Propulsion Conference, London, United Kingdom*, 2025. IEPC-2025-157.
7. G. Brivary-Couret, T. Yung, J. Scheiner, L. Herrero, F. Schäfer, R. Fritzsche, G Herdrich, M. Kubečka, A. Obrušník, E. Michaux, S. Mazouffre, F. Valensi, S. Dimitrov, V. Buchova. Experimental Characterization of the Injection and Discharge Processes on a Low-Power Pulsed Magnetoplasma dynamic Thruster. *39th International Electric Propulsion Conference, London, United Kingdom*, 2025. IEPC-2025-125.

Appendix B

Supplementary material

1 Thrust balance error budget

The error budget, expressed as a function of thrust level or impulse bit, is determined by assessing all potential sources of measurement error. All individual sources of error are summarized in table 2.1. The global standard deviation is calculated by combining the standard deviations of each term quadratically, under the assumption that the terms are independent. The uncertainty on the thrust then reads:

$$\sigma_T = \sqrt{\sum_1^N \left(\frac{\sigma_{X_i}}{X_i} \right)^2} \quad (2.1)$$

with X_i the balance parameter and σ_{X_i} the uncertainty on this parameter. According to the data presented in table 2.1, the total error for mean thrust measurements reads 5.7%.

Table 2.1: Budget error of the torsional thrust stand.

Parameter	σ_{X_i}/X_i
Balance mass	1.25×10^{-2}
Calibration arm	3.7×10^{-2}
Thruster arm	3.7×10^{-2}
Voice coil current	1×10^{-3}
Voice coil calibration error	3×10^{-2}
Capacitive sensor error	1×10^{-3}

2 Mean thrust measurement examples

Here are presented some additional examples of mean thrust measurements. In the following figures the PJP is pulsed at 1 Hz for 20 s starting from 10 s, F_1 refers to the force associated with this first deflection. Then the thruster is stopped for 20 s and operated again at 1 Hz for 20 s. The second deflection is referred to as F_2 .

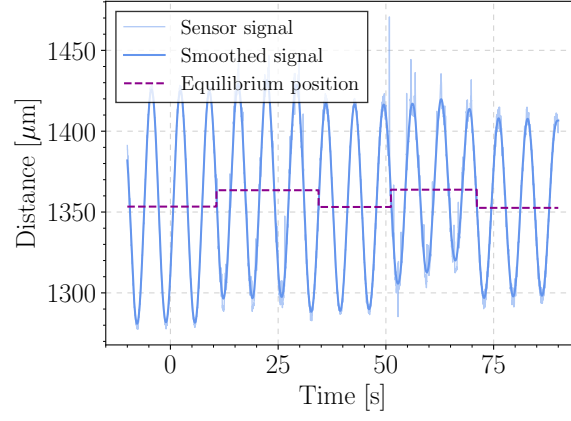


Figure 2.1: Example of thrust measurement with NiCr cathode. $F_1 = 47.7 \mu\text{N}$ and $F_2 = 50.3 \mu\text{N}$

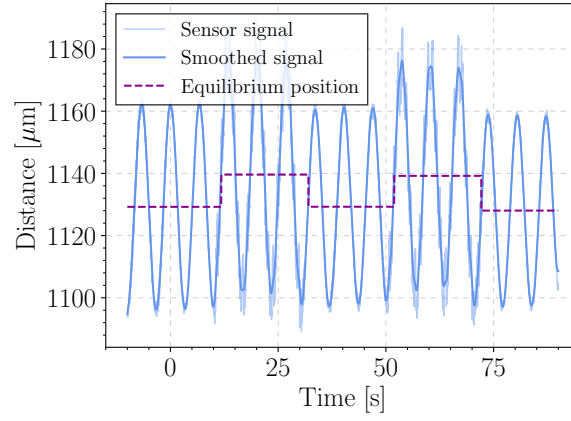


Figure 2.2: Example of thrust measurement with Al cathode. $F_1 = 48.8 \mu\text{N}$ and $F_2 = 46.7 \mu\text{N}$

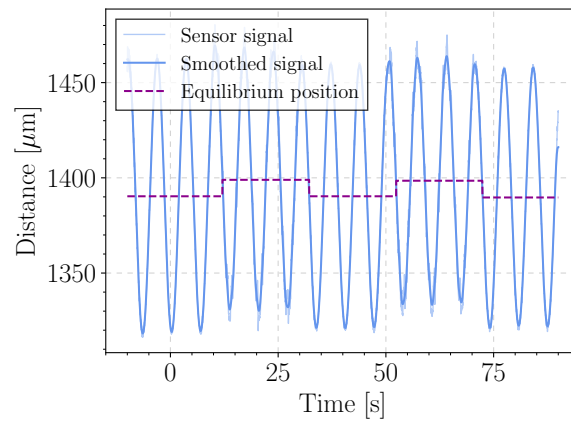


Figure 2.3: Example of thrust measurement with Fe cathode. $F_1 = 40.6 \mu\text{N}$ and $F_2 = 39.5 \mu\text{N}$

3 Influence of discharge duration on simulation outputs

Figure 2.5 shows the time and space profiles of the ion density n_i , neutral density n_a and ion mean charge $\bar{Z}$ for a discharge duration of $30\text{ }\mu\text{s}$ and the parameters given in table 7.2. The ion density peaks at the same location and at the same moment than in figure 7.4. Moreover the two densities are identical with a value of $5.9 \times 10^{23}\text{ m}^{-3}$. This is quite logical as it is the voltage ramp itself, i.e. the growth rate, that is expected to influence the plasma properties.

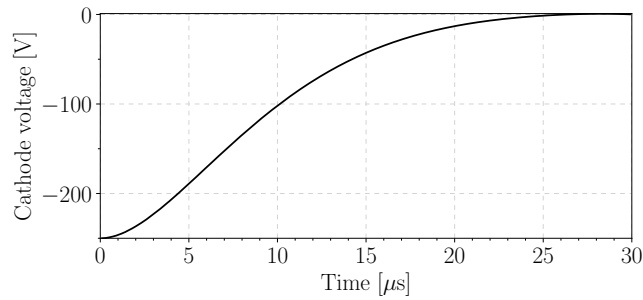
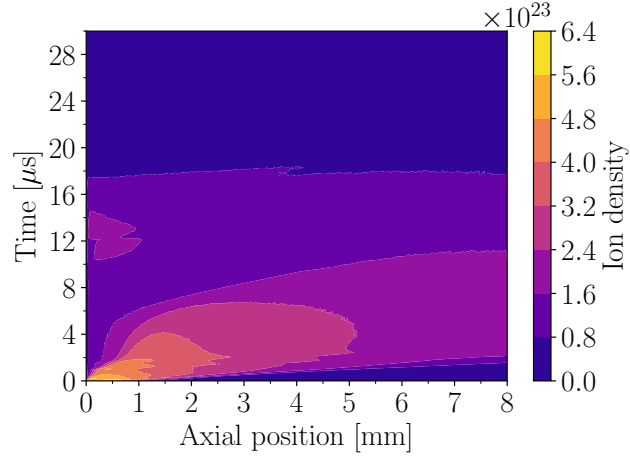
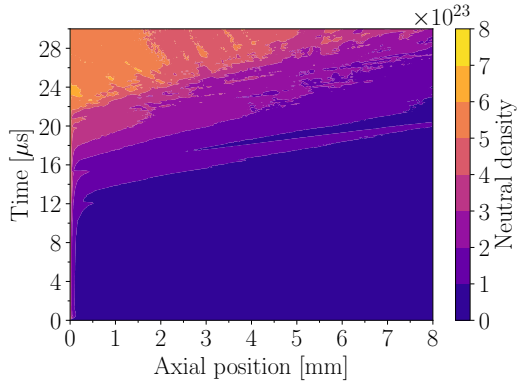


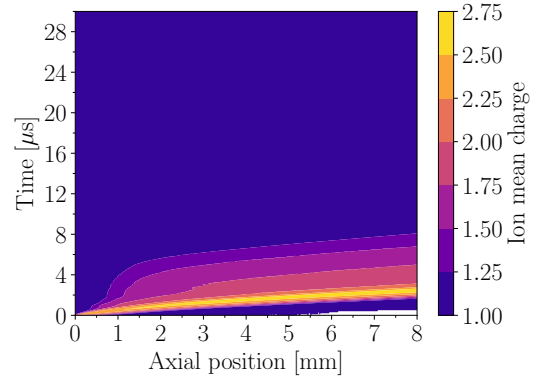
Figure 2.4: Ramp of cathode voltage for a discharge duration of $30\text{ }\mu\text{s}$.



(a) Ion density n_i .



(b) Neutral density n_a .



(c) Ion mean charge $\bar{Z}$

Figure 2.5: Time and space profiles of the ion density, neutral density and ion mean charge for a 30 μ s discharge duration.

Appendix C

Résumé en français

1 Introduction

La propulsion spatiale se base sur la loi de conservation de la quantité de mouvement. Pour qu'un satellite ou une fusée se déplace dans l'espace, il faut que le véhicule éjecte une masse de carburant, appelée ergol, à une certaine vitesse. Dans ce contexte, 2 grands paradigmes émergent. Le premier: éjecter une grande masse d'ergols à une vitesse assez faible. C'est dans ce paradigme que s'inscrit la propulsion dite chimique, utilisée pour propulser les lanceurs de type Ariane, Soyouz, Falcon, etc. Ce type de propulsion produit une grande poussée, ce qui absolument nécessaire afin de pouvoir s'arracher à la force de pesanteur ressentie au niveau terrestre. Pour donner un ordre de grandeur, on parle de brûler et d'éjecter des centaines de kg/s d'ergols à des vitesses de l'ordre de 2-5 km/s. D'un autre côté, une autre méthode consiste à éjecter une petite masse d'ergols à une vitesse élevée: c'est dans cette optique que s'inscrit la propulsion électrique. Dans le vide de l'espace et une fois affranchi de la force de pesanteur, éjecter une petite masse à grande vitesse sur un temps assez long permet à des satellites de réaliser tout type de manœuvres. Le carburant n'est pas ici brûlé mais ionisé, grâce à l'utilisation de l'énergie électrique générable à bord du satellite par le biais de panneaux solaires. A titre de comparaison par rapport aux propulseurs chimiques, les propulseurs électriques présentent un débit massique de l'ordre du $\mu\text{g/s}$ - mg/s pour une vitesse d'éjection des ergols de plusieurs dizaines de km/s. Il est à noter que cette vitesse d'éjection est théoriquement illimitée dans le cas de la propulsion électrique, car dépendante de l'énergie électrique injectée dans le système. La limite provient de la source d'énergie, extérieure. De l'autre côté, la limite théorique de vitesse d'éjection dans les propulseurs chimiques tourne autour des 5 km/s, car elle est liée à l'énergie contenue dans les liaisons chimiques de l'ergol. Cette limite est donc interne .

Un des grands avantages de la propulsion électrique est son utilisation efficace des ergols, et les possibilités de miniaturisation qu'elle propose. Nous remarquerons que différentes technologies de propulseurs électriques existent, définies par la manière dont les ergols sont ionisés. Certaines technologies présentent une plus grande poussée, comme les moteurs de Hall, d'autres une plus grande vitesse d'éjection comme les moteurs ioniques à grilles. D'autres encore se caractérisent par leur simplicité de fabrication et leur compacité. C'est le cas de la technologie de propulseurs étudiée dans cette thèse: les propulseurs à arc sous vide, ou VATs pour Vacuum Arc Thrusters en anglais.

Les VATs déclenchent un arc électrique entre deux électrodes. Dans le vide de l'espace, il n'y a pas de milieu à ioniser et c'est directement le matériau composant l'électrode négative, la cathode, qui est ionisé et éjecté à grande vitesse. On peut dire que le moteur est, en un sens, autophage: il érode son électrode pour se propulser. Bien que ce type de moteur soit assez facile à développer d'un point de vue ingénierie, il souffre encore de nombreuses inconnues sur le plan scientifique. En particulier, le comportement du plasma d'arc dans le vide, la manière dont il est créé et l'influence du matériau de la cathode sur ses propriétés constituent encore à ce jour des axes de recherche majeurs pour la communauté scientifique.

Une équipe russe a pu, dans les années 2000, proposer une théorie décrivant l'ensemble des phénomènes par lequel l'arc se crée. Ce modèle, appelé modèle ecton, propose que le déclenchement de l'arc se fait via un ensemble de micro-explosions à la surface de la cathode. Ces micros-explosions forment des zones d'émission de plasma localisées, de quelques μm de diamètre, qui sont nommés "pieds d'arcs" ou "cathode spots" en anglais. Un modèle plus tardif, proposé par Isak I. Beilis, développe le bilan d'énergie au dessus de ces pieds d'arc et les propriétés du plasma qui en est émis. Ce plasma est dense et très énergétique, ce qui le rend très difficile à sonder par des moyens expérimentaux. Enfin, aux alentours de 2005, André Anders propose le modèle fractal. Basé sur toute une suite d'observations, il déduit que les pieds d'arc sont auto-similaires, c'est à dire que chaque pied est composé d'un certain nombre de pieds plus petit, eux mêmes composés de pieds plus petits. A la manière d'une poupée russe, le pied d'arc semble composé d'une infinité de copies de lui-même.

La connaissance du comportement des pieds d'arcs et, d'une manière générale, du comportement du plasma de l'arc sous vide est actuellement un facteur limitant dans le développement des VATs. Cette thèse vise donc, principalement par le biais de techniques expérimentales, à caractériser le comportement du plasma créé par un VAT en fonction du matériau de sa cathode (son ergol). Le VAT ici étudié est le Plasma Jet Pack (PJP), un VAT à fort courant pulsé sur une courte période. Ses caractéristiques techniques ainsi les différents moyens expérimentaux mis en œuvre au cours de cette thèse sont détaillés dans le chapitre suivant.

2 Dispositif expérimental

Plasma Jet Pack

Le propulseur ici à l'étude est le Plasma Jet Pack PJP, un VAT de classe 30 W développé par la PME Toulousaine COMAT. Ce VAT a la particularité de développer jusqu'à 4 kA sur une période de 30 μs . Différentes versions du PJP ont été développées à ce jour pour répondre à différents besoins d'intégrateurs satellites. La version à l'étude dans cette thèse est le PJP H2020, prototype destiné à des laboratoires, dont les pièces peuvent facilement être interchangeables. En particulier, la cathode du PJP H2020 peut facilement être démontée et remontée, facilitant l'étude de différents matériaux.

Sondes électrostatiques

L'ensemble de sondes électrostatiques utilisées dans cette thèse comporte une sonde de Langmuir et un dispositif de sondes de temps de vol. La sonde de Langmuir est une tige de tungstène de 0.6 mm de diamètre et de 14 mm de longueur. Le dispositif de sondes de temps de vol est composé de 2 sondes électrostatiques. Une première sonde est constituée d'un fil de tungstène de 0.6 mm d'épaisseur, et forme une boucle de 15 mm de diamètre. Cette sonde capte une partie des particules chargées composant le plasma, et en laisse passer une grande partie en son centre. Les particules non captées par la première sonde vont ensuite toucher la deuxième sonde, une sonde plane en tungstène de 6 mm de diamètre. Les deux sondes sont polarisées à une tension égale de -35 V par la même alimentation. Ceci assure que seulement les ions soient captés, et la même tension de polarisation aux deux sondes assure qu'il n'y ait pas de champ électrique entre les deux sondes, pouvant accélérer ou ralentir les ions traversant le dispositif. Le courant récupéré par les sondes est mesuré par des boucles de Rogowski, qui sont reliées à un oscilloscope à haute résolution temporelle. La différence de temps entre les deux signaux permet de retrouver la vitesse moyenne des ions.

Imagerie

La répartition des pieds d'arcs sur la surface de la cathode du PJP est observée par deux moyens d'imagerie. Les images prises à exposition longue cartographient l'ensemble des pieds d'arcs sont initiés pendant la décharge. Ces images sont prises à l'aide d'un appareil photo Nikon D40. Pour explorer la manière dont les pieds se déplacent à la surface, il est nécessaire d'avoir une caméra à haute vitesse pouvant monter à une fréquence de plusieurs milliers d'images par secondes. Une Phantom TMX7510 a donc été utilisée dans cette thèse, permettant de prendre des images de la surface de la cathode toutes les 2.2 μ s.

Pour permettre un bon grossissement et une image précise de la cathode, un ensemble d'optiques a été installé sur ces deux appareils. Cet ensemble implique une lentille 200 mm pour la caméra rapide, et une lentille 180 mm ainsi qu'un téléconvertisseur 2X et deux tubes d'extension pour l'appareil photo Nikon. En amont des deux appareils, des filtres à densité neutre sont installés pour réduire la luminosité totale captée par les capteurs.

Balance de poussée

Une balance de poussée a également été développée dans le cadre de cette thèse. Son architecture en balance de torsion permet d'induire un mouvement oscillant à chaque impulsion du PJP. Cela permet de maximiser la visibilité dans le temps de la force générée par le PJP, qui est appliquée sur un temps très court. La balance a une longueur totale de 80 cm, et est soutenue par un ressort de torsion. Le PJP est placé à une extrémité du bras, et le mouvement est mesuré à l'autre extrémité grâce à un capteur capacitif. La balance est calibrée à l'aide d'une bobine à voix.

3 Mesures par sondes électrostatiques dans le jet de plasma

Sondes de Langmuir

Une des principales nouveautés que ce chapitre apporte est l'application de modèles pour sondes de Langmuir à des signaux pulsés. Il a donc été possible de tracer l'évolution au cours du temps de la densité électronique, de la température électronique et du potentiel plasma dans la plume du PJP selon 3 modèles: Orbital Motion Limited (OML), Allen-Boyd-Reynolds (ABR) et Bernstein-Rabinowitz-Laframboise (BRL). L'OML est le modèle le plus simple, qui considère que tous les ions sont attirés radialement à la sonde. L'ABR et le BRL résolvent l'équation de Poisson, et proposent une modélisation simplifiée de la gaine plasma. Le modèle BRL propose même l'existence de mouvement orbitaux des ions autour de la sonde, qui peuvent donc être captés ou non par la sonde en fonction de leur quantité de mouvement, de leur angle d'incidence et de la tension de polarisation de la sonde.

Une analyse comparative de ces 3 modèles est effectuée, sur les données obtenues dans la plume du PJP. Les valeurs pic de densité électronique, à 20 cm de l'anode (plan de sortie des ions), révèlent une densité allant de $1.1 \times 10^{18} \text{ m}^{-3}$ dans le cas de l'OML à $5 \times 10^{18} \text{ m}^{-3}$ pour le BRL. Les valeurs données par l'ABR se situent légèrement en dessous de celles de l'OML, à $8.5 \times 10^{17} \text{ m}^{-3}$. Pour chaque modèle, on observe que l'évolution de la densité électronique suit la forme du courant de décharge. Cependant, les deux modèles théoriquement les plus précis sont ceux qui donnent les valeurs les moins cohérentes avec ce qui est attendu. Une analyse plus fine de ces modèles révèlent qu'ils sont plus sensibles au bruit de mesure. L'ABR est également non adapté aux sondes cylindriques, or c'est cette géométrie de sonde qui est utilisée dans cette thèse. Il est à noter que ces grandes différences de valeurs ne sont observées que pour la densité électronique.

La température électronique et le potentiel plasma évoluent, eux, de concert. Il n'y a pas de différence notable de valeurs selon les modèles. On observe un pic au tout début de la décharge, avec une température électronique avoisinant les 4 eV et un potentiel plasma allant jusqu'à 17 V. Ces valeurs se stabilisent ensuite pour atteindre les 0.5 eV et 1 V, respectivement. On remarque une différence frappante avec la dérivée du courant de décharge. Ce pic au début de la décharge, ainsi que sa stabilisation, montrent bien la présence de deux régimes d'arcs distincts. Le premier, le "spark" ou étincelle en français, qui montre une température électronique haute pour une densité basse, est un mode transitoire du claquage vers l'état d'équilibre. Ensuite, l'arc se stabilise et on remarque une augmentation de la densité électronique, pour une baisse de la température des électrons. Ceci est tout à fait en phase avec le modèle ecton de G. Mesyats.

Vitesse des ions dans la plume

La vitesse des ions a été mesurée à différents endroits de la plume du PJP. Les mesures effectuées avec une cathode en titane révèlent une vitesse moyenne d'éjection autour de 32 km/s. Les ions cuivre montrent, eux, une vitesse moyenne avoisinant les 36 km/s. Dans les deux cas, l'écart type des valeurs trouvées se trouve dans les 15% et on remarque une répartition centrée des vitesses des ions. On remarque également que la vitesse moyenne n'évolue pas dans la plage de mesure, entre 9 cm

et 32 cm après le plan de sortie, indiquant l'absence de mécanisme d'accélération dans cette zone de la plume.

Il est cependant remarqué que la vitesse des ions évolue au cours de la décharge. Plusieurs populations sont observées, et on peut notamment dégager 3 populations principales. La première population qui émerge lors de la décharge est systématiquement plus rapide que la deuxième, elle même plus rapide que la troisième. L'explication proposée est la suivante: au fur et à mesure que la décharge évolue, l'espace inter-électrode se remplit de vapeur métallique et donc de neutres. Les ions créés à la fin de la décharge perdent donc de l'énergie par collision avec ces neutres.

Sur certaines décharges, des vitesses ioniques allant jusqu'à plus de 160 km s^{-1} sont observées. Bien que statistiquement rares, ces hautes vitesses sont très probablement causées par des instabilités de type pinch, connues pour être présentes dans les arcs sous vide à fort courant. Ces instabilités augmentent localement la densité de plasma et sa température, donnant lieu à des ions fortement multichargés qui sont par la suite éjectés à très haute vitesse. En plaçant un autre ensemble de sondes dans la plume, on remarque lors de quelques décharges que certaines populations d'ions éjectées avec un angle de 60° sont plus rapide qu'en ligne droite en sortie du propulseur. Cela démontre une forte anisotropie de la plume, ainsi qu'une directivité du jet de plasma.

4 Dimension fractale des pieds d'arc

La directivité du jet de plasma ne pouvant pas être expliquée directement par les mesures par sonde électrostatiques, le choix a été fait d'observer directement les zones d'émission du plasma à la surface de la cathode. Il est connu que les pieds d'arcs présentent un comportement hautement stochastique, et qu'ils se propagent de proche en proche en formant des motifs ramifiés.

La prise de photographies longue exposition (obturateur ouvert pendant toute la durée de la décharge), a permis de remarquer que les pieds d'arc ne couvraient pas l'entièreté de la surface de cathode. Certaines parties de la cathode émettent du plasma, tandis que d'autres ne sont pas ionisées. Ces zones émettrices changent d'une impulsion à l'autre et se répartissent, la plupart du temps, de manière asymétrique autour du centre de la cathode. Cette observation explique donc la directivité du jet. Une autre observation est faite: les motifs dessinés par les pieds d'arc sont très différents d'un matériau de cathode à l'autre. Les résultats obtenus sur un panel de 6 matériaux, à savoir titane, fer, cuivre, aluminium, nickel-chrome et laiton, mettent en lumière des différences flagrantes. Le titane, notamment, présente de larges branches qui s'étendent loin du centre alors que le cuivre et le laiton forment de petits îlots compacts et très isolés.

Une méthode pour quantifier cette différence de géométrie dans les motifs de pieds d'arc a donc été proposée. La propriété géométrique retenue est la dimension de Minkowski, utilisée en géométrie fractale pour caractériser la dimension d'une fractale. A noter que l'étude des pieds d'arc par le prisme de la géométrie fractale fait directement suite aux travaux d'Anders. Une fois les images nettoyées de phénomènes parasites, et transformées en matrices binaires comprenant les pieds d'arcs, on applique la méthode de Minkowski. Cette méthode consiste à recouvrir l'image d'une grille, compter le nombre de carrés comprenant des pieds d'arcs, et voir comment ce nombre évolue en faisant varier la finesse de la grille. On obtient ainsi

la dimension fractale, notée D_f . Plus D_f est élevée, plus la structure est complexe, ramifiée et remplit l'espace.

L'application de cette méthode à l'ensemble des données (50 images par matériau de cathode) confirme l'intuition initiale: le titane présente la dimension fractale la plus haute ($D_f \approx 1.45$), tandis que le Cuivre est autour 1.14. Il est intéressant de noter que ces valeurs ne varient que très peu pour chaque matériau, que la cathode soit neuve ou usée. La dimension fractale semble donc être particulièrement adaptée pour caractériser la géométrie des motifs en fonction du matériau, et ce indépendamment de la rugosité de surface.

Pour tenter de remonter à la source physique de ces différences de dimension fractale, un algorithme d'apprentissage automatique a été développé. La librairie scikit-learn pour Python a été utilisée, la sortie de l'algorithme a été définie comme la dimension fractale, les entrées étant des paramètres physiques d'intérêt du matériau cathodique, et la fonction à optimiser étant un polynôme multi-variables. Parmi les constantes physiques d'intérêt du matériau cathodique, le choix des entrées de l'algorithme s'est porté sur la conductivité thermique Λ , la conductivité électrique σ , la densité ρ , l'énergie de vaporisation E_{vap} , l'énergie de cohésion E_{coh} , l'énergie d'ionisation E_{ion} et le travail de sortie W_f . Ces propriétés ont été choisies car elles ne sont pas (ou très peu) couplées entre elles, et qu'elles ont déjà été identifiées dans la littérature comme étant liées à différents phénomènes au sein du pied d'arc. L'algorithme d'apprentissage automatique permet de voir que les propriétés dominantes dans l'évolution de la dimension fractale sont la fonction de sortie, à plus de 55%, ainsi que l'énergie de cohésion et l'énergie d'ionisation.

Par la suite, dans le but de compléter les résultats obtenus par le modèle, une analyse heuristique a été effectuée pour voir quelle combinaison de ces propriétés, sous forme de produit, pouvait donner le comportement le plus linéaire dans l'évolution de la dimension fractale. En d'autres termes, une relation sous la forme :

$$D_f \propto \prod_i x_i^k \quad (3.1)$$

a été recherchée. L'analyse révèle que la relation

$$D_f \propto \frac{E_{coh}^2}{W_f E_{ion}^2} \quad (3.2)$$

est la plus linéaire, avec un R^2 égal à 0.865. C'est un résultat fort car il permet, théoriquement, de prédire à quel point les pieds d'arcs recouvrent la surface de la cathode, simplement en connaissant les constantes physiques de l'ergol choisi.

5 Imagerie rapide de la propagation des pieds d'arc

Si l'analyse fractale présentée précédemment permet de comprendre la géométrie globale des pieds d'arc, elle ne nous renseigne pas sur son évolution temporelle au cours d'une décharge. Il est communément admis que les pieds d'arcs s'initient et s'éteignent, de proche en proche, présentant une évolution visuelle dans le temps qui pourrait s'apparenter à un mouvement. Ce mouvement apparent, quand l'arc n'est soumis à aucun champ magnétique extérieur, semble suivre une marche aléatoire sur un chemin auto-évitant.

Pour vérifier ce comportement sur la cathode du PJP, il est nécessaire de passer de la photographie à la prise de vidéos à haute fréquence d'acquisition. Ce chapitre se concentre donc sur l'analyse temporelle de la décharge via une caméra Phantom TMX7510, permettant d'atteindre une cadence de 450 000 images par seconde. Cela nous offre une résolution temporelle d'environ $2.2 \mu\text{s}$ par image, suffisante pour observer le mouvement apparent des pieds d'arcs sur une décharge complète.

Observations phénoménologiques

L'analyse porte sur les quatre matériaux métalliques purs de cette étude : le titane, l'aluminium, le fer et le cuivre. Dès les premières vidéos, des comportements très différents sont remarqués selon les différents matériaux de cathode, confirmant la forte influence du matériau sur la dynamique de l'arc.

Pour le titane, il est observé que les spots s'initient massivement au centre de la cathode, près du système d'ignition, avant de se propager sur le reste de la surface. Bien que le champ électrique soit radial, les pieds d'arc ne se contentent pas d'aller purement radialement vers le bord de la cathode: ils présentent des trajectoires complexes, parfois tangentielles, illustrant le caractère stochastique du processus. L'aluminium se distingue par des spots très diffus et peu lumineux, rendant leur détection difficile, mais révélant un comportement chaotique et rapide. À l'opposé, le fer et le cuivre montrent une dynamique beaucoup plus linéaire et prévisible. Les spots s'initient en cercle autour de l'isolant central et progressent de manière quasi-exclusivement radiale vers la périphérie de la cathode. Le cuivre, tout comme dans les images prises dans chapitre précédent, forme de petits îlots très lents, presque statiques à cette échelle de temps.

Lien entre courant et surface active

Une des premières analyses quantitatives a consisté à mesurer l'évolution de la surface instantanée couverte par les spots $S(t)$ et à la comparer avec la forme du courant de décharge $I_d(t)$. Le résultat est très clair: pour tous les matériaux, les deux courbes se superposent presque parfaitement. Les coefficients de corrélation de Spearman r_s ont été calculés pour chaque vidéo, et ils apparaissent très élevés pour chaque matériau. Pour le titane, r_s est même supérieur à 0.9, ce qui est impressionnant pour un résultat de physique expérimentale. Ceci prouve que la surface d'émission est tout à fait corrélée avec le courant de décharge.

Cette observation valide une hypothèse physique forte, déjà remarquée dans la littérature: chaque pied individuel ne peut supporter qu'une densité de courant limite. Lorsque le courant global augmente, le pied ne fournit pas plus de courant, mais il se divise ("spot splitting") pour créer de nouveaux sites d'émission. En divisant les valeurs de courant instantané par la surface active, nous retrouvons une densité de courant moyenne stable au cours de la décharge, et d'un matériau à l'autre. Cette valeur moyenne est de l'ordre de 10^9 A m^{-2} . Bien que cette valeur soit probablement sous-estimée du fait de la résolution optique de la caméra, qui augmente artificiellement la taille réelle des pieds d'arc, sa constance démontre que le mécanisme fondamental d'émission est similaire pour tous les métaux étudiés.

Regroupement des pieds d’arc

Il est important de préciser que les pieds d’arc observés par nos caméras ne sont pas des entités élémentaires indivisibles. Comme le suggère la classification de Siemroth [68], ce qui est capturé par la caméra correspond en réalité à des “macro-spots” ou des amas de spots, eux-mêmes composés d’une multitude de micro-spots de l’ordre du micromètre, invisibles à notre échelle d’observation. Pour quantifier le nombre et la taille de ces groupements de spots, nous avons développé un algorithme de type *Depth-First Search* (DFS) capable d’identifier et d’isoler automatiquement les groupements de pieds d’arc sur chaque image.

L’analyse de ces regroupements révèle une dynamique temporelle relativement contre-intuitive. Contrairement à ce que l’on pourrait penser, les regroupements les plus larges ne se forment pas au pic de courant, mais au tout début de la décharge. Dans les premiers instants, la forte densité de courant à l’allumage force les spots à rester regroupés. Ce n’est qu’ensuite, au fur et à mesure que l’arc se propage vers la périphérie, que ces grands ensembles se fracturent et se dispersent en une multitude de structures plus fines. Ce phénomène de fragmentation (“spot splitting”) confirme que la surface active n’est pas continue: elle est le résultat d’une somme de structures discrètes dont le nombre et la taille évoluent de manière dynamique. L’influence du matériau est ici encore déterminante. Là où l’aluminium tend à former de très larges groupes de pieds d’arc, jusqu’à 0.6 mm^2), le fer et le cuivre se distinguent par des structures beaucoup plus fragmentées et petites.

Vitesse de propagation et suivi des pieds d’arc

Pour aller plus loin, des algorithmes de suivi de particules (PTV pour Particle Tracking Velocimetry en anglais) ont été utilisés afin de reconstituer les trajectoires individuelles des pieds d’arc. Le logiciel libre *TracTrac* a été modifié et utilisé sur toutes les vidéos afin d’en extraire des statistiques de vitesse apparente sur les pieds d’arcs isolés.

Les résultats classent les matériaux dans un ordre qui nous est désormais familier. Le titane présente les vitesses apparentes les plus élevées, avec une moyenne autour de 60 m s^{-1} et des maximums dépassant les 100 m s^{-1} . L’aluminium suit de près, avec une vitesse moyenne de propagation de l’ordre de 50 m s^{-1} bien que la nature diffuse de ses pieds d’arc rende le suivi délicat. Le fer est plus lent, autour des 37 m s^{-1} , et le cuivre ferme la marche avec des vitesses apparentes aux alentours de 19 m s^{-1} .

Il est intéressant de noter que cet ordre de vitesse ($\text{Ti} > \text{Al} > \text{Fe} > \text{Cu}$) est exactement le même que celui de la surface totale couverte par les pieds d’arc, et de la dimension fractale trouvées au chapitre précédent. Si le titane présente une structure fractale si étendue et complexe, c’est parce que ses pieds d’arc se déplacent vite. Pour une décharge de durée fixe, en l’occurrence $30 \mu\text{s}$, des pieds d’arc à propagation rapide ont le temps de couvrir une plus grande surface et de créer des arborescences plus complexes que des pieds d’arc lents comme ceux du cuivre.

6 Mesures de poussée

Après avoir exploré la physique fondamentale de l'arc et la dynamique des pieds d'arc, il convient de s'intéresser à la finalité de tout propulseur : la génération de poussée. Ce chapitre se consacre à la caractérisation des performances du PJP à l'aide d'une balance de torsion, développée spécifiquement pour cette étude. L'enjeu est de mesurer des forces de l'ordre du micronewton sur des temps d'impulsion extrêmement brefs, ce qui nécessite une méthodologie adaptée.

La principale difficulté réside dans la faiblesse du signal généré par une impulsion unique. L'impulsion unitaire, ou Impulse bit (I_{bit}), est si faible qu'elle se perd souvent dans le bruit thermique et mécanique du système de mesure. Pour contourner cet obstacle, deux méthodes de mesure sont proposées: une mesure de l'impulse bit par résonance et une mesure de poussée moyenne.

Le principe de la méthode de résonance repose sur l'excitation de la balance à sa fréquence naturelle d'oscillation. La balance de torsion se comporte comme un oscillateur harmonique amorti, caractérisé par une fréquence propre f_0 et mesurée expérimentalement autour de 147 mHz. En synchronisant les tirs du PJP avec cette fréquence, sur le front montant des oscillations de la balance, un phénomène de résonance est créé. Chaque impulsion vient transférer sa quantité de mouvement au système, amplifiant progressivement l'amplitude des oscillations jusqu'à atteindre un régime établi. Cette méthode présente un deux principaux avantages. D'une part, elle agit comme un filtre mécanique naturel. En augmentant l'amplitude du signal, on augmente naturellement le ratio signal/bruit. D'autre part, cette méthode permet de remonter à la valeur moyenne de la poussée et à l'impulse bit en analysant l'enveloppe de croissance des oscillations, modélisée par l'équation du mouvement d'un pendule forcé.

La mesure de poussée moyenne vise à déclencher le propulseur à une fréquence bien supérieure à la fréquence naturelle de la balance. Ce faisant, la force appliquée en continu sur le pendule le fait dévier de sa position d'origine. En calculant le décalage entre sa position en régime établi et sa position d'équilibre quand le propulseur n'est pas activé, on peut retrouver la force exercée par le propulseur. La mesure de poussée moyenne est une méthode plus simple d'un point de vue théorique, mais nécessite une bonne connaissance des incertitudes sur les différents sous-systèmes. Elle nécessite notamment une calibration précise de la balance de poussée.

Performances globales

Les campagnes de mesures ont été menées sur la gamme de matériaux métalliques précédemment étudiée: Ti, Al, Fe, Cu and NiCr. En ce qui concerne la poussée moyenne à 1 Hz, les valeurs s'étendent de 38.6 μ N pour le Cuivre à 71.7 μ N pour le Titane. La hiérarchie des matériaux est encore une fois très claire: le titane domine, suivi de l'aluminium avec 50.8 μ N et du nickel-chrome avec 50.7 μ N, le fer (42.4 μ N) et le cuivre fermant la marche.

Pour l'impulse bit déterminé par la méthode de résonance, les valeurs varient entre environ 13 μ Ns pour le cuivre et 44 μ Ns pour le titane. Si le classement des matériaux reste globalement le même, le titane générant toujours l'impulsion la plus forte, une anomalie notable est observée pour l'aluminium. Alors qu'il est performant en poussée moyenne, les valeurs d' I_{bit} trouvées par la méthode de résonance

sont très faibles, de l'ordre de $13.0\ \mu\text{Ns}$, ce qui suggère un problème mécanique spécifique lors de cette acquisition.

Une comparaison critique des deux méthodes de mesure révèle une divergence systématique. Théoriquement, pour une fréquence de 1 Hz, la valeur de la poussée moyenne en μN devrait être identique à celle de l'impulsion unitaire en μNs . Or, nous observons un facteur de divergence allant de 1.3 à 2: la méthode par résonance sous-estime systématiquement la performance par rapport à la méthode moyenne. L'analyse suggère que le modèle mathématique de l'oscillateur forcé atteint ici ses limites. Le système oscillant rejoint son régime établi plus vite que prévu, faussant l'hypothèse d'un régime transitoire purement amplifié.

Enfin, au sein même de la méthode par résonance, deux approches de calcul ont été confrontées. Une première méthode, dite d'ajustement, vise à ajuster une fonction comprenant le modèle d'oscillations forcées sur le signal complet. Un algorithme d'optimisation ajuste les variables d'intérêt du modèle au signal expérimental, permettant de retrouver l'impulse bit. Une deuxième méthode, dite des incréments, vise à calculer la différence d'amplitude entre les pics à chaque période. En implémentant un modèle mathématique adapté, cette méthode permet de retrouver l'impulse bit produit par le propulseur à chaque période. Cette dernière méthode se trouve être la plus robuste dans nos conditions expérimentales. Elle permet en effet d'identifier et d'exclure les tirs ratés du PJP, à savoir les moments où le PJP ne se déclenche pas. Cette problématique est particulièrement présente avec le cuivre et a déjà été dénotée lors des précédents chapitres. La méthode d'ajustement global tente de lisser ces échecs, introduisant une erreur dans le calcul final. C'est donc la méthode des incréments qui est retenue pour l'analyse fine de la dispersion des valeurs d'impulse bit.

Influence du matériau de la cathode

C'est en comparant les différents matériaux de cathode que les liens avec les chapitres précédents sont établis. Nous observons une hiérarchie claire dans les performances de poussée :

- Les matériaux légers comme le titane et l'aluminium offrent les poussées les plus élevées.
- Les matériaux plus denses, comme le cuivre et le laiton, produisent systématiquement les poussées les plus faibles.
- Le fer et le nickel-chrome se situent dans une gamme intermédiaire.

Cette classification rejoint parfaitement les observations faites sur la dimension fractale et la vitesse apparente des pieds d'arc. Le titane, dont nous avons vu qu'il générerait une structure de pieds d'arcs complexe et étendue, est aussi celui qui produit le plus de poussée. À l'inverse, le cuivre, avec ses pieds d'arc lents et regroupés, est le moins performant pour de la génération de poussée.

Ce résultat est assez contre-intuitif quand on regarde le modèle de J. Polk, proposant une expression de la poussée générée par un arc sous vide basée sur la conservation de quantité de mouvement [192]. Le modèle propose que la poussée générée par un pied d'arc unitaire peut s'écrire sous la forme:

$$F = \frac{m_i f_i I_d v_i}{e \bar{Z}} C_t(\bar{L}, \bar{r}_a) \quad (3.3)$$

avec m_i la masse de l'ion, f_i la fraction de courant ionique par rapport au courant total $I_i = f_i I_d$, v_i la vitesse d'éjection des ions, $\bar{Z}$ la charge moyenne des ions et $C_t(\bar{L}, \bar{r}_a)$ un facteur de correction dépendant principalement de la géométrie des électrodes.

La hiérarchie des matériaux en terme de poussée prédite par ce modèle ne correspond pas à celle observée pour le PJP, et la forte différence mesurée entre les différents matériaux n'y est pas prédite. Cette différence entre poussée théorique et réelle peut s'expliquer par plusieurs facteurs. Premièrement, ce modèle repose sur des données obtenues pour des arcs fonctionnant à un courant de décharge d'environ 100 A et une durée d'impulsion de 250 µs. Le PJP opère avec un courant près de 30 fois plus élevé et des impulsions presque 10 fois plus courtes. Par ailleurs, des paramètres clés tels que f_i , $\bar{Z}$ et la vitesse ionique v_i sont fortement dépendants du courant et de son taux de croissance, et diffèrent donc significativement dans le cas du PJP. Ceci qui souligne l'influence déterminante des conditions de décharge sur les résultats du modèle.

Surtout, ce modèle est fondé sur le comportement d'un pied d'arc unitaire, unique, et ne tient pas compte de la dynamique collective des nombreux pieds générés par un arc sous vide à fort courant. Or, les résultats montrent que la poussée évolue de la même manière que la surface totale couverte par pieds d'arc, laquelle dépend principalement du courant de décharge. De plus, malgré des dispersions importantes, une corrélation nette apparaît entre la vitesse de propagation des pieds d'arc et la poussée générée par le propulseur. Cela indique que les phénomènes macroscopiques liés aux groupements de pieds d'arc jouent un rôle plus déterminant que les mécanismes microscopiques à l'échelle d'un pied d'arc individuel.

Positionnement face à l'état de l'art

Enfin, il est intéressant de placer le PJP dans le paysage mondial des VATs. La plupart des propulseurs à arc sous vide développés à ce jour montrent des valeurs d'impulse bit de l'ordre du µNs, si ce n'est moins. Ces faibles valeurs sont compensées en atteignant des fréquences d'opération plus hautes, de plusieurs dizaines de Hertz. Il est cependant à noter que les VATs actuels opèrent dans des gammes de courant de quelques dizaines à quelques centaines d'ampères. De plus, la plupart des VATs de la littérature présentent des temps de décharge de quelques centaines de µs. Le PJP, avec ses pics à 4 kA et son impulsion de 30 µs, se situe dans une classe à part: celle des arcs à fort courant et courte durée d'impulsion. La comparaison montre que si le PJP partage les mécanismes fondamentaux des VATs classiques, son régime de fonctionnement intense induit des vitesses d'éjection supérieures et une plus grande poussée. Le taux de croissance du courant $dI/dt \approx 10^9 \text{ A s}^{-1}$ est le principal moteur de ces hautes vitesses, le fort courant étant quant à lui la cause de la plus grande poussée.

En conclusion, ce chapitre valide expérimentalement l'intérêt du titane comme ergol de choix pour ce type de propulseur. Il démontre également que la dimension fractale et la dynamique rapide des pieds d'arc, loin d'être de simples curiosités physiques, peuvent être considérées comme des paramètres à étudier pour prédire les performances des VATs.

7 Simulations numériques MC-PIC

Les chapitres précédents nous ont permis de faire le lien entre le comportement du propulseur au niveau macroscopique et les phénomènes caractérisant l'arc sous vide au niveau microscopique. Cependant, la compréhension fine des mécanismes régissant l'expansion du plasma dans le vide, en particulier l'évolution des états de charge des ions, reste difficilement accessible par la seule expérience. Les sondes perturbent le milieu, et la spectroscopie est complexe à mettre en œuvre sur des échelles de temps aussi courtes.

Ce chapitre propose une approche numérique pour étudier le comportement du plasma inter-électrodes. Le choix s'est porté sur l'adaptation d'un code de simulation hybride, couplant une méthode Particle-In-Cell (PIC) pour les ions à une approche fluide pour les électrons. Les collisions entre espèces sont gérées, elles, par une méthode de Monte Carlo (MC).

Modèle hybride

La simulation de l'arc sous vide pose un défi majeur : la disparité des échelles de temps et d'espace entre les électrons, qui sont très légers et rapides, et les ions. Une simulation entièrement PIC, qui traiterait cinétiquement toutes les particules, serait trop coûteuse en temps de calcul pour simuler le plasma sur plusieurs centimètres. Le compromis retenu est donc celui du modèle hybride MC-PIC. Dans cette architecture:

- **Les ions** sont traités comme des particules cinétiques. On suit leur trajectoire individuellement, selon les lois de Newton et la force de Lorentz.
- **Les électrons** sont traités comme un fluide sans masse.

Le modèle utilisé dans ce chapitre prend en compte l'existence de deux populations d'électrons. Une première population d'électron est considérée à basse énergie, et décrite par la relation de Maxwell Boltzmann

$$\phi = \phi_{ref} + T_e \ln \left(\frac{n_e}{n_{ref}} \right) \quad (3.4)$$

où T_e est la température électronique, supposée constante ici, V le potentiel, et n_{ref} une densité de référence calculée à l'anode (où $V = 0$). Une deuxième population d'électrons est considérée assez énergétique pour ioniser les neutres. Cette deuxième population est traitée avec une procédure de Monte Carlo, et agit comme un terme source dans la création de nouvelles paires électron-ion, par ionisation. La densité de cette population d'électrons énergétiques est considérée négligeable par rapport à la première population d'électrons, et sa contribution à l'équation de Poisson est négligée.

Le volet Monte Carlo (MC) du code gère les interactions collisionnelles. Trois types de collisions sont pris en compte pour reproduire les interactions dans le plasma d'arc: les collisions élastiques, l'échange de charge (CEX), et l'ionisation par étapes, par impact d'électron. C'est ce dernier mécanisme qui nous intéresse particulièrement car il est responsable de la création des ions multichargés caractéristiques des arcs sous vide.

Le domaine de simulation est à une dimension d'espace (1D), représentant la zone entre la cathode et l'anode, et son évolution au cours du temps est simulée. Une attention particulière a été portée à l'injection des particules. Les ions sont injectés au niveau d'une demi-sphère virtuelle centrée sur la cathode, représentant le pied d'arc.

Les paramètres d'entrée du code comme la vitesse initiale des ions, la température des électrons, ou la proportion des états de charge sont tirés de la littérature, et notamment des travaux d'André Anders. Le courant de décharge simulé est de 100 A par spot élémentaire, une valeur typique pour un groupe de pieds d'arc dans le régime de fort courant du PJP. Le matériau de cathode simulé est le titane.

Résultats

Le cas de référence est défini par une cathode en titane portée à une température de $T_c = 3200$ K, un potentiel cathodique de $V_c = -250$ V, un écart inter-électrode de 8 mm et une température électronique fixée à $T_e = 1$ eV. Afin de limiter le coût de calcul, la durée de la décharge simulée est fixée à $15 \mu s$

Dans cette configuration de référence, une ionisation significative des neutres est observée durant les premières $4 \mu s$ de la décharge. La densité ionique atteint son maximum, $5.9 \times 10^{23} \text{ m}^{-3}$, très rapidement au cours de la décharge. Cette densité d'ions maximum se retrouve à proximité immédiate de la gaine cathodique, là où l'énergie des électrons est maximale. L'état de charge moyen des ions $\bar{Z}$ culmine également au début de la décharge, entre 0 et $2 \mu s$. Au-delà de $8 \mu s$, la réduction de l'émission électronique entraîne naturellement une baisse des collisions et des réactions d'ionisation. Ceci se traduit par une chute de la densité d'ions vers 10^{21} m^{-3} tandis que la densité de neutres augmente, sous l'effet de l'évaporation continue de la cathode.

L'étude paramétrique révèle une forte sensibilité du modèle à la tension appliquée entre les électrodes. En passant à une tension cathodique de $V_c = -850$ V, le pic de densité ionique s'élève à $9.23 \times 10^{23} \text{ m}^{-3}$. Contrairement au cas de référence où les ions multichargés forment un pic localisé dans le temps et l'espace, une tension plus élevée engendre une saturation de l'état de charge moyen à la valeur limite du modèle, à savoir $\bar{Z} = 4$, entre 1 et $4 \mu s$. Ces ions hautement chargés occupent alors l'ensemble de l'espace inter-électrode et les processus d'ionisation par étape persistent jusqu'à $10 \mu s$.

Enfin, la température électronique joue un rôle déterminant dans la dynamique du plasma. L'augmentation de T_e à 3 eV ralentit la décroissance de la densité ionique, maintenant le milieu ionisé plus longtemps et augmentant significativement la charge moyenne près de l'anode ($\bar{Z} \approx 3.6$). Parallèlement, l'augmentation de la température de cathode à $T_c = 3500$ K met en évidence une boucle de rétroaction critique: l'augmentation de l'évaporation densifie le plasma, ce qui amincit la gaine et renforce le champ électrique, amplifiant in fine l'émission électronique par effet Schottky. Ce couplage conduit à une augmentation d'un ordre de grandeur de la densité moyenne du plasma, et porte la charge moyenne $\bar{Z}$ à plus de 2.2 contre 1.2 pour le cas à 3200 K [135].

Limites du modèle et perspectives

Bien que l’approche hybride MC-PIC permette de simuler l’intégralité de l’espace inter-électrode sur des échelles de temps réalistes, une analyse détaillée met en lumière plusieurs limitations physiques et numériques.

Une première restriction majeure concerne les processus collisionnels. Le modèle actuel néglige la recombinaison à trois corps électron-ion. Or, pour une température électronique de 1 eV, ce processus devient dominant dès que la densité de plasma excède $3 \times 10^{19} \text{ m}^{-3}$. Les simulations présentant des densités ioniques de l’ordre de 10^{24} m^{-3} , l’absence du processus inverse à l’ionisation par impact conduit probablement à une surestimation du degré d’ionisation final. Parallèlement, le modèle utilise des sections efficaces d’ionisation statiques, calculées pour des atomes isolés. À ces niveaux de densité, le plasma est fortement couplé et l’écrantage du noyau par les charges environnantes devrait abaisser le potentiel d’ionisation. La prise en compte de cet effet favoriserait les mécanismes d’ionisation par étapes.

Les simplifications géométriques et thermodynamiques imposent également des contraintes. La nature unidimensionnelle du code empêche intrinsèquement la modélisation de la constriction magnétique de la colonne d’arc, autrement connue sous le nom de “pinching”. Ce phénomène tridimensionnel est pourtant connu pour accroître localement la densité et la température, favorisant l’apparition d’états de charge très élevés, au-delà de $Z = 4$, observés expérimentalement [236]. De plus, l’hypothèse d’une température électronique globale constante constitue une simplification forte. En réalité, T_e varie sous l’effet du chauffage par effet Joule et du refroidissement par expansion adiabatique ou collisions inélastiques.

Enfin, la modélisation des conditions aux limites présente des carences. Au niveau de la cathode, l’absence de modélisation des aspérités à sa surface conduit à sous-estimer le champ électrique local et, par conséquent, l’émission électronique par effet de champ. À l’anode, le modèle considère une paroi passive, négligeant la pulvérisation causée par le bombardement ionique. Cette pulvérisation pourrait injecter des neutres supplémentaires dans l’espace simulé. Ces limitations sont accentuées par le manque de données fondamentales sur les sections efficaces du titane, qui contraint artificiellement la simulation à ne pas dépasser l’état de charge Ti^{4+} .

8 Conclusion

Cette thèse étudie les propriétés du plasma d’un propulseur à arc sous vide (VAT) à fort courant et pulsé sur une courte durée, au moyen de différentes méthodes expérimentales. L’objectif de ce travail était de quantifier et de mieux comprendre les facteurs qui influencent les performances de ce type de propulseur.

À cette fin, trois approches expérimentales principales ont été présentées: l’utilisation de sondes électrostatiques dans le jet de plasma, l’imagerie de la surface de la cathode, et la quantification des performances à l’aide d’une balance de poussée. Chacune de ces approches est complémentaire, et des corrélations ont été établies entre les résultats obtenus par les différentes méthodes.

La première partie de cette thèse fournit un aperçu des concepts fondamentaux couramment utilisés pour décrire les arcs sous vide. Elle présente également un bref panorama des principaux domaines d’application des arcs sous vide. L’état de l’art est plus particulièrement axé sur la propulsion spatiale, qui constitue le cadre

de cette thèse. Le propulseur Plasma Jet Pack (PJP), développé par la société COMAT, y est présenté. Il se caractérise par sa capacité à générer un courant de décharge d'environ 3 kA, sur une durée de décharge de 35 μ s. Ces conditions de décharge rendent le PJP assez unique parmi l'ensemble des propulseurs à arc sous vide existants. Les détails techniques du PJP, ainsi que les différents dispositifs expérimentaux utilisés pour étudier son plasma, sont également présentés.

Des mesures résolues en temps à l'aide de sondes de Langmuir révèlent l'évolution temporelle de la densité de plasma, de la température électronique et du potentiel plasma dans le panache du PJP équipé d'une cathode en titane. Les mesures montrent que la densité de plasma suit généralement la forme du courant de décharge, atteignant des valeurs allant jusqu'à 10^{18} m^{-3} . L'évolution temporelle du potentiel plasma et de la température électronique met en évidence la présence de deux régimes d'arc distincts : le régime d'étincelle (spark), caractérisé par une température électronique élevée, et le régime d'arc, caractérisé par une forte densité de particules. Une brève comparaison de différents modèles d'analyse des sondes de Langmuir indique que le modèle Orbital Motion Limited (OML) est le plus adapté à l'analyse de plasmas denses et pulsés, bien qu'il soit le moins complet sur le plan théorique. Les deux autres modèles considérés ici, à savoir Allen-Boyd-Reynolds (ABR) et Bernstein-Rabinowitz-Laframboise (BRL), se révèlent plus sensibles au bruit de mesure, ce qui conduit à des erreurs. Un ensemble linéaire d'électrodes a été utilisé pour mesurer la vitesse d'éjection des ions, donnant des valeurs d'environ 35 km/s pour le PJP avec une cathode en titane et environ 40 km/s avec une cathode en cuivre. Bien qu'initialement surprenant, étant donné que les atomes de cuivre sont plus lourds que ceux de titane, ce résultat démontre l'influence du taux de croissance du courant de décharge sur la vitesse des ions. En particulier, au-delà de 10^8 A/s , des effets de friction électron-ion sont rapportés, entraînant un transfert significatif d'énergie cinétique des électrons vers les ions. L'observation d'ions très rapides, avec des vitesses atteignant jusqu'à 160 km/s, suggère également la présence d'instabilités de type pinch dans le plasma du PJP.

Des photographies de la surface de la cathode prises lors des décharges révèlent une structure fractale des motifs de pieds d'arc. La dimension de ces motifs est déterminée par la méthode de Minkowski. Il est constaté que la dimension fractale varie considérablement en fonction du matériau de la cathode. Ces différences ne peuvent pas être attribuées à la dispersion typique sur plusieurs décharges, ni aux conditions de rugosité de la surface. Pour une même énergie libérée lors de la décharge, D_f s'étend de 1.14 ± 0.02 pour le cuivre à 1.45 ± 0.03 pour le titane. La surface totale couverte par les pieds d'arc au cours d'une décharge présente la même variation et le même ordre selon les matériaux. Deux méthodes ont été proposées pour étudier le rôle des propriétés physiques du matériau de la cathode, en relation à la fois avec la dimension fractale et la surface totale. Une approche par apprentissage automatique ainsi qu'une analyse heuristique montrent que le travail de sortie, l'énergie d'ionisation et l'énergie de cohésion du matériau jouent un rôle important dans la géométrie des motifs fractals.

L'imagerie à grande vitesse révèle le processus de formation de ces motifs. Bien que la direction de propagation des pieds d'arc soit connue pour être aléatoire, les valeurs de surface totale couverte (ou surface émissive) sont reproductibles avec un écart-type relativement faible. Combiné au fait que les trajectoires sont finalement assez rectilignes, on peut remarquer une dualité de comportement. La littérature

montre que le mouvement apparent d'un pied d'arc isolé est aléatoire, mais les expériences présentées ici révèlent que certains paramètres de l'arc sous vide produit par le PJP peuvent néanmoins être quantifiés. La surface émissive évolue linéairement avec le courant de décharge, et la densité de courant au niveau des pieds d'arc reste donc constante, autour de 10^9 A/m^2 , pour tous les matériaux étudiés. Ce comportement est vérifié sur l'ensemble des jeux de données et pour tous les matériaux de cathode étudiés. Il est également observé que la vitesse de propagation des pieds d'arcs dépend du matériau de la cathode, allant d'environ 19 m/s pour le cuivre à plus de 59 m/s pour le titane. En fait, il apparaît clairement que la dimension fractale est corrélée à la vitesse de propagation des pieds d'arc sur la surface de la cathode. Sur les 35 μs de la décharge, les matériaux présentant les pieds d'arc dont le mouvement est le plus rapide montrent également la dimension fractale la plus élevée. À un autre niveau, le regroupement maximal des pieds d'arc au début de la décharge, combiné à la position centrale du système d'ignition, implique que la cathode n'est pas érodée de manière homogène sur l'ensemble de sa surface. Plus important encore, les images révèlent que l'arc se développe de manière asymétrique autour de l'isolant central. Cette asymétrie conduit à une vectorisation de poussée non désirée, qui a également été observée dans les mesures par sondes électrostatiques. Cela peut donc constituer une limitation technique du PJP, liée à une utilisation faiblement optimisée de la cathode et à un manque de précision lors de manœuvres à impulsion unique pour le satellite.

Enfin, les mesures de poussée confirment, à l'échelle macroscopique, les observations réalisées à l'échelle microscopique, à savoir :

- le PJP produit un niveau de poussée différent d'une impulsion à l'autre.
- les matériaux de cathode présentant des dimensions fractales plus élevées, et par conséquent des vitesses de propagation des pieds d'arc plus importantes, génèrent des niveaux de poussée plus élevés.

Cela indique que la poussée produite par les VAT est principalement déterminée par la surface totale couverte par les pieds d'arc au cours d'une décharge ou, de manière équivalente, par leur dimension fractale plutôt que par la masse atomique du matériau de la cathode. Du moins, pour les tailles de cathodes et les niveaux de courant de décharge considérés ici, ce paramètre semble dominer les autres. La surface totale des zones d'émission de plasma et la dimension fractale sont gouvernées par la vitesse à laquelle ces pieds se propagent à la surface, et sont donc probablement liées au travail de sortie, à l'énergie d'ionisation et à l'énergie de cohésion des matériaux. Cette dernière est systématiquement utilisée par André Anders pour établir des corrélations entre les différents paramètres plasma des arcs sous vide. Ainsi, l'énergie de cohésion semble émerger non pas simplement comme un paramètre, mais comme le fil conducteur de nombreux phénomènes régissant les plasmas d'arc sous vide.

8.1 Perspectives

Bien que six matériaux de cathode aient été étudiés dans cette thèse, il serait intéressant de mesurer la dimension fractale, la vitesse des pieds d'arc, la vitesse des ions et le niveau de poussée produits avec d'autres matériaux. En particulier, il serait pertinent d'effectuer des mesures similaires avec des matériaux non métalliques tels que

le graphite, afin de déterminer si la poussée reste corrélée à la dimension fractale. Il pourrait également être intéressant d'étudier plus en détail l'influence d'autres propriétés du matériau de la cathode, notamment des aspects tels que sa structure cristalline. L'impact précis de la forme d'onde du courant de décharge, de son taux de croissance et de la durée d'impulsion sur les performances du propulseur constitue également un sujet clé dans le contexte de la propulsion. L'établissement d'une relation claire et robuste entre ces différents paramètres permettrait d'optimiser la conception des VATs, en développant des géométries et des compositions de cathodes précisément adaptées aux conditions de décharge souhaitées. Une telle approche permettrait de concevoir des propulseurs à arc sous vide optimisés à la fois en termes de poussée et d'impulsion spécifique, et adaptés aux exigences de la mission souhaitée.

Une autre direction de recherche, de nature plus théorique, consisterait en la modélisation détaillée du mouvement apparent des pieds d'arc. Il a été observé que leur schéma de propagation suit un comportement de marche aléatoire, et forme des motifs similaires à ceux générés par agrégation limitée par diffusion (DLA ou Diffusion Limited aggregation en anglais) ou par les algorithmes de type Hastings–Levitov. Ces algorithmes peuvent être caractérisés par une constante de diffusion, qui dirige la forme finale des motifs. Déterminer cette constante de diffusion en fonction du matériau de la cathode serait particulièrement précieux. Si une loi d'évolution peut être établie, il deviendrait possible d'identifier les matériaux les mieux adaptés à des applications en propulsion. Un modèle précis, qu'il soit purement numérique/théorique ou empirique, ouvrirait la voie à des conceptions de cathodes plus optimisées. Il serait alors possible de développer des alliages de différents métaux, avec des proportions optimisées, afin d'obtenir à la fois une poussée élevée et une faible dispersion de l'impulse bit.

Enfin, le principal facteur limitant l'utilisation des VATs est la contamination des surfaces isolantes par la pulvérisation du matériau de la cathode. Comme observé dans cette thèse, des décharges répétées conduisent à la formation d'une couche conductrice sur les surfaces isolantes, ce qui induit des courts-circuits inévitables après un certain temps d'utilisation. L'une des solutions proposées à ce jour consiste à appliquer un champ magnétique externe à l'arc, le forçant à se propager dans la direction $-\mathbf{j} \times \mathbf{B}$ et homogénéisant ainsi l'érosion de la cathode. L'application d'un champ magnétique semble également affecter la manière dont le matériau érodé est redéposé, ainsi que la microstructure de la couche déposée elle-même [233–235]. Cela représente une direction de recherche clé pour augmenter la durée de vie, et donc l'impulsion totale, des propulseurs à arc sous vide. Néanmoins, l'interaction entre l'arc et un champ magnétique externe demeure un sujet de recherche à part entière, comportant de nombreuses questions ouvertes et un potentiel considérable pour des recherches futures.

Etienne MICHAUX

Impact du matériau de la cathode sur les performances et le comportement du plasma d'un propulseur à arc sous vide à fort courant

Résumé :

Les propulseurs à arc sous vide représentent une technologie de propulsion spatiale innovante, légère et compacte. Ils fonctionnent en générant des arcs électriques qui, dans le vide de l'espace, vaporisent et ionisent le matériau composant la cathode. Ce dernier est expulsé à grande vitesse ce qui génère ainsi une poussée dans la direction opposée. Ces propulseurs sont particulièrement adaptés aux petits satellites grâce à leur simplicité, leur faible consommation énergétique et leur utilisation d'un ergol solide, ce qui minimise leur masse. Cependant, la physique des arcs sous vide dans un contexte de propulsion reste encore partiellement méconnue. En particulier, la dynamique de l'érosion de la cathode nécessite des travaux de recherche approfondis pour comprendre, et donc optimiser, l'utilisation des ergols.

Dans cette thèse, le comportement du plasma généré par le PJP, un propulseur à arc sous vide développé par l'entreprise COMAT, est analysé à l'aide de divers moyens expérimentaux. Ce propulseur se distingue par sa compacité et sa capacité à développer un fort courant sur une impulsion très brève. Des campagnes de mesures résolues en temps par sondes électrostatiques montrent une densité de plasma élevée, ainsi qu'une vitesse des ions supérieure à celle trouvée dans d'autres propulseurs du même type. Différentes méthodes d'imagerie révèlent que les pieds de l'arc électrique forment des motifs fractals à la surface de la cathode. La façon dont ces motifs se forment, et la manière dont le matériau de la cathode influence leur géométrie, est analysée et interprétée. Enfin, la corrélation entre les phénomènes observés à l'échelle microscopique et les performances globales du propulseur est examinée. Les conclusions de ce travail posent les bases d'une compréhension plus aboutie des processus physiques à l'œuvre dans les propulseurs à arc sous vide, ouvrant la voie à leur optimisation future.

Mots clés : Propulsion électrique, arc sous vide, plasma, fractales, imagerie, performance

Impact of cathode material on performance and plasma behavior of a high-current vacuum arc thruster

Summary :

Vacuum arc thrusters are a compact and lightweight form of electric propulsion. They operate by generating electrical arcs which, in vacuum, vaporize and ionize the cathode material. The resulting plasma is ejected at high speed, producing thrust in the opposite direction. These thrusters are well suited for small satellites due to their simple design, low power requirements, and use of a solid propellant which reduces the system mass. The physics of vacuum arcs in the context of space propulsion is still not fully understood. In particular, the mechanisms governing cathode erosion require further investigation to improve propellant utilization and thruster performance.

In this work, the plasma produced by the PJP, a vacuum arc thruster developed by COMAT, is studied using several experimental diagnostics. The PJP is characterized by its compact geometry and its short discharge pulses developing a high current. Time-resolved measurements with electrostatic probes show high plasma densities, and ion velocities exceeding those typically reported for similar thrusters. Imaging reveal that the plasma emission sites on the cathode surface form fractal patterns. The formation of these structures and the influence of the cathode material on their geometry are analyzed and discussed. Finally, the link between microscopic phenomena and thruster performance is examined. The findings of this work provide new insight into the plasma dynamics governing vacuum arc thruster performance, and contribute to their future optimization.

Keywords : Electric propulsion, vacuum arc, plasma, fractals, imaging, performance