

UNIVERSITÉ D'ORLÉANS
ÉCOLE DOCTORALE *Mathématiques, Informatique,
Physique Théorique et Ingénierie*

Laboratoire PRISME

THÈSE

présentée par

Nacer YOUSFI

soutenue le **3 avril 2026**

pour obtenir le grade de

Docteur de l'Université d'Orléans

Discipline : **Sciences et Technologies Industrielles**

**Blind Nonlinear Source Separation for Rotating
Machine Health Monitoring**

THÈSE dirigée par :

M. Karim Abed-Meraim Professeur, Université d'Orléans
M. Mohammed El-Badaoui Professeur, expert en traitement du signal, SAFRAN TECH

RAPPORTEURS :

M. Jérôme Antoni Professeur, Université de Lyon
M. Gilles Chardon Professeur, CentraleSupélec

EXAMINATEURS :

Mme Thi-Bich-Hanh Dao Professeure, Université d'Orléans (présidente du jury)
M. François Guillet Professeur, Université Jean Monnet, Saint-Étienne
M. Rémy Boyer Professeur, Université de Lille

“Vibration is the language of machines.”

Nacer YOUSFI

Séparation Aveugle de Sources Non Linéaire pour la Surveillance de l'État de Santé des Machines Tournantes

Résumé :

Cette thèse porte sur l'analyse des signaux vibratoires pour la surveillance des machines tournantes. Elle vise à développer des méthodes permettant d'interpréter automatiquement les composantes spectrales du signal liées aux phénomènes d'engrènement et de rotation. Le signal vibratoire mesuré est considéré comme le résultat du mélange de plusieurs sources d'excitation périodiques se propageant à travers le système mécanique jusqu'au capteur. La propagation, caractérisée par la fonction de transfert du système, modifie les caractéristiques des signaux et les informations directement associées à l'état de santé des composants. Afin d'accéder à ces informations, une méthodologie de modélisation automatique est proposée, fondée sur des formulations analytiques compactes du contenu fréquentiel. Cette modélisation permet l'identification et la séparation des composantes vibratoires élémentaires associées aux fréquences caractéristiques. Une méthode spécifique d'estimation de la fréquence d'engrènement est développée et intégrée dans un modèle de séparation de sources, permettant d'analyser les contributions des sources vibratoires élémentaires et leurs variations en présence de défauts.

Mots clés : Modélisation, Régression symbolique, Réseau de neurones, Séparation de sources, Identification de fonction de transfert, Analyse vibratoire.

Blind Nonlinear Source Separation for Rotating Machine Health Monitoring

Summary :

Vibration data analysis plays a vital role in condition monitoring of rotating machines by extending service life, preventing costly repairs, and improving safety. An effective analysis, however, requires a proper understanding of vibration characteristics. In this thesis, an automatic modeling approach is proposed, based on symbolic analysis to interpret and understand the frequency content of rotating machines by leveraging analytical formulas, aiming to replace traditional manual behavior modeling methods. These concise models are grounded on system mechanisms and are crucial to separate the vibration components and to identify potential fault sources. The health state information in vibration data is often distorted by the transfer function effects of system transmission paths, which hinder direct access. In this work, we propose methodologies to remove these effects from measurements. In one approach, the effects are identified during the automatic modeling process. In the other approach, identification is performed after modeling, using the deterministic component of the vibration to estimate it. The latter deterministic part of the vibration is also used to estimate the fundamental carrier frequency, which is then employed in the source separation scheme. Our contributions are validated using both simulated and real data from planetary gear transmission sets, demonstrating strong potential in addressing vibration modeling, source separation, and system identification applications.

Keywords: Modeling, Symbolic regression, Neural network, Source separation, Transfer function identification, Vibration analysis.



Laboratoire PRISME
12 rue de Blois, Orléans 45100, France



Contents

Abstract	13
Résumé	17
Résumé substantiel	21
1 Introduction	33
1.1 Motivation	33
1.2 Scope of the thesis	34
1.3 Contributions	35
1.4 Publications	37
1.5 Structure of the Thesis	39
2 Background	43
2.1 Introduction	43
2.2 Gear transmission set	45
2.2.1 Fixed-axis gearbox	45
2.2.2 Epicyclic gear set	46
2.2.3 Perceived signal	48
2.3 Phenomenological modeling	50
2.3.1 Vibration signal	53
2.3.2 Planetary gearboxes characteristic frequencies	56
2.4 Source separation	58
2.4.1 Vibration separation methods	59
2.4.2 Blind source separation	61
2.5 Transfer function effects	62
2.5.1 Signal processing methods	62
2.5.2 Operational Modal Analysis methods (OMA)	64
2.6 Problem statement	67

3	A Phenomenological modeling via symbolic regression	70
3.1	Problem statement	72
3.2	Neural network based symbolic regression	74
3.2.1	Related work	74
3.2.2	Multivariate regression	75
3.2.3	Network architecture	76
3.2.4	Sparsity regularization	80
3.2.5	Training procedure	81
3.2.6	Model selection	81
3.3	Robust Modeling through Binary Neural Network-Based Symbolic Regression	82
3.3.1	New target data	83
3.3.2	Spectral peaks detection algorithm	84
3.3.3	Binary Neural Network	85
3.4	Modeling with Transfer Function effects removing	86
3.4.1	Finite impulsive response	86
3.4.2	Infinite impulsive response	87
3.5	Stacked Neural Network-based Symbolic Regression	89
3.5.1	Network architecture	89
3.6	Simulations	90
3.6.1	Signal modeling with transfer function effects compensation	93
3.6.2	Robust solution for signal modeling	96
3.6.3	MISO system	97
3.6.4	Real world dataset	98
3.7	Conclusion	103
4	Blind System Identification	106
4.1	Problem statement	107
4.2	Estimation Bound	109
4.2.1	Finite Impulse Response (FIR)	109
4.2.2	Infinite Impulse Response (IIR)	110
4.2.3	Fisher matrix	111
4.2.4	Identifiability Analysis	111
4.3	Neural Network-based System Identification	112
4.4	System identification with Structured Line Spectrum	113
4.4.1	Spectrum line magnitudes estimation	114
4.4.2	FIR case	114
4.4.3	IIR case	115

4.4.4	Least square	117
4.4.5	Alternating least square	117
4.4.6	Simulations	119
4.5	Conclusion	127
5	Frequency estimation based on line spectrum structure	129
5.1	Introduction	129
5.2	Vibration spectrum structure	131
5.3	Problem statement	132
5.4	Proposed method	133
5.4.1	Peaks detection	133
5.4.2	Weighted Least Squares	133
5.4.3	RANSAC-Based Estimation	135
5.5	Frequency refinement	137
5.6	Simulation	138
5.7	Application: planetary gearbox meshing frequency estimation	139
5.8	Conclusion	142
6	Source Separation and Tracking of Periodic Signals	144
6.1	Introduction	144
6.2	Problem Setup	145
6.3	Neural network-Based Source Separation	146
6.4	Signal Harmonic Selection	147
6.5	Maximum Likelihood (ML) Solution with ALS Optimization	147
6.6	Alternating Least Squares	149
6.6.1	Algorithm's initialization	150
6.7	Adaptive algorithm	150
6.8	Simulation results	151
6.8.1	Neural network	151
6.8.2	ALS	155
6.8.3	Application: Planetary gearbox	156
6.9	Conclusion	158
7	Conclusion	160
7.1	Contributions	161
7.2	Perspectives	164
7.2.1	Near-Term	164
7.2.2	Future works	164

List of Figures

2.1	Fixed-shaft gearbox	45
2.2	Schematic of a 3-planet planetary gearbox	46
2.3	Typical vibration spectrum	50
2.4	PG with single planet	54
2.5	PG with 4 planet gears, with phase settings ψ_i for each gear as in the traditional modeling.	56
3.1	MISO system	73
3.2	Example of EQL network for SR where f_b and f_u are the binary and unary operators. The red and green colors represent the skip connections. Bias nodes are not shown for clarity.	77
3.3	EQL network with sinusoidal inputs	79
3.4	Spectrum peaks interpretation with Neural network-based symbolic regression	83
3.5	Proposed regularization with two minimas $\{0, 1\}$	86
3.6	Neural network-based symbolic regression with a convolution layer used for transfer function removal.	87
3.7	Neural network-based symbolic regression with two convolu- tion layers used for rational transfer function removal.	88
3.8	Architecture of the 3-stacked neural network-based symbolic regression. The neural network architecture in the middle box is the same as in Fig. 3.6.	89
3.9	A synthesized vibration data of a three-planet gear.	91
3.10	Signal reconstruction	93

LIST OF FIGURES

3.11	One of the final neural networks after training and thresholding, where only the desired inputs (harmonics) and a few connections are activated. For clarity, only 3 out of 4 sources are displayed as activated. Also, the exact number of harmonics per source signal is obtained after convergence, while the actual number of overestimated harmonics is 20 per signal. The first hidden layer estimates the elementary sources $s_{1 \leq i \leq 3}(t)$, where the source s_4 , associated with the sun's rotation frequency, is successively deactivated by our method.	94
3.12	Source harmonics activation in the estimated sources.	94
3.13	Signal reconstruction	95
3.14	Comparison between the convolution layer output and the three filters used to generate the synthesized data.	95
3.15	Detected spectral peaks from synthesized vibration data	96
3.16	Signal reconstruction zoomed around the first meshing harmonic with a stacked Symbolic regression based-neural network	98
3.17	Signal reconstruction zoomed around the first meshing harmonic with stacked Symbolic regression based-neural network	99
3.18	The spectra of both the original and reconstructed signals.	100
3.19	Spectral representation of the reconstructed signals	100
3.20	The signal $\mathbf{x}$, derived from the detected peaks of the raw signal $\mathbf{y}$, used for our multivariate regression.	102
3.21	Reconstruction of the spectrum peaks in signal $\mathbf{x}$ using the proposed neural network, denoted as $\hat{\mathbf{x}}$ around the fourth meshing harmonic.	102
4.1	Amplitude spectrum illustration zoomed around a single carrier.	113
4.2	*	121
4.3	The estimated IIR filter.	122
4.4	Filter estimation results for the MISO system at 20 dB.	124
4.5	Filter estimation results for the MISO system at 0 dB.	124
4.6	Comparison of the Performance via Spectral Residual of ADDM-Based and SVD-Based Methods as a Function of Filter Tap Length	125
4.7	Spectral residual as a function of SNR at an FIR length of 15	126
4.8	FIR example reconstruction.	126
4.9	Reconstruction example of $\hat{\boldsymbol{\alpha}} = \hat{\mathbf{a}} \otimes \hat{\mathbf{b}}$.	127

LIST OF FIGURES

5.1	Planetary gear set vibration power spectrum illustration. . .	132
5.2	Signal reconstruction without dictionary refinement for fundamental frequency perturbation $\delta = 0.5$ with $SNR = 0$ dB.	140
5.3	Signal reconstruction with dictionary refinement for fundamental frequency perturbation $\delta = 0.5$ with $SNR = 0$ dB. .	140
5.4	Power spectrum peaks detection.	141
5.5	Measured and Model-Predicted Spectral Peaks	142
6.1	Harmonic selection and robust modeling	148
6.2	Zoomed portion of the signal reconstruction at the first carrier harmonic.	152
6.3	Source separation results at 0 dB.	153
6.4	Source separation results at 20 dB.	154
6.5	Our approach-based ALS performance against the CRB with a low level of SNR over 100 runs.	155
6.6	Tracking of source parameters over time with $SNR = 10$ dB. Each plot shows the tracking of five harmonics of the considered source.	156
6.7	The root-mean-square (RMS) of the estimated sources s_1, s_2, s_3 . The modulation signal s_1 exhibits a monotonic trend correlated with the increasing planetary gear degradation, especially in the final measurements, where an artificial crack is introduced along the planetary gear. In contrast, the meshing and modulation signal s_2 fluctuates over time without a clear monotonic pattern.	157

List of Tables

3.1	Equations at 0 dB.	97
4.1	Estimation performance of the sources after training the neural network.	121
4.2	Estimation performance of the sources in the presence of an IIR filter.	122
4.3	FIM results	125
4.4	IIR filter Spectra Residual as a function of SNR with $N = 4$	126
5.1	Performance of LASSO reconstruction before and after dictionary refinement for different perturbation values δ at $SNR = 0$ dB.	139
6.1	Correlation between true and predicted signals for different networks at various SNR.	152
6.2	Neural Network equations at 0 dB.	154

Abstract

Blind Nonlinear Source Separation for Rotating Machine Health Monitoring

Safety and reliability are critical challenges in the aeronautical sector, affecting aircrafts, helicopters, and other complex mechanical systems such as wind turbines and large vehicles, where continuous operation and high availability are essential. Condition monitoring has become a major research focus for early machine fault detection. In this context, vibration-based condition monitoring is widely adopted, as vibration signals encode the dynamic behavior of complex mechanical systems and the interactions among their components. Moreover, vibration measurements are simple and low-cost to implement, which explains their widespread use in industry. This thesis aims to develop new methods for decomposing data from complex rotating machines to extract informative indicators of their health that cannot be measured directly due to system complexity. Vibration model-based approaches typically comprise several steps, among

which phenomenological modeling plays a central role. This modeling aims to unveil analytical expressions that fit the accelerometer data by exploiting prior knowledge of system dynamics, such as characteristic rotational frequencies. Traditionally, this process relies on manual matching between characteristic frequencies and spectral content, which is both time-consuming and highly dependent on expert knowledge. To address these limitations, this research proposes an approach grounded in symbolic analysis to automatically identify mathematical expressions that describe the measured vibration data, given the machine’s rotational speed and kinematic parameters. Specifically, a neural network-based symbolic regression framework is introduced to unveil compact and interpretable analytical formulas selected through a learning process incorporating sparsity constraints.

Health-related information in vibration signals often appears as weak, short-duration patterns in the time domain and is significantly distorted by the system transfer function, resulting from the propagation of vibrations from internal and external components to the sensor location. Consequently, a preprocessing step is commonly employed to mitigate these effects in both phase and amplitude, enabling the reconstruction of signals that are more representative of the true health state. While cepstrum-based approaches focus on the stochastic components of the vibration signal, we propose a methodology leveraging the deterministic part of the vibration signal and its structured spectral information to estimate the system’s impulse response and recover health-related patterns.

Accurate rotational-speed estimation is essential for fine vibration analysis, particularly under fluctuating operating conditions. Existing multi-order probabilistic approaches exploit several harmonics of a dominant frequency to refine the speed estimate. In this thesis, we propose a complementary method based on a structured line-spectrum model derived from amplitude modulations identified through phenomenological modeling. This model is used to estimate the gear-meshing frequency associated with the machine rotational speed. The approach begins by estimating spectral harmonic peaks and their amplitudes, followed by a robust matching optimization procedure that aligns the detected peaks with predefined normalized harmonic frequency patterns, improving the estimation without assuming the highest-energy line represents the target frequency.

Finally, a source separation approach is proposed to isolate the elementary vibration components, enabling direct access to the information reflecting the system’s health state. A maximum likelihood estimator is proposed based on the phenomenological model using alternating least-squares optimization

followed by a gradient-like step to track the components' variation. These contributions, validated through simulations and experimental data, demonstrate significant potential to enhance fault detection pipelines.

Keywords: Modeling, Symbolic regression, Neural network, Blind source separation, Transfer function identification, Vibration analysis, Frequency estimation, CRB.

Résumé

Séparation Aveugle de Sources Non Linéaire pour la Surveillance de l'État de Santé des Machines Tournantes

La sécurité et la fiabilité sont cruciales en aéronautique et dans d'autres industries telles que les éoliennes ou les véhicules lourds, où un fonctionnement continu est essentiel. La surveillance de l'état des machines est un domaine clé pour la détection précoce des défauts. Dans ce contexte, la surveillance vibratoire est largement utilisée pour l'évaluation de l'état de santé des composants. Cette thèse vise à développer de nouvelles méthodes de décomposition des signaux vibratoires afin d'extraire des indicateurs pertinents de l'état de santé du système, inaccessibles par des mesures directes. Les approches basées sur des modèles vibratoires comprennent plusieurs étapes principales, parmi lesquelles la modélisation phénoménologique occupe une place centrale. Cette modélisation vise à mettre en évidence des

expressions analytiques adaptées aux données vibratoires, en exploitant les connaissances a priori sur la cinématique du système. Traditionnellement, ce processus repose sur une correspondance manuelle entre les fréquences caractéristiques et le contenu spectral, ce qui est à la fois chronophage et fortement dépendant de l'expertise. Cette recherche remédie à ces limitations en proposant une approche basée sur l'analyse symbolique, capable d'identifier automatiquement des expressions mathématiques décrivant les données vibratoires, en tenant compte de la vitesse de rotation de la machine et de la cinématique. Une méthode de régression symbolique basée sur les réseaux de neurones est introduite afin de découvrir des formules analytiques compactes et interprétables, sélectionnées via un processus d'apprentissage intégrant des contraintes de parcimonie. Les informations relatives à l'état de santé contenues dans les signaux vibratoires apparaissent souvent sous forme de motifs faibles et de courte durée dans le domaine temporel, et sont déformées par la fonction de transfert du système, due à la propagation des sources d'excitation jusqu'au capteur. Par conséquent, une étape de prétraitement est utilisée pour atténuer ces effets, permettant ainsi de reconstruire des signaux plus représentatifs de l'état de santé réel. Dans ce contexte, contrairement aux approches basées sur le cepstre, qui exploitent les composantes stochastiques du signal vibratoire, nous proposons une contribution originale fondée sur la composante déterministe des données vibratoires mesurées. La méthode proposée exploite les informations spectrales structurées pour estimer la réponse impulsionnelle du système, permettant ainsi de récupérer les motifs liés à l'état de santé. De plus, la connaissance précise de la vitesse de rotation des machines est essentielle pour une analyse fine des vibrations, en particulier pour compenser les fluctuations de vitesse. Cette exigence motive la recherche visant à extraire l'information sur la vitesse de rotation directement à partir des mesures vibratoires, évitant ainsi l'installation coûteuse d'encodeurs. Contrairement aux approches probabilistes multi-ordres (MOPA), qui exploitent les harmoniques multiples d'une fréquence d'intérêt, la méthode proposée utilise un modèle phénoménologique pour caractériser les spectres structurés où l'information se répartit sur plusieurs raies en raison de phénomènes de modulation. La méthode estime d'abord les pics harmoniques, puis les aligne via une optimisation robuste sur des modèles harmoniques normalisés, améliorant ainsi l'estimation de la vitesse de rotation sans supposer que la raie d'énergie maximale correspond à la fondamentale, comme dans MOPA. Enfin, une méthode de séparation de sources associées aux composantes caractéristiques du système est proposée afin d'isoler et d'extraire directement les informations liées à son état de

santé. À cette fin, un estimateur du maximum de vraisemblance fondé sur le modèle phénoménologique est mis en œuvre. Son estimation repose sur une optimisation par moindres carrés alternés, complétée par une étape de type gradient permettant le suivi temporel des variations des sources.

Mots-clés : Modélisation, Régression symbolique, Réseau de neurones, Séparation de sources, Identification de fonction de transfert, Analyse vibratoire.

Résumé substantiel

Introduction générale

Le secteur aéronautique a connu une évolution technologique significative au cours des dernières décennies, portée par des exigences croissantes en matière d'efficacité, de performance et de fiabilité. Cette évolution est particulièrement visible dans les systèmes de propulsion avancés pour avions et hélicoptères, ainsi que dans d'autres machines tournantes à haute performance telles que les éoliennes, les véhicules lourds et les équipements industriels. Ces systèmes se caractérisent par une densité de puissance, une vitesse de rotation et une complexité mécanique sans cesse croissantes. Ces machines tournantes sont constituées de multiples sous-systèmes interconnectés, les systèmes de réduction par engrenages, les roulements et les arbres tournants. Cette complexité structurelle et fonctionnelle rend le diagnostic au niveau des composants indispensable pour la détection des défauts, la fiabilité du système et la sécurité de fonctionnement.

La surveillance de l'état de ces systèmes est particulièrement complexe. Les stratégies de maintenance actuelles reposent largement sur la maintenance corrective, fondée sur un modèle de panne-réparation, ainsi que sur la maintenance préventive, qui implique des inspections périodiques après plusieurs milliers d'heures de vol, durant lesquelles le moteur est démonté pour une inspection détaillée composant par composant. Bien qu'efficaces, ces interventions sont coûteuses et chronophages, ce qui souligne la nécessité de diagnostics avancés et d'une surveillance de l'état en temps réel.

D'un point de vue économique, la maintenance prédictive est essentielle pour optimiser l'état de santé du système, réduire les temps d'immobilisation des moteurs et garantir la fiabilité opérationnelle. Dans l'industrie aérospatiale, les modèles économiques évoluent de plus en plus de l'achat direct de

moteurs vers des dispositifs de location. Dans ce contexte, la surveillance de l'état devient une priorité stratégique, car elle permet de maintenir les moteurs dans un état optimal, permettant ainsi aux entreprises de gérer efficacement les actifs loués et de minimiser les risques opérationnels.

Cette thèse propose un ensemble de méthodes pour la séparation de sources de signaux vibratoires d'engrenages, articulées autour de la modélisation des données, de l'atténuation des effets de propagation et de l'estimation de la fréquence porteuse. Après un état de l'art sur l'analyse vibratoire des trains épicycloïdaux, un cadre de modélisation phénoménologique automatique fondé sur la régression symbolique par réseau de neurones est introduit, produisant des expressions analytiques interprétables. La réponse impulsionnelle du système est ensuite identifiée à partir des composantes harmoniques déterministes du signal, par une approche intégrée à la modélisation ainsi que par optimisation non linéaire exploitant l'asymétrie des bandes latérales. Un problème d'estimation structurée de raies spectrales, appliqué à la détermination de la fréquence d'engrènement, est également traité. Enfin, une solution de séparation et de suivi de sources est proposée, fondée sur un estimateur du maximum de vraisemblance atteignant la borne de Cramér–Rao, avec un raffinement adaptatif permettant de suivre la dégradation progressive des éléments du système.

Background

Les systèmes de transmission de puissance sont des composants critiques dans les applications industrielles telles que les ensembles de transmission par engrenages. La surveillance de l'état de santé de ces machines tournantes devient de plus en plus importante pour répondre à la demande de l'industrie en dispositifs permettant de réduire les temps d'immobilisation dans des applications critiques telles que les hélicoptères, les aéronefs, les éoliennes et les véhicules lourds.

Les stratégies de maintenance traditionnelles sont généralement réalisées de manière périodique selon des plans de maintenance fixes ou lors des arrêts machine, impliquant souvent le remplacement de composants entiers même lorsque certaines pièces restent intactes. Les objectifs actuels de l'industrie visent à améliorer le fonctionnement des moteurs et la planification de la maintenance afin d'assurer une performance efficace et fiable tout au long

de la durée de vie de la machine. Les futures générations de moteurs à haute puissance utilisant des technologies de pointe nécessitent des systèmes de surveillance de l'état avancés, capables de suivre l'état de la machine et de planifier automatiquement des actions de maintenance ciblées sous des conditions de fonctionnement variables. Par conséquent, l'industrie s'oriente de plus en plus vers des systèmes de diagnostic capables de détecter et de localiser les composants défaillants à un stade précoce, réduisant ainsi les temps d'arrêt et les pertes économiques. Dans les machines tournantes, la maintenance repose souvent sur l'analyse du bruit, de manière analogue à un médecin écoutant le battement de cœur d'un patient. Bien que ces machines génèrent naturellement du bruit, la surveillance de l'état met l'accent sur l'identification des changements dans les caractéristiques du bruit qui indiquent l'apparition de défauts.

Les systèmes d'engrenages épicycloïdaux (systèmes d'engrenages planétaires) constituent un axe de recherche majeur en raison de leur large utilisation dans les applications de transmission de puissance, telles que les transmissions de rotor principal d'hélicoptère, les éoliennes et les boîtes de réduction des moteurs d'aéronefs, où des rapports de réduction de couple ou de vitesse élevés sont requis dans un encombrement compact. De plus, ces ensembles d'engrenages fonctionnent souvent dans des conditions de travail sévères, rendant les dents d'engrenage susceptibles à la fissuration par fatigue et, dans les cas graves, à la rupture de dent. Comparés aux ensembles d'engrenages à axe fixe conventionnels, les ensembles d'engrenages épicycloïdaux présentent des caractéristiques uniques en raison de leur structure complexe. Cette complexité se traduit par des signaux vibratoires mesurés présentant des motifs distincts et de nombreuses composantes spectrales, contrairement à ceux observés dans les boîtes de vitesses à axe fixe. Par conséquent, les techniques conventionnelles de détection de défauts développées pour les ensembles d'engrenages à axe fixe ne peuvent pas être directement appliquées aux systèmes d'engrenages épicycloïdaux. Ce défi a ainsi motivé les chercheurs à approfondir l'étude et la compréhension des motifs vibratoires des ensembles d'engrenages épicycloïdaux ainsi que de l'influence de leur mouvement sur les signaux vibratoires mesurés.

Par conséquent, de nombreuses techniques basées sur l'analyse vibratoire ont été développées pour les ensembles d'engrenages épicycloïdaux. Les chercheurs se sont soit concentrés sur le préconditionnement des signaux vibratoires afin de permettre une analyse similaire à celle des ensembles

d'engrenages à axe fixe, permettant ainsi l'utilisation de méthodes conventionnelles de détection de défauts, soit ont proposé de nouvelles approches pour mieux comprendre les motifs vibratoires associés au mouvement épicycloïdal. Cela donne lieu à un ensemble de méthodes qui se concentrent sur la modélisation du signal vibratoire et qui proposent des équations explicites pour décrire la structure spectrale, ce qui contribue à la compréhension des caractéristiques spectrales permettant des techniques de détection de défauts plus efficaces. De plus, le signal mesuré est généré par des excitations qui se propagent à travers un chemin de transmission depuis la source d'excitation jusqu'au point de mesure. Par conséquent, l'état de santé du système ne peut pas être mesuré directement, ce qui limite l'efficacité des méthodes conventionnelles de détection de défauts. Pour résoudre ce problème, les techniques basées sur l'analyse vibratoire incluent souvent une étape de prétraitement visant à éliminer les effets du chemin de transmission ou à identifier la réponse impulsionnelle de la fonction de transfert du système.

Modélisation phénoménologique par régression symbolique

Les travaux de cette thèse s'inscrivent dans le domaine de la surveillance et du diagnostic de l'état de santé des machines rotatives, et plus particulièrement des transmissions de type engrenages planétaires/épicycloïdaux. L'objectif principal est de développer des indicateurs de santé à visée préventive, permettant d'anticiper l'apparition de défaillances et d'améliorer la fiabilité des systèmes. L'approche méthodologique retenue repose sur l'analyse des signaux de vibration. Des techniques avancées de traitement du signal sont mises en oeuvre afin d'extraire les informations pertinentes révélatrices de l'état de fonctionnement des organes mécaniques.

Le principe central consiste à décomposer le signal de vibration global en composantes élémentaires associées aux différents éléments mobiles du machine. Chacune de ces composantes est caractérisée par des fréquences caractéristiques propres au comportement de l'élément considéré, traduisant sa contribution individuelle à la signature vibratoire globale de la machine. Cette modélisation par contributions individuelles permet non seulement d'identifier avec précision l'origine des défauts susceptibles d'émerger au cours du fonctionnement, mais également d'en assurer le suivi dans des conditions de régime variable en vitesse et en charge.

Les méthodes développées s'articulent autour d'une analyse conjointe des fréquences caractéristiques et des amplitudes associées, offrant ainsi un cadre robuste pour l'évaluation de l'évolution de l'état de santé des systèmes étudiés. Le signal de vibration des machines rotatives présente un caractère quasi-périodique en raison du mouvement de rotation des organes mécaniques. Cette périodicité engendre une organisation structurée du contenu spectral, généralement caractérisée, comme décrit dans la littérature, par la présence de multiples phénomènes de modulation entre les différentes fréquences caractéristiques de la machine, notamment la fréquence d'engrènement et les fréquences de rotation des éléments mobiles. Traditionnellement, l'interprétation de cette structure spectrale repose sur une analyse manuelle et visuelle des raies fréquentielles, visant à mettre en correspondance les composantes observées avec un modèle mathématique fondé sur des modulations d'amplitude ou de fréquence. Toutefois, cette approche reste fortement dépendante de l'expertise de l'analyste et ne permet pas toujours d'identifier automatiquement ou robuste des signatures de défauts.

Du point de vue du contenu spectral du signal de vibration, l'objectif est d'attribuer automatiquement chaque raie fréquentielle soit directement à la fréquence d'engrènement, soit à une composante issue d'un phénomène de modulation entre cette fréquence d'engrènement et une fréquence caractéristique associée à un organe mécanique. Cette approche s'inscrit dans une analyse principalement menée sur l'axe fréquentiel.

Le principe consiste à développer une méthodologie automatique prenant en entrée une mesure de vitesse de rotation ainsi que le modèle cinématique de la machine (notamment le rapport d'engrènement), afin d'estimer les différentes fréquences caractéristiques attendues. À partir de ces informations, un modèle mathématique est généré dans le but de reproduire fidèlement la structure spectrale observée, en justifiant la présence de chaque raie par une interaction identifiable entre les composantes cinématiques de la machine. Une telle modélisation offre également la possibilité de révéler des interactions faibles ou complexes qui ne sont pas détectables à l'oeil nu lors d'une simple inspection visuelle du spectre. La sélection du modèle retenu repose sur un principe de parcimonie : parmi l'ensemble des modèles envisageables, on privilégie celui qui décrit le mieux le signal tout en conservant une complexité minimale.

Le premier travail réalisé a consisté à tenter de reconstruire le signal de vibration à partir des fréquences caractéristiques, en utilisant un réseau

de neurones symbolique. Dans ce type de réseau, les fonctions d'activation classiques sont remplacées par des opérateurs mathématiques représentant les termes primitifs de l'expression finale du modèle. La reconstruction du signal est influencée par le fait que, dans la réalité, le signal de vibration intègre également l'effet des canaux de transmission, c'est-à-dire la fonction de transfert du système entre le point d'engrènement et la position du capteur.

En revanche, la fonction de transfert du système agit principalement sur l'amplitude et l'énergie des raies, sans modifier leur position fréquentielle. Or, les méthodes classiques de modélisation des signaux vibratoires se basent uniquement sur la position des raies sur l'axe fréquentiel, et négligent ainsi l'effet de la fonction de transfert sur l'énergie du signal. Pour y remédier, nous proposons une approche combinant un algorithme d'estimation des fréquences, capable d'extraire automatiquement toutes les positions des raies, et un réseau de neurones qui génère une approximation du modèle expliquant l'ensemble des raies observées, tout en prenant en compte les fréquences caractéristiques de la machine.

De plus, le signal de vibration constitue une combinaison des contributions provenant de différents points d'engrènement, chacun étant relié au capteur par son propre canal de transmission. Par ailleurs, en présence d'imperfections dans les roues ou de défauts naissants sur certains éléments de la machine, ces points peuvent générer des mélanges variés de fréquences caractéristiques, des amplitudes différentes, ainsi que des réponses affecté par des fonctions de transfert distinctes.

Pour répondre à ces complexités, nous proposons d'utiliser un réseau de neurones symbolique pour chaque point de contact (engrènement), permettant de modéliser le mélange des fréquences caractéristiques. Chaque bloc est ensuite suivi d'une convolution spécifique, destinée à modéliser les différents canaux de propagation entre le point d'engrènement et le capteur. Cette architecture permet ainsi de prendre en compte à la fois les interactions entre fréquences caractéristiques pour chaque point de contact, les variations d'amplitude liées aux fonctions de transfert, ainsi que les contributions élémentaires dans chaque point d'engrènement à la vibration globale. Les différents mélanges identifiés pour chaque point d'engrènement peuvent ensuite servir d'indicateurs de santé. En présence de défauts naissants, de nouvelles interactions apparaissent dans le spectre (émergence de raies spécifiques), ce qui permet de localiser précisément l'origine des défauts.

Identification aveugle de systèmes

Les informations relatives au défaut contenues dans les signaux vibratoires sont souvent modifiées et atténuées par les effets de la fonction de transfert du système, en particulier pour les défauts locaux, tels que les fissures, où la durée de l'excitation est courte. La forme d'onde de l'excitation au point d'engrènement est importante pour détecter et localiser les éléments défaillants. Plusieurs travaux ont récemment été proposés pour identifier et supprimer les effets des fonctions de transfert afin de reconstruire les fonctions de force d'engrènement au point d'action.

En revanche, ce travail exploite les bandes latérales déterministes situées au voisinage de la fréquence d'engrènement (porteuse) et de ses harmoniques dans le signal vibratoire mesuré sur des ensembles d'engrenages à mouvement épicycloïdal, tels qu'un train épicycloïdal (*planetary gear set*). Le signal vibratoire est généralement modélisé à l'aide d'un modèle de modulation d'amplitude permettant de décrire les bandes latérales observées dans le spectre vibratoire. Étant donné les fréquences caractéristiques et un modèle qui explique les raies du spectre, nous avons entamé le travail en exprimant les amplitudes des raies du contenu spectral en fonction des paramètres du filtre (FIR ou IIR) et des amplitudes des signaux élémentaires liés aux fréquences caractéristiques. Dans ce cadre, nous obtenons un système d'équations dont le nombre d'équations est supérieur au nombre de paramètres lorsque le spectre contient un grand nombre d'harmoniques. Nous avons développé un système linéaire qui, via un solveur linéaire basé sur la décomposition en valeurs singulières (SVD), permettait d'estimer les paramètres du filtre. Cette linéarisation échoue lorsque le mémoire du filtre est large, ou lorsque l'ordre du filtre est élevé, rendant l'approche linéaire inefficace. Nous avons donc proposé de traiter le problème via une optimisation alternée, plutôt que de passer par une étape de linéarisation, pour mieux gérer les filtres de grande mémoire et d'ordre élevé.

Estimation de fréquence basée sur la structure du spectre de raies

Dans les machines tournantes, les méthodes d'analyse spectrale sont généralement utilisées pour identifier les défauts survenant au niveau des composants rotatifs de la machine, notamment la boîte de vitesses, les arbres, etc. Leur

application est toutefois limitée par la contrainte que représente l'installation d'un codeur (encoder) ou d'un tachymètre, permettant de mesurer la vitesse de rotation des machines en fonctionnement sous conditions opérationnelles variables. En raison des économies potentielles qu'elle permet, l'utilisation d'un accéléromètre en remplacement des codeurs a suscité un intérêt croissant au sein de la communauté de la surveillance de l'état (condition monitoring).

L'estimation de la vitesse de rotation constitue une exigence essentielle pour la plupart des méthodes de surveillance de l'état basées sur les vibrations, telles que les méthodes fréquentielles et les méthodes de rééchantillonnage angulaire. Des méthodes basées sur l'analyse vibratoire ont été explorées afin de réaliser l'estimation de la vitesse de rotation ; cependant, en raison de la présence de multiples harmoniques dans les signaux vibratoires, de la résonance structurelle et du faible rapport signal-sur-bruit, l'applicabilité des méthodes existantes, telles que les méthodes d'estimation bayésienne et la transformée de Hilbert, demeure limitée et difficile. En outre, plusieurs méthodes ont été proposées, telles que la démodulation de phase, la démodulation de phase itérative, la démodulation de phase multi-harmonique, le suivi de maximum combiné à un filtre de Vold-Kalman, l'approche probabiliste multi-ordre. Les trains épicycloïdaux (hélicoptères, aéronefs, éoliennes) présentent des bandes latérales spectrales autour de la fréquence d'engrènement, même en état sain, reflétant les modulations avec la fréquence de rotation des éléments mobiles (détails au chapitre Modeling). Or, la méthode MOPA n'exploite que la fréquence de rotation et ses multiples, sans tirer parti de ces bandes latérales. Ce travail propose donc une nouvelle approche exploitant à la fois les harmoniques au voisinage de la fréquence d'engrènement et ses multiples, afin d'améliorer l'estimation de la fréquence de rotation à partir des informations cinématiques de la machine.

L'objectif est de concevoir un dispositif de séparation de sources fonctionnant sans mesure de la vitesse de rotation de l'arbre d'entrée ou de sortie, tout en permettant le calcul des fréquences caractéristiques de la machine. Ces fréquences serviront ensuite aux algorithmes de séparation et de suivi. Dans le prolongement de ce travail, il est envisagé de proposer, dans le cadre d'un brevet, une méthode d'estimation de la fréquence d'engrènement exploitant l'ensemble des informations contenues dans les raies du spectre du signal. En examinant les raies du spectre, toutes leurs positions peuvent être exprimées uniquement en fonction de la fréquence d'engrènement et des paramètres cinématiques de la machine. La démarche se déroule en plusieurs étapes : la première consiste à estimer toutes les raies du spectre, puis à chercher à ex-

primer chacune d'entre elles en fonction de la fréquence d'engrènement et des paramètres cinématiques. L'étape essentielle consiste à faire correspondre les raies du spectre avec une grille représentant le spectre théorique, construite à partir des paramètres cinématiques. De plus, la gestion des raies erronées ou non détectées constitue le point central de ce travail. Une première approche consiste à adopter une méthode itérative, en testant les premières raies du spectre afin d'identifier celles qui minimisent les erreurs par rapport à la grille du spectre. Une autre approche consiste à utiliser les techniques de Sparse Bayesian Learning pour estimer la fréquence d'engrènement et identifier avec précision la grille spectrale appropriée.

Séparation de sources et suivi de signaux périodiques

Il est essentiel de distinguer avec précision les composantes de modulation liées à un défaut de celles qui n'y sont pas associées, afin d'assurer un diagnostic et une localisation efficaces des défauts dans les boîtes de vitesses. Les méthodes traditionnelles telles que l'analyse d'enveloppe (envelope analysis), la démodulation de Hilbert et la décomposition modale empirique peinent souvent à séparer les contributions de modulation provenant des différents composants du système dans les signaux vibratoires des boîtes de vitesses, ce qui peut entraver la surveillance des composants défaillants.

En partant d'un modèle mathématique qui ajuste bien les raies du spectre (expliqué par une modulation multiple) et avec une estimation des fréquences caractéristiques, nous avons proposé d'utiliser un estimateur du maximum de vraisemblance non linéaire pour estimer les paramètres liés aux signaux élémentaires de vibration. Vu la non-linéarité du modèle, nous avons procédé par une optimisation alternée pour ajuster les paramètres du modèle : À chaque itération, on fixe tous les paramètres d'un signal élémentaire sauf un à estimer, et on répète ce processus pour tous les ensembles de paramètres. Cette méthode a permis d'obtenir une estimation fiable, proche de la limite théorique de précision déterminée par la borne de Cramér-Rao. Ensuite, en supposant que la dégradation du défaut est lente, nous appliquons un gradient descendant pour affiner les paramètres de chaque signal élémentaire à mesure que de nouvelles données arrivent, permettant un suivi adaptatif en temps réel.

Conclusion

Une surveillance efficace de l'état des machines tournantes, telles que les systèmes de transmission par engrenages, à partir de l'analyse vibratoire repose sur une compréhension approfondie des mécanismes de génération des vibrations ainsi que sur une extraction fiable des caractéristiques informatives. Dans les machines tournantes, le signal vibratoire acquis présente généralement un spectre structuré, composé des fréquences d'engrènement, de leurs harmoniques, et d'autres composantes dominantes situées dans leurs bandes latérales. L'interprétation des relations entre ces composantes spectrales et leurs origines physiques, en particulier leur association avec les vitesses de rotation des éléments mobiles, constitue depuis plusieurs décennies un axe majeur de la recherche en surveillance de l'état et diagnostic des machines.

Malgré la puissance des techniques conventionnelles de séparation et d'analyse vibratoire conçues pour la détection de défauts, leur applicabilité demeure limitée lorsqu'il s'agit de traiter des architectures mécaniques complexes. Les systèmes de transmission épicycloïdaux, en particulier, se caractérisent par de multiples sources d'excitation mobiles, ce qui complique considérablement l'estimation des composantes vibratoires élémentaires ainsi que l'identification de la localisation des défauts. Comparées aux systèmes à axe fixe, pour lesquels ces techniques sont généralement efficaces, les méthodes conventionnelles ne semblent pas adaptées à l'extraction de caractéristiques dans les systèmes d'engrenages épicycloïdaux en raison de la mobilité des sources d'excitation. De plus, les multiples sources d'excitation sont modifiées par leurs chemins de propagation à travers la structure du système avant d'atteindre le point de mesure.

Dans cette thèse, nous avons étudié la modélisation phénoménologique des données vibratoires issues de machines tournantes, en portant une attention particulière aux systèmes de transmission par engrenages à mouvement épicycloïdal. Ce travail a notamment porté sur l'interprétation des harmoniques de la fréquence porteuse du spectre ainsi que des harmoniques dominantes situées à leur voisinage, en lien avec la dynamique du système. Une méthode automatique fondée sur l'analyse symbolique a été proposée afin de décrire le signal vibratoire à partir des fréquences caractéristiques de la machine. Par ailleurs, l'identification des effets de la fonction de transfert a été examinée à la fois au cours du processus de modélisation et en tant qu'étape post-modélisation, sur la base d'un modèle prédéfini du signal vibratoire. Une méthode d'estimation de la fréquence porteuse fondamentale, fondée sur l'analyse de la structure des bandes latérales, a également été introduite.

Enfin, une approche de séparation de sources fondée sur un modèle a été proposée afin de reconstruire les composantes vibratoires élémentaires et de suivre leurs variations.

Chapter 1

Introduction

1.1 Motivation

The aeronautical sector has undergone significant technological evolution over recent decades, driven by increasing demands for efficiency, performance, and reliability. This evolution is particularly evident in advanced propulsion systems for aircraft and helicopters, as well as in other high-performance rotating machinery such as wind turbines, heavy vehicles, and industrial equipment. These systems are characterized by ever-increasing power density, rotational speed, and mechanical complexity.

Modern aero-engines illustrate the extreme operating conditions associated with such machines. A contemporary CFM LEAP (Leading Edge Aviation Propulsion) turbofan engine produces approximately 30 MW of mechanical power during cruise. In comparison, a typical nuclear power plant produces about 1,500 MW of electrical power, meaning that a nuclear reactor generates roughly 50 times more total power than a single aircraft engine. Despite this large difference in total power, aircraft engines must deliver their power from a much smaller and lighter system, resulting in an exceptionally high power density and requiring operation under severe thermal, mechanical, and environmental constraints.

These rotating machines consist of multiple interconnected subsystems, including turbine stages, gear reduction systems, bearings, and rotating shafts. This structural and functional complexity makes component-level diagnosis indispensable for fault detection, system reliability, and safe operation.

Condition monitoring of such systems is particularly challenging. Current maintenance strategies largely rely on corrective maintenance, based on a

failure-and-repair model, and on preventive maintenance which involves periodic inspections after several thousand flight hours, during which the engine is disassembled for detailed component-by-component inspection. Although effective, these interventions are costly and time-consuming, highlighting the need for advanced diagnostics and real-time condition monitoring.

From an economic perspective, predictive maintenance is crucial for optimizing system health, reducing engine downtime, and ensuring operational reliability. In the aerospace industry, business models are increasingly shifting from outright engine purchases to leasing arrangements. In this context, health monitoring becomes a strategic priority, as it ensures engines are maintained in optimal condition, enabling companies to manage leased assets efficiently and minimize operational risks.

1.2 Scope of the thesis

In this research, gear transmission systems are the main focus, as they are fundamental components of mechanical systems and are often subjected to high loads and demanding operating conditions. Many diagnostic approaches rely on lubricant analysis or acoustic measurements. On the other hand, vibration analysis has become increasingly crucial for the health monitoring of mechanical systems, driven by advances in signal processing techniques originally developed in telecommunications and structural dynamics, including source separation, modal analysis, time-frequency methods, and adaptive filtering.

The aim of vibration analysis approaches is to extract precise information about the system's health, which generally appears as weak signals in the observations. The measurements are often corrupted by noise and undesired signatures originating from other mechanical components. Consequently, the extraction of reliable system health information is particularly challenging.

In the signal processing community, a mechanical system can be likened to an orchestra, where each component acts as a musical instrument contributing its own notes (signatures) to a collective response. A predictive maintenance system functions like a conductor, monitoring the notes of each instrument and identifying any that are out of tune.

Importantly, these vibration signatures are not independent. Interactions between components lead to coupled dynamics, such that the overall response depends on the collective behavior of the system rather than on isolated

elements. Analyzing these interactions enables predictive maintenance approaches to detect early signs of degradation or faults, thereby helping to ensure reliable and sustained system performance.

Epicyclic gear sets, compared with fixed-shaft gearboxes, can provide a larger transmission ratio in a more compact package and they are useful for machinery with a high output power requirement. However, due to their heavy load and tough working environment, a failure of the gearbox may cause shutdown of the entire train and result in major economic losses. They receive considerable attention in condition monitoring and fault prevention studies, to avoid unexpected failures that would necessitate shutdowns for repair and potentially disrupt or halt overall production. Although fixed-shaft gear transmission systems have shown satisfactory diagnostic performance for a wide range of faults over several decades, fault diagnosis in epicyclic gear sets remains less mature.

The effectiveness of vibration-based monitoring relies on interpreting the characteristics of measured signals. Typically, accelerometers are used to capture the vibrations of gear sets, which contain structured spectral patterns reflecting the dynamic behavior of the system. In epicyclic gear systems, these patterns are influenced by their kinematic configuration, operating conditions, manufacturing imperfections, and the presence of faults introduced during service. This complexity contrasts with fixed-shaft transmissions, where vibration spectra tend to be more regular and predictable.

A key challenge is to understand and interpret the spectral content of epicyclic gear vibrations, identifying the origin of each harmonic and the internal force it represents. By analyzing the contributions of individual gears to the overall vibration, it is possible to detect changes caused by faults or varying operating conditions. Such analysis forms the basis for phenomenological models that describe the vibration signals and allow for source separation of the contributions from different rotating elements. Furthermore, understanding these spectral characteristics requires examining how excitations propagate through the complex structure of planetary gear systems, which determines the weak vibration patterns produced by early-stage faults.

1.3 Contributions

The contributions of this thesis are developed along four main lines.

1.3. CONTRIBUTIONS

First, we contribute to phenomenological modeling by proposing an automatic framework to assess whether existing phenomenological models adequately represent the observed vibration responses, or to discover new models that capture previously unobserved interactions within measured vibrations. This framework is based on a neural-network-driven Symbolic Regression (SR) approach, which, to the best of our knowledge, has not previously been applied to the modeling of gearbox vibration signals. The proposed SR method yields concise analytical expressions that explain the physical behavior by revealing the relationships between spectral harmonics and the system characteristic frequencies. By embedding interpretable mathematical operators within the neural network architecture, the resulting models maintain a level of interpretability consistent with the physical behavior of the systems. The method can be interpreted as identifying structured nonlinear mappings that govern harmonic interactions in the frequency domain.

Second, we introduce a transfer function identification method based on the deterministic components of the observation, in contrast to recent spectrum-based approaches that rely primarily on stochastic broadband components in the cepstral domain after separating the deterministic harmonic part of the vibration signal. The approach exploits the fact that the system excitations at the interaction points propagate through the structure and induce amplitude asymmetries in the spectral sidebands around the carrier frequency and its harmonics. The proposed approach estimates the transfer function by restoring the original symmetry of the sidebands through a nonlinear least-squares optimization.

Third, we propose a parametric signal decomposition and tracking method aimed at isolating the contributions of fault-related characteristic frequencies from the observed vibration signal. Given the characteristic frequencies, a nonlinear parametric model is introduced, and parameter estimation is performed via a maximum likelihood estimator using an alternating minimization strategy. The model is linear with respect to the carrier-frequency-related component and nonlinear with respect to cross-frequency interaction terms that describe the modulations between the carrier and characteristic frequencies. A theoretical analysis of parameter estimation bounds is provided. Assuming that system degradation evolves continuously and slowly, this naturally motivates the use of gradient-descent-type algorithms for online parameter tracking, without the need for explicit continuity regularization.

Fourth, we present a carrier (meshing) frequency estimation method that exploits the line-structured nature of vibration spectra. The proposed estimator leverages prior knowledge of the algebraic relationships between spectral lines to infer the fundamental carrier frequency. This formulation reframes carrier frequency estimation as a structured spectral inference problem under deterministic line constraints. This enables the use of a dictionary-learning-based strategy for joint frequency and amplitude tracking via gradient-descent optimization.

The main contributions of this thesis can be summarized as follows:

1. An automatic and generalized phenomenological modeling framework for vibration signals of planetary gearboxes based on interpretable symbolic regression.
2. A parametric decomposition and tracking method for isolating characteristic-frequency contributions using maximum-likelihood estimation and on-line optimization.
3. A deterministic spectral transfer function identification technique based on sideband symmetry restoration.
4. A carrier frequency estimation method exploiting line-structured spectra and prior algebraic relationships among spectral components.

1.4 Publications

As a result of these contributions, the following publications have been produced:

Journal papers

1. Nacer Yousfi, Karim Abed-Meraim, Yosra Marnissi, Maxime Leiber, and Mohamed El-Badaoui. *An End-to-End Modelling and Source Separation Solution for Vibration Signal Analysis. Mechanical Systems and Signal Processing*, 2025. [⟨hal-05304734⟩](#).
2. Nacer Yousfi, Karim Abed-Meraim, and Mohamed El-Badaoui. *Blind System Identification of Rotating Machines with Modulated Periodic*

1.4. PUBLICATIONS

Input Signals. Submitted to *Journal of Signal Processing*, Elsevier, 2025.

Conference papers

1. N. Yousfi, K. Abed-Meraim, Y. Marnissi, M. Leiber, and M. El-Badaoui. “Neural Network-Based Symbolic Regression for Empirical Modeling of the Behavior of a Planetary Gearbox.” In *ICASSP 2024 – IEEE International Conference on Acoustics, Speech and Signal Processing*, Seoul, Republic of Korea, 2024.
2. N. Yousfi, K. Abed-Meraim, M. El-Badaoui, and Y. Marnissi. “Blind Identification of Mechanical Systems with Modulated Periodic Input Signals.” In *32nd European Signal Processing Conference (EUSIPCO)*, Lyon, France, 2024.
3. N. Yousfi, K. Abed-Meraim, and M. El-Badaoui. “Robust Modeling of a Class of Vibration Signals Through Binary Neural Network-Based Symbolic Regression.” In *33rd European Signal Processing Conference (EUSIPCO)*, Palermo, Italy, 2025.
4. N. Yousfi, K. Abed-Meraim, M. Leiber, and M. El-Badaoui. “Tracking Multi-Modulated Periodic Sources for Rotating Machines Diagnostics.” In *Proceedings of the 30th GRETSI Symposium on Signal and Image Processing (GRETSI 2025)*, Strasbourg, France, 2025.
5. N. Yousfi, K. Abed-Meraim, Y. Marnissi, and M. El-Badaoui. *A Sparse Coding Approach for Planetary Gearbox Vibration Signal Decomposition*. In *Proceedings of the 26th Conference of the International Society for Air Breathing Engines (ISABE 2024)*, Toulouse, France, September 2024. <hal-05336842>.

Patents

1. B-029478 : Procédé de modelisation automatique des vibration des machines tournantes , N. Yousfi, K. Abed-Meraim, Y. Marnissi, M. Leiber, and M. El-Badaoui.
2. B-031401 : Procédé et Dispositif d’extraction automatique de la mesure vibratoire de balourds de turbomachines, , Y. Marnissi, M. Leiber, N. Yousfi, Jean-Frédéric Diebold, and M. El-Badaoui.

3. B-029472 : Dispositif de suivi de vibrations des elements rotatifs dans un engrenage, Benoit Malezieux, Y. Marnissi, M. Leiber, N. Yousfi, and M. El-Badaoui.

Talks

1. SURVISHNO 2025: Dictionary Learning for Periodic Signals: Application to Rotating Machine Elementary Vibration Source Tracking. N. Yousfi, K. Abed-Meraim, Maxime Leiber, M. Elbadaoui. Paris, France
2. Journée LALALA 1: An End-to-End Modelling and Source Separation Solution for Vibration Signal Analysis. LASPI laboratory, Roanne, France

1.5 Structure of the Thesis

As stated previously, the primary objective of this thesis is to develop novel methods for vibration signal source separation. Achieving this objective requires preliminary modeling of the vibration data, mitigation of propagation path effects present in the measurements, and estimation of the modulation carrier frequency, in order to enable effective separation of the underlying gear-related source signals.

In chapter 2, we provide a detailed background on epicyclic gear set vibration signal analysis, illustrating the system behavior and presenting an overview of traditional phenomenological modeling approaches that aim to describe the distinct sideband behavior of epicyclic gear set vibration signal, related source separation frameworks in recovering the signal of interest, and the recent methods for transmission path effects removal.

Chapter 3 introduces an automatic, generalized framework for phenomenological modeling based on symbolic regression. The proposed method aims to recover analytical expressions that fit the measured data using a given set of input features and mathematical functions. The focus is on identifying phenomenological models for the observed data in order to assess the fidelity of traditional modeling approaches. We employ a machine-learning technique based on neural-network-driven symbolic regression to model vibration signals. Unlike conventional neural networks, this approach uses predefined mathematical functions instead of standard activation functions, yielding interpretable hidden-layer representations. The learning process involves

a trade-off between data fidelity and the compactness and sparsity of the induced analytical expressions. We show the performance of the approach with polynomial analytical expressions.

In chapter 4, we address the recovery of the system transfer function’s impulsive response from modulated vibration measurements. Recent approaches often rely on Cepstrum-based modal analysis, which uses the linearization of the logarithm of the spectrum and exploits the broadband frequency content of random signal components. In contrast, our method focuses on the harmonic deterministic components of the signal rather than the random part to reconstruct the system response. Impulsive response identification is approached in two ways. On one hand, it is performed simultaneously with vibration data modeling using symbolic analysis in an end-to-end manner, facilitating the reconstruction of the spectrum carrier frequency sidebands. On the other hand, after modeling, the spectral sidebands exhibit amplitude asymmetry, most likely due to system propagation effects, which is exploited to recover the system transfer function’s impulse response using nonlinear least-squares optimization, applied to both finite and infinite impulse response filter models. This approach ensures accurate identification of the system dynamics while leveraging the structured deterministic content of the measurements.

In Chapter 5, we consider a structured spectral line estimation problem in which a single unknown parameter governs multiple spectral lines. Unlike classical harmonic models, where standard algorithms are applied without explicit constraints linking the harmonics, the proposed approach explicitly exploits the underlying line structure. This enables robust parameter estimation in the frequency domain within a simplified single-group setting. Furthermore, we show the performance of the proposed method for determining a gear set meshing frequency from a vibration measurement with prior knowledge of the system’s kinematic model.

Chapter 6 presents the proposed solution for source separation and tracking, applied to the decomposition of the vibration sideband spectrum. Building on the preceding modeling and transfer function identification, we introduce a likelihood estimator based on the nonlinear phenomenological model. To estimate the coefficients of the Fourier basis associated with the system’s characteristic frequencies, we employ an alternating optimization algorithm. The estimated parameters achieve the theoretical Cramér–Rao

1.5. *STRUCTURE OF THE THESIS*

Bound with simulated data. Since it is well known that the source parameters are related to the elements' health state and their degradation typically increases gradually, we refine the estimator weights for incoming data using a few gradient iterations based on the likelihood estimators obtained from an initial data segment.

Finally, we draw some conclusions and perspectives in Chapter 7.

Chapter 2

Background

2.1 Introduction

Power transmission systems are critical components in industrial applications such as gear transmission sets. The health monitoring of these rotating machines is becoming more and more important to follow the demand of the industry for devices that reduce their shutdown time in critical applications such as helicopters, aircraft, wind turbines, and heavy vehicles.

Traditional maintenance strategies are typically performed periodically according to fixed maintenance plans or during machine shutdowns, often involving the replacement of entire components even when some parts remain undamaged. Current industry aims are to improve engine operation and maintenance scheduling to ensure efficient and reliable performance throughout the machine's service life. Future generations of high-power engines using cutting-edge technologies require advanced condition-monitoring systems capable of tracking machine condition and automatically scheduling targeted maintenance actions under varying operating conditions. Consequently, the industry is increasingly focused on diagnostic systems that can detect and localize faulty components at an early stage, reducing downtime and economic losses. In rotating machinery, maintenance often relies on noise analysis, analogous to a doctor listening to a patient's heartbeat. While such machines naturally generate noise, condition monitoring emphasizes identifying changes in noise characteristics that indicate developing faults.

Epicyclic gear systems (planetary gear systems) are a major focus due to their large use in power transmission applications, such as helicopter main

rotor transmissions, wind turbines, and aircraft engine reduction gearboxes, where high torque or speed reduction ratios are required within a compact package. In addition, these gear sets often operate under harsh working conditions, making the gear teeth susceptible to fatigue cracking and, in severe cases, tooth break. Compared to conventional fixed-axis gear sets, epicyclic gear sets exhibit unique characteristics due to their complex structure. This complexity results in measured vibration signals with distinct patterns and numerous spectral components, unlike those observed in fixed-axis gearboxes. As a result, conventional fault detection techniques developed for fixed-axis gear sets cannot be directly applied to epicyclic gear systems. Consequently, this challenge has motivated researchers to further investigate and understand the vibration patterns of epicyclic gear sets and the influence of their motion on the measured vibration signals.

Therefore, numerous vibration-based techniques have been developed for epicyclic gear sets. Researchers have either focused on preconditioning vibration signals to allow analysis similar to that of fixed-axis gear sets, thereby enabling the use of conventional fault detection methods, or have proposed novel approaches to better understand vibration patterns associated with epicyclic motion. This gives a bunch of methods that concentrate on modeling the vibration signal that report explicit equations to describe the spectral structure, which helps the understanding of the spectral features that enable more effective fault detection techniques. In addition, the measured signal is generated by excitations that propagate through a transmission path from the excitation source to the measurement point. Consequently, the system's health state cannot be measured directly, which limits the effectiveness of conventional fault detection methods. To address this issue, vibration-analysis-based techniques often include a preprocessing step to remove the effects of the transmission path or to identify the impulse response of the system transfer function.

In the following, the epicyclic gear transmission system is introduced, its features are compared with those of fixed-axis gear sets, and the different vibration sources captured by the sensors are described. In addition, an overview is provided of the modeling approaches developed to characterize the spectral structure of planetary gear vibration signals using explicit formulas. Moreover, a review of vibration source separation methods for the planetary gear system vibration signal is presented. Finally, we introduce the widely used techniques in the literature for removing the effects of the system

transmission paths.

2.2 Gear transmission set

2.2.1 Fixed-axis gearbox

Gear sets are used to transmit motion between shafts. A basic fixed-axis gearbox consists of a pinion–gear pair, in which one gear acts as the driving gear and the other as the driven gear, Fig 2.1.

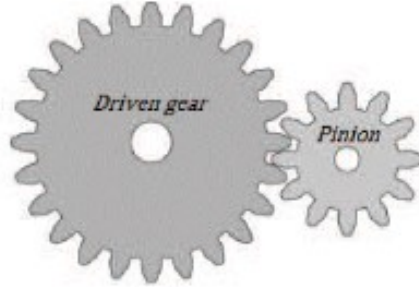


Figure 2.1: Fixed-shaft gearbox

The power transmitted by the system is a function of the shaft rotational speed and the applied load. A key characteristic of any gearbox is the transmission ratio, which is defined as the ratio between the input-shaft and output-shaft angular velocities. This ratio depends on the number of teeth of the meshing gears and is given by

$$r = \frac{Z_1}{Z_2}, \quad (2.1)$$

where Z_1 and Z_2 denote the numbers of teeth on the driving and driven gears, respectively.

The vibration signal measured on a gearbox by an accelerometer results from excitation forces generated at the gear-meshing point and shaped by the transfer function associated with the transmission path from the meshing location to the sensor position. Variations in the excitation functions are induced at the meshing point due to deformations caused by manufacturing

errors, tooth geometric profile errors, and tooth faults, etc. These perturbations are confined to short time windows (such as a tooth crack) during which the faulty tooth is in mesh. The resulting gear meshing forces therefore contain clear information related to the faulty element. However, in practice, these perturbations are transmitted through the structural path of the gearbox before being measured, thereby generating the observed vibration response. Compared with a fixed-shaft gearbox, an epicyclic gearbox exhibits a significantly more complex structure and vibration characteristics that involve multiple gear meshing points.

2.2.2 Epicyclic gear set

This section provides a brief overview of a single-stage epicyclic transmission system, describing its structure, operating modes, and the different sources of system vibration. An epicyclic gear set can provide large transmission ratios due to its advantageous structural characteristics. By utilizing P identical planet branches, the transmitted power is divided into P parallel paths, thereby achieving high power density in a compact packaging. This adaptability makes epicyclic systems highly suitable for demanding applications, such as a reducer in the new generation of aircraft engines.

The gear set is formed by a sun gear, a ring gear, and P identical planet

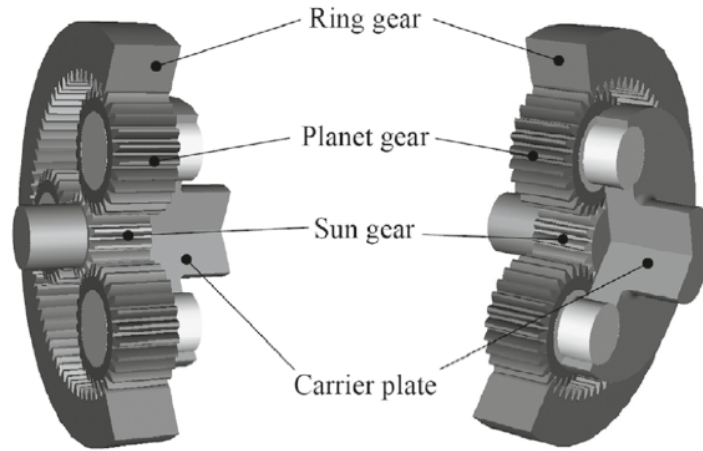


Figure 2.2: Schematic of a 3-planet planetary gearbox

gears equally distributed around the sun gear at distinct angular positions, as illustrated in Fig. 2.2. The planet gears are held by a common structure

called a planet carrier through planet bearings. Planet gears act as idler gears with no connections to the outside structures of the gearbox. The sun gear, ring gear, and planet carrier serve as the three principal members of a planetary gearbox, any of which may act as input, output, or fixed member depending on the transmission configuration, while being supported by the transmission housing.

Epicyclic gears can have three different modes on the basis of which the component is fixed, with the exception of a differential gear (with three drive shafts), in which all the parts are in motion and which therefore needs two drives. The three modes of epicyclic gears are:

- planetary: where the ring gear is the fixed component,
- star: where the carrier plate is the fixed component,
- solar: where the sun gear is the fixed component.

A vibration sensor, typically an accelerometer, is mounted on the gearbox housing or on the outer ring gear in order to capture the structural response of the system to internal dynamic excitations. The measured vibration reflects the dynamic behavior of the gearbox's gears as well as the transmission paths of the mechanical structure.

The transmission by planetary gear sets has some unique characteristics compared to fixed-shaft gear sets such as:

1. the epicyclic motion of planet gears revolving around the sun gear makes the vibration transfer paths between the gears meshing positions and the mounted sensor are time-varying.
2. multiple gear pairs are meshing simultaneously, e.g., each planet gear is meshing with the ring and the sun gears at the same time.
3. the structure of a planetary set is more complicated than that of a fixed-axis gear transmission.

As a result, time-varying vibration paths, multiple meshing sources, and complex structure make planetary gearbox vibrations rich in components and modulations. This complicates the identification of tooth-fault-related modulation sidebands, as many spectral components appear in the vibration spectrum.

2.2.3 Perceived signal

In a geared transmission system, vibration is generated by several mechanical excitation mechanisms, the most significant of which are:

1. Shaft-related excitations such as mass unbalance, angular or parallel misalignment, and shaft cracks,
2. Periodic excitation forces induced by gear meshing,
3. Localized defects in rolling element bearings, including spalls and pitting,
4. Background noise

Each excitation source produces dynamic forces that propagate through the gearbox structure before reaching the sensor location. The measured vibration signal is therefore not a direct measurement of the excitation forces, but rather the result of their propagation through complex transmission paths involving shafts, bearings, gears, and the housing. This system can be modeled as a multiple-input single-output (MISO) system.

Let $x_i(t)$ denote the excitation signal generated by the i -th source at the meshing point, and let $h_i(t)$ denote the impulse response of the corresponding transmission path between this source and the sensor location. The observed vibration signal $y(t)$ can then be expressed as:

$$y(t) = \sum_{i=1}^m h_i(t) \star x_i(t) \quad (2.2)$$

where $\star$ denotes the convolution operator and m is the number of active excitation sources. This formulation highlights that the measured vibration is a linear superposition of the individual source contributions, each shaped by its own transmission path, each shaped by its own transmission path dynamics.

Gearbox vibration signals consist of both deterministic and stochastic components. The deterministic component is primarily associated with periodic excitations due to continuous gear meshing. These excitations generate harmonic components at the gear mesh frequency and its multiples, which may also be modulated by shaft rotational frequencies due to load variations,

manufacturing tolerances, and gear tooth faults.

The stochastic component arises from random or quasi-random processes such as rolling element bearing interactions, surface roughness, tooth profile deviations, and frictional effects. These phenomena give rise to broadband vibration signatures that are often modeled as cyclostationary or non-stationary random processes. Additionally, measurement noise and environmental disturbances contribute further stochastic components to the observed signal. Consequently, the measured vibration signal can be interpreted as a composite signal containing structured periodic information embedded within a noisy background. This mixture complicates fault detection and diagnosis, as fault-related features may be masked by transmission path effects or by dominant vibration sources.

In this work, the harmonic components of the vibration signal are of primary interest because they are directly linked to the kinematics of the gear set and may convey information about gear health. Although random components can also contain fault-related information, the present thesis focuses mainly on the deterministic harmonic structure. If the tooth profiles of a gear transmission set were perfect involute and the teeth were non-deformable, then no vibration would be generated due to the tooth gear meshing process. However, this does not occur. In reality, tooth profiles differ from the perfect involute due to, for example, fabrication errors, deformation under load, wear, cracks, etc. Consequently, vibrations resulting from tooth meshing are inherent in gear transmission sets. Due to the meshing of successive pairs of teeth from the conducted and conduced gear wheels, the excitation sources are periodic functions equal to the gear mesh frequency.

The spectral patterns of the vibrations present lines at the mesh frequency and its harmonics. Sidebands spaced at the rotational frequencies of the rotating gears are usually present around the gear mesh frequency and harmonics. Some visualizations also prove that spectral lines at the rotational frequencies of the rotating gears appear in the spectrum. This spectral pattern represents the normal spectrum of the vibrations measured on any fixed-shaft gear transmission. In the case of planetary (mode of epicyclic gear set) gear transmissions, different structures of the vibration spectrum can be found, thus making it impossible to define a unique spectral pattern. In what follows, we focus on the planetary gear set to review traditional phenomenological models of spectral patterns.

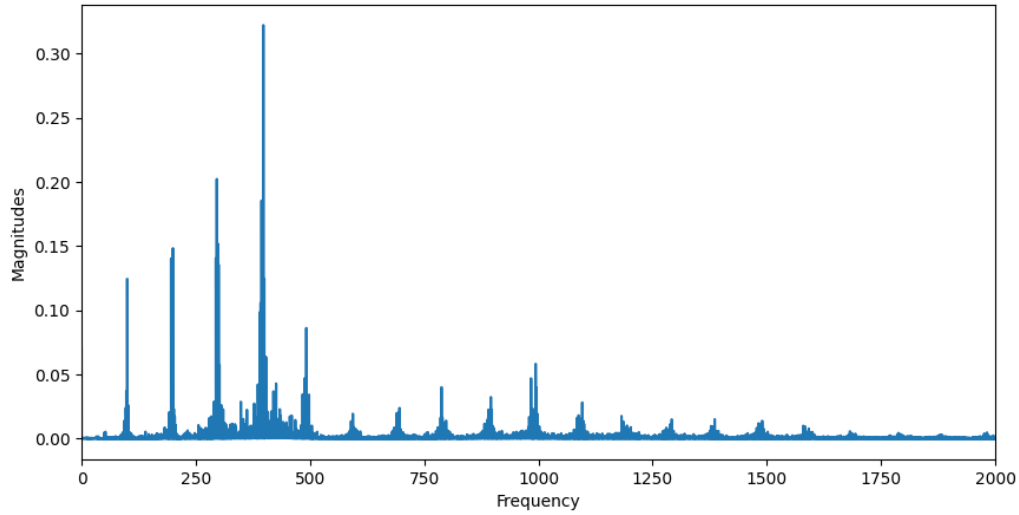


Figure 2.3: Typical vibration spectrum

Spectral analysis of the vibration responses has become a widely adopted technique in the field of machinery fault diagnosis. A detailed understanding of the spectral characteristics of planetary gearbox vibration in healthy and abnormal cases enables more accurate and efficient fault detection.

2.3 Phenomenological modeling

The multiple meshing excitations that take place simultaneously in a planetary gearbox with an epicyclic motion result in time-varying transmission paths between the meshing points and the sensor location. This makes the structure of the vibration spectrum always exhibit a complex characteristic. Misunderstandings of these characteristics can cause a misinterpretation of the vibrational behavior and even impact the fault diagnosis. Thus, having a reasonable and physical meaning of the unique characteristics in the structure vibration responses can provide a proper interpretation for spectrum analysis. One of the most distinctive behaviors observed in vibration responses of planetary gearboxes is the occurrence of modulation sidebands. These sideband harmonics typically appear in the vicinity of the gear-mesh frequency and

its integer multiples. In this part, we present an overview of the vibration response spectra characteristics related to healthy components (normal vibration) and with the faulty elements, including the examination of the effects of the structure on the propagation of the excitations.

Researchers have investigated the sideband behavior in planetary gearbox vibration spectra, where they attempt to establish a general explanation for the harmonic components within the sidebands and the causes of their asymmetric distribution around the tooth meshing frequency in both amplitudes and frequency in some planetary gear set configurations. This is well known as a phenomenological modeling, which offers a meaningful and efficient description of vibration measurements. Unlike dynamic models [1] that incorporate detailed and complex interactions such as friction, lubrication effects, and parametric excitations.

Phenomenological models primarily focus on the system's kinematic modulations and transmission mechanisms that govern its vibration behavior. They are functioning as a kind of mathematical-analytical model. Instead of using differential equations, phenomenological models use periodic functions based directly upon empirical observations and are aimed at providing a more convenient way to describe system responses than dynamic models offer.

The first phenomenological model was introduced by [2]. McFadden and Smith contribute to advancing the understanding of the phenomena in planetary (epicyclic) gears, whereby the sidebands surrounding the meshing frequency dominate the vibration spectrum instead of the meshing frequency and its harmonics. An analytical model was developed to explain the asymmetry around meshing frequencies, showing that phase differences among planet gears may cause the asymmetry sidebands generated by the planet carrier motion. This model was used after by [3] with Fourier series analysis to predict the dominant components in the vibration spectrum. Also, in [4, 5], the authors investigate the influence of the different meshing phases, produced by the planets and their contact with the ring and the sun, on the vibration spectrum. The normal vibration of different geometries of planetary gearboxes (planet gear number, gear teeth number, planet equally or unequally spaced) has been investigated, leading to a classification of gearboxes into groups based on their spectral structure in [6, 5, 7]. The above works treat a normal system vibration where the authors consider only the amplitude modulation effect caused by the rotation motion of the

planet carrier, and do not take into account the variation of meshing stiffness or gear damage that may cause a frequency modulation. The latter was studied in [8] where the authors introduce the frequency modulation in the phenomenological model due to manufacturing errors. After, Feng et al. in [9] investigate further the spectral characteristics of planetary gear set vibration signals with local and distributed damage. They consider in their model the Amplitude Modulation and Frequency Modulation (AMFM) effects caused by both the gear damage and the AM effect due to time-varying vibration transfer paths. In addition, [10, 11], the authors extended previous work by developing analytical models to establish more robust phenomenological models of faulty gearboxes, which convincingly explain the variations in peak positions of the vibration spectrum. In [12], the robustness of the multiplicative model (AM) for explaining the structure of the measured vibration spectrum is investigated.

Furthermore, the authors in [13] overlooked the characteristics of the vibration spectrum to improve the understanding of the behavior of such systems. It is shown that the asymmetry of the spectrum structure should be attributed to the magnitude randomness of gear meshing vibration rather than different phase angles of multiple-planet gears. They argue that modeling of the vibration magnitude as random provides a more accurate representation of real systems, due to the inevitable manufacturing imperfections, assembly errors, uneven load distribution among the planets, etc.

For decades, extensive theoretical works have attempted to develop reasonable phenomenological models that interpret the spectrum structure of the planetary gearbox vibration measurements, either the frequency contents or the amplitudes of the sidebands. It is of vital importance to focus on the sideband amplitudes, which are directly induced by the gear faults. However, some of the analyzing sidebands were not only induced by a gear fault but also by other modulation sources, such as the planet carrier rotation frequency and the shaft rotation frequency. These modulations are not exclusive fault indications and will enhance the uncertainty in fault diagnosis. To this end, [11] contributes by using a Sideband Energy Ratio (SER) for monitoring the gear health in planetary gearboxes. It is a ratio between the summation amplitudes of the first six sideband peaks on both sides of the gear-meshing frequency and the peak of the meshing frequency. This method may lose its ability if the sidebands are inappropriately selected. Also, condition indicators such as Root Mean Square (RMS), Kurtosis for fault diagnosis are studied with an appropriate study and theoretical support [14].

Next, a brief overview of the traditional phenomenological modeling approach is presented, along with the vibration models of the planetary gear set that have been investigated.

2.3.1 Vibration signal

A phenomenological model describes vibration components arising from all gear-mesh interactions (planet–sun and planet–ring pairs), where meshing vibrations are modeled as periodic functions with a fundamental frequency equal to the gear mesh frequency f_{mesh} [2]. Due to the sensor’s proximity to the ring gear, the measured vibration is dominated by the planet–ring gear meshing, as it has the most direct transmission path to the sensor. The modeling process typically begins by analyzing the gearbox kinematics, often considering a single planet gear for simplicity, before extending the formulation to multiple planet gears. Gear meshing processes and transmission effects are the two primary factors that must be taken into account

Single planet gear

Consider first a simplified planetary gear system composed of a sun gear, a ring gear, and a single planet gear, as shown in Fig 2.4. In this work, the planetary gearbox configuration considered has a fixed (stationary) ring gear, which represents the most common practical setup. Accelerometers are usually mounted either on the gearbox casing or directly onto the ring gear. The vibration component associated with the ring-planet mesh, denoted by s_i^r , contains the gear-mesh frequency and its harmonics. These components arise from periodic meshing forces caused by tooth-profile deviations, manufacturing errors, elastic deformation under load, and possible local defects.

At the meshing point, when a damaged tooth comes into contact with its mating gear, it produces a localized perturbation of the meshing force. This perturbation gives rise to amplitude modulation at the characteristic frequency of the damaged gear and may also induce instantaneous frequency modulation of the meshing signal. Under stationary operating conditions, the amplitude modulation (AM) and frequency modulation (FM) terms can be expressed as a Fourier series whose fundamental frequency is the characteristic rotational frequency of the damaged component. Let f_{mesh} denote the gear-mesh frequency and f_{ch} the characteristic frequency associated with the damaged gear. Considering H significant mesh harmonics, the vibration

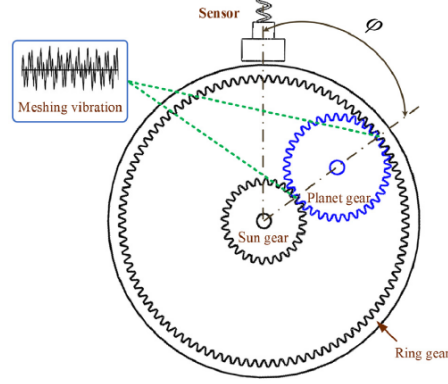


Figure 2.4: PG with single planet

component can be modeled as

$$\mathbf{s}(t) = \sum_{i=1}^H a_i^{ch}(t) \sin(2\pi i f_{mesh} t + b_i^{ch}(t) + \phi_i) \quad (2.3)$$

where

$$a_i^{ch}(t) = 1 + \sum_{l=1}^M A_{i,l} \sin(2\pi l f_{ch} t + \psi_{i,l}) \quad (2.4)$$

$$b_i^{ch}(t) = \sum_{l=1}^N B_{i,l} \sin(2\pi l f_{ch} t + \zeta_{i,l}) \quad (2.5)$$

where $a_i^{ch}(t)$ and $b_i^{ch}(t)$ denote the AM and FM functions, respectively, with M and N significant harmonics. The coefficients $A_{i,l}$ and $B_{i,l}$ are the corresponding positive modulation amplitudes, while ϕ_i , $\psi_{i,l}$, and $\zeta_{i,l}$ denote the associated initial phases.

Next, in the perceived signal, we consider the effect of vibration transfer paths with an additional AM term due to time-varying transfer paths caused by the rotation of the planet gear... In the signal perceived by the sensor, the vibration transfer paths must also be considered. In planetary gearboxes, these paths may be time-varying because the planet gears rotate around the sun gear. This additional modulation affects the measured amplitudes and may differ depending on whether the fault is located on the ring gear, a planet gear, or the sun gear.

When damage occurs in the ring gear, the transfer path from the damaged tooth to the sensor location remains fixed because the ring gear is stationary.

Although the planet carrier rotates, it does not alter this transfer path. Therefore, the planet carrier does not conventionally cause amplitude modulation in the vibration induced by the damaged ring tooth. Consequently, the captured vibration at the sensor location is:

$$\mathbf{s}(t) = \sum_{i=1}^H a_i^{ring}(t) \sin(2\pi i f_{mesh} t + b_i^{ring}(t) + \phi_i) \quad (2.6)$$

Suppose the damage occurs in the planet gear. As the planet carrier rotates, the meshing location of the damaged area, either at the ring-planet or sun-planet contact, will change relative to the fixed sensor. This motion changes the transfer path of the damage-induced vibration from the meshing location to the sensor position. Thus, the vibration at the meshing location will be amplitude modulated with the planet carrier rotating frequency. The vibration signal at the sensor location is then:

$$\mathbf{s}(t) = \sum_{i=1}^H a_i^{carrier}(t) a_i^{planet}(t) \sin(2\pi i f_{mesh} t + b_i^{planet}(t) + \phi_i) \quad (2.7)$$

This investigation is valid when damage occurs in the sun gear. The damage-induced vibration from each planet-sun meshing will be modulated with the rotating frequency of the sun gear.

The authors above, by analyzing the sideband structure and examining whether the sideband locations correspond to any of the characteristic frequencies of the gearbox gears, were able to detect faults and determine their locations. Herein, we will describe how to calculate these characteristic frequencies based on the gearbox configuration and operating speed, referring to the work of [9].

Multiple planet gear

In modeling multiple planet gears, the vibrations produced by each gear are usually treated as identical in amplitude, period, and transfer function, differing only in phase. Thus, a sensor capturing the vibration from the i -th planet gear records the same signal as that from the first gear, but delayed by a time t_i . Accordingly, the observed vibration can generally be modeled as the linear superposition of the vibrations produced by each mesh pair, namely the ring-planet and sun-planet interactions, as follows:

$$y(t) = \sum_{i=1}^P s_i^r(t) + s_i^s(t), \quad (2.8)$$

where $s_i^s(t)$ is the signal that originates from the meshing of the sun-planet pair, and P is the number of planet gears.

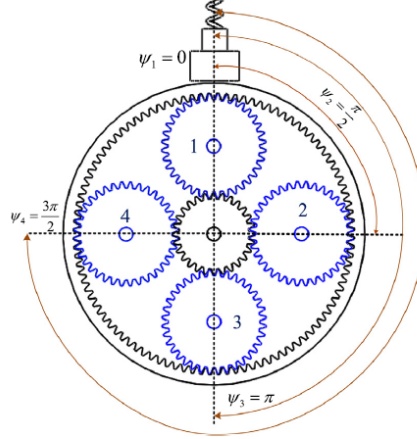


Figure 2.5: PG with 4 planet gears, with phase settings ψ_i for each gear as in the traditional modeling.

2.3.2 Planetary gearboxes characteristic frequencies

In this part, we define the gear mesh frequency and the characteristic fault frequencies, and describe how they can be computed from the gearbox shaft speed, the number of planet gears, and the gear tooth numbers. The characteristic frequency of a faulty gear depends on whether the damage is localized in a single gear tooth or distributed across the gear teeth and the configuration of the gearbox (what part is fixed).

Localized gear damage produces meshing vibration modulation at the repeated frequency of the sudden changes when the damaged tooth contacts its mating gear. Accordingly, the characteristic frequency of a faulty gear in planetary gearboxes is determined by how many times the damaged tooth meshes with mating gears per unit time. While a distributed gear damage modulates the gear meshing vibration over each complete revolution of the faulty gear relative to the carrier, producing periodic amplitude variations at that frequency. Thus, the characteristic frequency corresponds to the relative rotational frequency of the damaged gear with respect to the planet carrier. A single-stage planetary gearbox, the two types of gear pair, sun-planet and planet-ring, have identical meshing frequency. The meshing frequency of a

gear of interest means how many gear teeth get involved in meshing per time, then the meshing frequency equals the product of planet carrier rotating frequencies $f_{carrier}$ and the number of ring gear teeth Z_{ring}

$$f_{mesh} = f_{carrier} Z_{ring}. \quad (2.9)$$

And the relative rotating frequency of a gear of interest is equal to :

$$f_{relative} = \frac{f_{mesh}}{Z} \quad (2.10)$$

where Z represents the number of teeth of the gear of interest. The characteristic frequency of the faulty gear is proportional to the relative rotational frequency of the gear and depends on the meshing points considered, as well as on whether the damage is local or distributed.

Faulty planet gear

Suppose a single damage occurs on the surface of a planet gear tooth. If the damage exists on only one side of the tooth, the modulation in the vibration is caused by the contact of the damaged area with the mating gear (either the sun gear or the ring gear) once per relative rotation cycle of the planet gear with respect to the planet carrier. Accordingly, the characteristic frequency of a faulty planet gear is

$$f_{planet} = \frac{f_{mesh}}{Z_{planet}} \quad (2.11)$$

where the Z_{planet} is the number of planet gear.

However, if the damage exists on both sides of the gear tooth, the damaged planet gear tooth meshes with both mating gears. This may further influence the vibration pattern. Then, the characteristic frequency of the planet gear becomes:

$$f_{planet} = 2 \frac{f_{mesh}}{Z_{planet}} \quad (2.12)$$

For a distributed damage case, the characteristic frequency for the faulty planet gear is the same as 2.11 since the distributed damage produces periodical changes as every damaged tooth gets in mesh, but the envelope amplitude of these periodical changes fluctuates at the relative rotating frequency with respect to the planet gear.

Faulty sun gear

With the same analogy, [9] computes the characteristic frequency of the sun gear for local damage as:

$$f_{sun} = \frac{f_{mesh}}{Z_{sun}} N_{planet} \quad (2.13)$$

where Z_{sun} and N_{planet} are the number of sun gear teeth and the number of planet gears respectively. The factor N_{planet} in 2.13 arises from the fact that, during one relative rotation cycle with respect to the planet carrier, the damaged area will mesh with all the N_{planet} gears.

Similar to the distributed case of planet gear, the characteristic frequency of the distributed damage of the sun gear is equal to

$$f_{sun} = \frac{f_{mesh}}{Z_{sun}} \quad (2.14)$$

Faulty ring gear

Similar to the case above, for local damage, the characteristic frequency is given by:

$$f_{ring} = \frac{f_{mesh}}{Z_{ring}} N_{planet} \quad (2.15)$$

where Z_{ring} is the number of teeth on the ring gear.

For distributed damage, the characteristic frequency is:

$$f_{ring} = \frac{f_{mesh}}{Z_{ring}} \quad (2.16)$$

2.4 Source separation

The vibration signals captured from rotating machines, such as gear transmission sets, are contaminated with heavy background noise. Furthermore, the health state information is mixed with the vibration source of normal components such as gear meshing, shaft rotations, etc, making it difficult to identify useful fault vibration characteristics. Thus, the extraction and separation of specific signal components related to the defective part from the overall machine vibration measurement is essential to directly access to the system health state.

2.4.1 Vibration separation methods

The vibration signal of planetary gear sets has some unique characteristics compared to fixed-axis gear sets, as stated earlier, such as the time-varying vibration transfer paths, multi-meshing vibration sources, and complex structure. These features make the observed vibration signal rich in components, which in turn complicates the identification of tooth-fault-related modulation sidebands in planetary gear sets [15, 6].

For decades, vibration separation has been a central topic in vibration analysis. Researchers aim to isolate specific vibration components of interest that are often obscured by strong interfering components in accelerometer measurements. Numerous vibration separation techniques based on signal processing have been developed for fault detection in epicyclic gear sets. These include time synchronous averaging methods [16, 17]; autocorrelation-based time synchronous averaging methods for wind turbine fault diagnosis [18]; phase and amplitude demodulation [19]; stochastic resonance methods for weak fault signal detection in planetary gearboxes [20]; feature selection algorithms for fault severity diagnosis of planetary gearboxes [21]; adaptive optimal kernel time–frequency analysis methods that address nonstationary operating conditions in wind turbine planetary gearboxes [22]; time–frequency demodulation approaches based on iterative generalized demodulation [23]; and ensemble empirical mode decomposition based methods [24], among others [25, 26, 27].

Many existing methods were based on the powerful techniques of time synchronous averaging (TSA), phase and amplitude demodulation for fault detection, where firstly applied to a fixed-axis gear, which are not applicable directly for gear fault detection in planetary gear sets. Then, other techniques based on TSA and demodulation were established on vibration analysis of epicyclic gear sets to reduce the effect of the time-varying vibration transfer path and make the vibration picked up from a planetary gearbox be analyzed like that from a fixed-axis gearbox by utilizing the conventional approaches for gear fault detection. Therefore, we consider it worthy here to make a reminder of the main principles of the time synchronous averaging and demodulation techniques.

Time synchronous averaging (TSA)

In a noisy condition, synchronous averaging (SA) is a powerful tool to improve the signal-to-noise ratio, in which the signal components synchronous with

the frequency of a reference shaft are remained. On the contrary, random noises and other periodic interferences non-synchronous with the frequency of interest are removed at the same time. A synchronous averaging is generally employed as a pre-processing step in the narrow amplitude and phase demodulation for localized gear fault detection of a gearbox since the captured vibration signal contains a lot of interferences and random noise contributed by shafts, bearings, and other unknown sources.

TSA is widely used in the vibration analysis of a fixed-axis gearbox if the shaft rotating speed is almost constant. The central idea of the TSA algorithm is as follows. The vibration data series $x(t)$ is divided into P data blocks with M points in each data block by a synchronizing reference signal, which is phase-locked with the angular position of a specified reference shaft. Let M correspond to an integer multiple of the period of the synchronous signal. Then, the TSA result signal defined as $\bar{x}(m)$ is given by [28]:

$$\bar{x}(m) = \frac{1}{P} \sum_{p=1}^{P-1} x(m + pM), \quad (m = 0, \dots, M - 1) \quad (2.17)$$

The key to performing a successful SA is to keep the phase alignment of synchronous components in each data block. For example, components with an integer multiple period of the reference shaft rotating speed in data series $x(m + pM)$ should have the same phases at every corresponding point in blocks, by which the synchronous components are remained and non-synchronous components are attenuated greatly after the SA.

However, the vibration observed from planetary gear sets differs from that of fixed-axis gear sets because it is influenced by time-varying vibration transfer paths caused by the epicyclic motion of the planet gears. This motion leads to time-varying phase shifts in the measured vibrations. Consequently, phase alignment of vibrations in planetary gearboxes is a challenging task that must be addressed before applying TSA. To overcome this, a generalized time-domain averaging method has been proposed for separating the vibrations of the planet and sun gears in planetary gear sets [29, 27], etc.

Narrowband demodulation

As discussed earlier, in the section of phenomenological modeling of vibration signal, a gear tooth fault contributes to the amplitude and phase modulation phenomenon. It enriches the sidebands in the vicinity of the meshing

frequency and its harmonics in the vibration spectrum. This is explained by the meshing stiffness variation due to the faulty gear tooth, such as a cracked or broken tooth. In this context, a first work of narrow demodulation was proposed by McFadden [19]. The demodulation is processed only on selected meshing harmonics after applying a band-pass filter around it, where the amplitude modulation function is calculated with the amplitude of the analytical signal (computed with Hilbert transform), and the phase modulation function is the argument of the analytical signal of the selected meshing harmonics and its sidebands [19, 30, 27].

Network-based methods

Recently, neural networks have attracted considerable attention from the machine diagnostics community, particularly in the context of planetary gear sets. Neural network-based methods aim to directly construct a mapping from raw signals to fault conditions by training the network, rather than relying on traditional signal processing techniques such as FFT (Fast Fourier Transform), STFT (Short-Time Fourier Transform), or WT (Wavelet Transform) [31, 32, 33]. These methods typically require large volumes of data to ensure a well-trained model suitable for industrial applications. Moreover, there remains a lack of interpretability regarding the physical meaning of the model structure and its parameters.

2.4.2 Blind source separation

Several methods can be considered to achieve elementary source recovery of the system's moving parts from the overall vibration signal. Much research has been carried out on the general topic of recovering a set of signals once they have been mixed together [34, 35, 36, 37, 38]. This problem, known as Blind Source Separation (BSS), is not specific to mechanical systems and actually takes its roots in audio processing and telecommunications rather than in vibration analysis. BSS frameworks are generally applied to estimate linearly mixed signals (with unknown mixture matrix), and gave rise to an abundant literature [39, 40]. Most proposed methods rely on statistical independence criteria of the sources, such as the widely used Independent Component Analysis (ICA); however, these approaches require a number of sensors equal to the number of sources. Unfortunately, such assumptions are typically not valid in mechanical systems, as the sources associated with system components are generally dependent due to their kinematic interactions.

Additionally, only a limited number of sensors are usually available to record the nonlinear mixtures of vibration signals [41].

On the other hand, other works in the literature have investigated different scenarios of non-linear BSS (e.g., quadratic mixtures, polynomial mixtures) and post-linear BSS [42, 43] in an overdetermined (i.e., more sensors than sources) context. Our present work addresses a distinct mixture scenario, where we delve into one single factorized form of a polynomial mixture (multi-modulations model). In brief, our BSS problem is both nonlinear and underdetermined (i.e., we use only one sensor). In general, this is a difficult, almost untractable, problem.

2.5 Transfer function effects

As stated earlier, vibration measurements comprise both the forcing functions at the meshing points (excitations) and the path effects of the complex structure of the rotating machine. Most of the gear health state information is contained in the excitations rather than in the transfer functions [44]. The complexity of gearbox structures is typically characterized by a transfer function rich in resonances, where excitation forces (e.g., small defects such as tooth cracks) are convolved with the system's impulsive response. Additionally, non-stationary operating conditions modify the transfer function effects, which in turn influence the system response. This affects the ratio between the frequencies of the force functions (garmesh frequencies and shaft rotation frequency, etc) and the structure resonance. Thereby, one needs to perform an identification of the transfer function effects to successfully interpret the vibration origin at the meshing point, which means isolating the contributions of gear meshing frequencies, and the changes may be caused by a faulty element of the epicyclic gearbox. In the sequel, we briefly introduce the well-known vibration signal-based methods that aim to remove the transfer function effects from the vibration signal.

2.5.1 Signal processing methods

Minimum entropy deconvolution (MED)

For vibration signals, a MED-based method identifies an optimum set of filter coefficients, which provides the inverse-filtered signal with maximum kurtosis. The MED-based methods remove the transfer function effects based

on the assumption that the forcing function is impulsive, which is reasonable for gear cracks, but not necessarily the case for other types of faults [45, 46].

Average-Log-Ratio (ALR)

Several techniques have been developed that do not require the assumptions made in the MED-based methods of impulsive faults. A frequency-domain ALR-based method was proposed that takes the log ratio of the rotational-harmonic amplitudes before and after damage, through which the common transfer function in the two cases could be removed. However, this method is only valid under constant speed conditions, where TSA can give the whole gear-related response signal, and it cannot reconstruct the time-domain waveform of the forcing function [47].

Exponential liftering

The exponential liftering method has been proposed as a useful technique for removing modal information from gear vibration response spectra and can be applied under both constant- and variable-speed conditions. The method employs an exponential short-pass lifter on the real cepstrum of the response, from which the approximate amplitude of the transfer function can be inferred. The modal information is subsequently removed from the signal amplitude spectrum, and the resulting edited amplitude spectrum is recombined with the original phase spectrum of the response [47, 48].

Adaptive Clutter Separation (ACS)

The ACS-based methods apply the adaptive clutter separation (ACS) method to separate sharp spectral peaks, related to rotating components, from the background noise, which tends to have a shape resembling the transfer function, and smooths the background noise spectrum. The transfer function effects can be removed, and the edited signal can be reconstructed. The estimation of the optimal parameters for ACS was automated in later research by using the ACS and cepstrum liftering in parallel. The ACS method was also further extended to non-stationary conditions [49, 47].

A key limitation of existing transfer-function removal methods, such as exponential liftering and ACS methods, is that they only remove the amplitude effects of the transmission path, meaning they cannot correct the waveform itself, only the spectrum. However, phase information is crucial

for reconstructing the time-domain forcing functions at meshing points. Furthermore, the transfer-function magnitude estimated using exponential liftering and ACS often retains the color of the excitation, so it cannot recover the true time-domain forcing function at the meshing point when the excitation is non-white. Additionally, as noted by the authors in [47, 50], for minimum-phase systems, the phase is given by the negative Hilbert transform of the log-magnitude transfer function, but this property has not been exploited in exponential liftering or ACS methods to remove transfer-function effects in both amplitude and phase. This limitation is one of the main factors motivating the development of cepstrum-based methods. Operational Modal Analysis (OMA) has also been explored for removing transfer-function effects from response measurements, as it can identify modal parameters and reconstruct the transfer function, including both amplitude and phase, thereby mitigating transmission effects [50].

2.5.2 Operational Modal Analysis methods (OMA)

Pole-residue based methods

Over the decades, operational modal analysis (OMA) has been a well-established research field. Classical OMA techniques typically assume that the excitation is white, at least near the resonance frequencies. More recent methods have extended these approaches to account for harmonic components or non-white excitations. OMA-based techniques can accommodate various types of excitation, including white, colored, and harmonic inputs, thus extending their applicability to the analysis of more complex systems. However, these methods provide unscaled mode shapes without the input measurement since they are based on pole-residue models. Several methods have been proposed to overcome this limitation in classical OMA. However, they demand additional requirements that are hard to achieve when the system is in service, such as changing the local mass of the structure, adjusting the operating speed to provide harmonics at desired frequencies, or a finite element model. The scaling of modes, at least the relative scaling between adjacent modes, is important for the reconstruction of the waveform of forcing functions.

In short, most conventional OMA-based techniques rely on a pole-residue representation of the transfer function impulse response. As a result, they cannot preserve or estimate the relative scaling among the identified modes. The unknown color of the excitation means that the residues (relative strengths) of

the poles cannot be determined, so that the mode shapes are unscaled. This limits the true reconstruction of the transfer function in order to have direct access to the forcing functions. This is widely used in structure vibration analysis (beam vibration). However, it's rarely leveraged for reconstructing the excitation functions of machines such as gearboxes.

Pole-zero based methods

By contrast, cepstrum-based OMA adopts the pole-zero model, where the inclusion of zeros provides the relative scaling between adjacent modes. Cepstrum-based OMA is able to reconstruct the transfer function from the estimated poles and zeros, up to an overall scaling factor. In addition, cepstrum-based OMA only requires the excitation to be smooth in the frequency domain and relatively flat on a log amplitude scale, which further relaxes the restriction of white noise excitation in conventional OMA techniques. Therefore, cepstrum-based OMA has a significant advantage in regenerating the transfer function for force reconstruction purposes, compared to other OMA techniques. The cepstrum-based OMA method is proposed to identify the transfer functions from a single measurement, enabling the correction of both amplitude and phase transfer function effects, and then reconstructing the waveforms of the excitations.

Early studies on cepstrum-based operational modal analysis (OMA) were developed to extract modal information from systems with a single input and multiple outputs (SIMO). These methods were initially validated on simple beam structures subjected to impact excitation. Subsequently, the approach was extended to systems with multiple inputs and multiple outputs (MIMO). For a comprehensive overview of the theoretical background and early developments, readers are referred to [44].

Recently, cepstrum-based OMA has been applied to vibration signals measured under constant and varying speed conditions in a single-stage spur gearbox. The method relies on several key assumptions. First, the mechanical system is assumed to be minimum-phase, which is generally reasonable for structural and mechanical systems and enables estimation of the transfer function phase. Second, the system excitation is assumed to be broadband and frequency-smooth. Third, the measured response is considered to be dominated by a single excitation source. Under these assumptions, the broadband random components of the vibration response are exploited to identify the transfer function, as these components are continuous in the frequency domain. Deterministic harmonic components are removed during

a preprocessing stage. A nonlinear least-squares algorithm is then employed to perform curve fitting over the quefrency range of interest. Afterward, the deterministic components of the forcing function are used to reconstruct the excitation forces associated with gear-tooth meshing [51, 47]. In this framework, the random component of the signal is assumed to arise predominantly from stochastic gear-mesh forces, while contributions from other sources—such as bearings (assumed to be healthy)—are neglected. Consequently, the response measurement is treated as being driven by a single effective input associated with the gear mesh. It is worth noting that existing OMA-based approaches were primarily developed for transfer function identification in a single gear pair, corresponding to a single excitation source. This assumption does not hold for complex epicyclic (planetary) gearboxes, where the measured vibration response is influenced by multiple simultaneous excitation sources.

In this work, we propose a novel methodology for identifying the transfer functions of a gearbox, focusing on the harmonic components of the vibration signal in contrast to cepstrum-based approaches. The sidebands of the vibration signal in the vicinity of the meshing-frequency harmonics often exhibit asymmetry, which may originate from transmission-path effects. Therefore, our methodology focuses on recovering the symmetry of these sidebands to identify the system transfer functions.

To conclude this review, existing techniques for mitigating transfer-function effects in measured vibration signals differ in their ability to address the following challenges:

1. Removal of both the phase and amplitude effects of the signal transmission path. Accurate phase information is essential for reconstructing the time-domain waveform of forcing functions.
2. Compensation for the amplitude effects of the transfer function without distorting the spectral “color” of the forcing function. Methods that assume white excitation may not reconstruct the true forcing function under non-white excitations.
3. Applicability to different types of fault, not only those where the forcing function is impulsive, which is typically the case for gear cracks.
4. Operation under varying-speed conditions, where both excitation and transmission path characteristics may change.

5. Implementation using a single measurement channel.
6. Handling of multi-input single-output (MISO) systems, i.e., systems with multiple simultaneous forcing functions and a single measured response.

2.6 Problem statement

Epicyclic gear sets exhibit unique spectral characteristics in the measured vibration signal, making the application of conventional vibration analysis techniques challenging due to the epicyclic motion of the system. The vibration spectrum has a distinct structure in varying configuration modes and operation conditions. Furthermore, the complexity of the system obscures the excitation forces caused by tooth meshing and faulty components as they propagate through the system. Therefore, the localization and detection of the faulty sources from the complex measured vibration signal poses a significant challenge.

Phenomenological modeling, whether healthy or faulty, of epicyclic gear-boxes is the cornerstone for exploring the vibration behaviors and fault identification. Such modeling aims to explain the sidebands of the spectrum and the origins of asymmetries in both spectral structure and amplitude. The decomposition of the spectrum sidebands is essential to track the latent components induced by the faulty gear. Furthermore, it is essential to take into account the propagation effects on the amplitudes of the sidebands for an appropriate faulty track. To this end, we identify three main primary focuses.

1. The fidelity of existing phenomenological models requires further investigation. In particular, the origins of sideband asymmetries and the appearance and disappearance of certain harmonics are not yet fully understood. Furthermore, the influence of the distinct angular positions of the planet gears and transmission effects on the measured vibration signal remains to be fully understood in an automated manner [6, 9].
2. The changes induced by a gear fault are short-duration events that are convolved with the system's transfer function impulse response, producing the observed vibration response. This convolution complicates the identification of gear faults. Therefore, the objective of reconstructing the meshing force functions is to obtain more indicative information

about the faults. In contrast to the approach in [47], which relies on random components to identify the transfer function impulse response after separating it from the deterministic part. However, our method focuses on the valuable information contained within the deterministic component. We aim to directly utilize the deterministic component, which highlights the asymmetric sidebands around the meshing frequency and its harmonics. By recovering these asymmetries near the meshing frequencies, we can identify the system's transfer functions.

3. A time-domain source separation and tracking module is proposed using alternating optimization, accompanied by a theoretical analysis of the corresponding estimation bound. The objective of this framework is to isolate the signal components associated with the faulty characteristic frequency from the so-called healthy components, such as those arising from the meshing frequency, within the global vibration response.

Chapter 3

A Phenomenological modeling via symbolic regression

The vibration behavior of a planetary (epicyclic) gear set, under both healthy and defective operating conditions, has been investigated using a phenomenological modeling approach. Vibration components associated with different excitation sources are expressed as periodic functions of the rotational motion, taking into account the kinematic relationships of the system. The phenomenological modeling approaches aim to constitute an analytical–mathematical model that facilitates understanding of the relationship between the spectrum’s dominant frequencies and the system’s characteristic frequencies. This, in turn, facilitates the interpretation of spectral features, such as asymmetry and the appearance or disappearance of spectral components, which reflect the underlying dynamic behavior of the system.

The modeling process requires domain expertise and is time-consuming. This is because different configurations of planetary gearboxes exhibit distinct sideband spectra, and in the presence of manufacturing errors or under faulty conditions, the resulting vibrations become increasingly complex. Moreover, the analytical model may vary across operating conditions (load, torque), introducing additional complexity to the modeling task. This motivates the need for automatic modeling tools capable of extracting the characteristics of the observed data. These phenomenological models interpret the underlying dynamics of power transmission systems and can be used to explain the system behavior across a wide range of system conditions and configurations. This helps afterwards in predicting unseen data using the underlying system

laws.

On the other hand, symbolic analysis has shown significant progress in extracting the underlying laws of physical systems from experimental data, producing analytical models that accurately represent the system’s behavior. Symbolic Regression (SR) is a data-driven analytical method that seeks to identify the most appropriate and simplest mathematical expression that best fits observed data with high accuracy. It has been used extensively in many nonlinear data-driven modeling tasks. Most popular frameworks for symbolic regression use genetic programming (GP), a method that evolves a population of candidate solutions through a number of generations. This gradient-free learning process is driven by a selection strategy that prefers high-quality solutions to poor ones, and new candidate solutions are created by genetic operators of crossover and mutation.

Contrary to data-hungry approaches such as neural networks, symbolic regression (SR) can reveal accurate models even from small datasets. Moreover, SR is particularly well-suited for incorporating prior knowledge, especially when the training data are limited or insufficiently informative. Integrating prior knowledge into the learning process enables the derivation of models that not only achieve low training error but also exhibit strong compliance with the physical properties of the system under study. However, GP approaches do not scale well to the number of variables and samples since an entire population of formulas has to be evolved and evaluated repeatedly through many generations. [52].

SR has also seen growing interest from the Deep Learning community, motivated by the fact that neural networks are effective at identifying qualitative patterns. Recent works in [53] and [54] show that NNs are able to learn formulas by means of gradient-based optimization. They propose the modifications to feed-forward networks to include interpretable components, i.e. replacing usual activation functions by primitive mathematical functions (binary or unary operators) $\{\times, +, \sin, \exp, \dots\}$. [53, 55].

The gradient-based learning process aims to minimize the training error while reducing the number of active units, which means the weights above a threshold are used in the final NN output. Thereby, the formulas obtained with the NN after training are simple and concise. The major difference between the recent approaches is the learning process and the sparse regularization used.

In this work, we adapt a neural-network-based symbolic regression ap-

proach to uncover the vibration behavior of a rotating machine while accounting for the system kinematics. The input variables of the symbolic regression model consist of the fundamental characteristic frequencies and their harmonics, with an overestimation number introduced to capture the rich harmonic structure of the data. Consequently, the number of input variables is relatively large, which motivates the use of neural-network-based symbolic regression rather than traditional genetic-programming-based approaches.

We align the interpretability of symbolic layers with the dynamics of the systems. Indeed, the primitive mathematical functions used in the NN hidden layers translate the physical interactions between gears within the system and their dynamic motion. That data-driven approach aims to identify explicit analytical expressions that explain the "mixing" process of the gears meshing sources of a planetary gearbox, which means the origin of each activated harmonic in the given data. The focus of our work is on applying symbolic regression to the planetary gearbox vibration analysis, where the power spectrum of the vibration signal shows distinct complex characteristics coming from the meshing sources and the many propagation path distances. This motivates the development of an end-to-end architecture that decodes the vibration patterns with analytical formulas in an automatic fashion for varying operation conditions and faulty cases.

This work leverages neural network-based symbolic regression to derive a compact analytical expression for gearbox vibration modeling. By incorporating system kinematics, an interpretable neural network is trained under sparsity constraints to identify closed-form expressions that accurately represent the observed data.

3.1 Problem statement

As described earlier, an epicyclic gear transmission consists of a central sun gear that meshes with P planet gears positioned at different angular locations around it. Planetary gear sets are most commonly used in practical applications in configurations where the ring gear is fixed.

The observed vibration signal is modeled as a multi-input single-output (MISO) system, where each input corresponds to an equivalent vibration source associated with a meshing interaction, as illustrated in (Fig 3.1). The

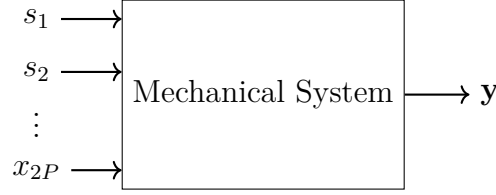


Figure 3.1: MISO system

measured signal can be expressed as

$$y(t) = \sum_{p=1}^{2P} (\mathbf{h}_p * s_p)(t) \quad (3.1)$$

$$s_p(t) = \mathcal{F}_p(\mathbf{s}_{1,p}(t), \dots, \mathbf{s}_{m,p}(t)) \quad (3.2)$$

where h_p denotes the transfer path between the p -th meshing interaction and the sensor, and $s_p(t)$ denotes the equivalent vibration source associated with this interaction. The function $\mathcal{F}_p$ models the nonlinear mixing of elementary periodic components $s_{1,p}, \dots, s_{m,p}$. These components are linked to the gear-mesh frequency, the carrier rotation frequency, and the characteristic frequencies of the sun and planet gears. These quantities should be interpreted as latent periodic components providing a physically meaningful representation of the vibration-generation mechanism, rather than as directly measurable elementary sources.

In a healthy planetary gearbox, the sun-planet and ring-planet contacts are associated with the same fundamental mesh frequency. However, their measured contributions may differ because of modulation effects, different transfer paths, and attenuation mechanisms related to the sensor location. Therefore, the contribution of the sun-planet mesh may be considered weaker at the measurement point and neglected in a first approximation, provided that this assumption is validated experimentally.

First, our modeling process begins with the assumption that the global vibration signal is generated by $2P$ copies of the vibration produced at the ring-planet meshing points, where one looks for a single function $\mathcal{F}$ that is valid for the different meshing points. The accelerometer is assumed to be mounted near the ring gear. This is close to a healthy scenario, and with manufacturing errors where the meshing points exhibit similar patterns, but with a time delay of the excitations' propagation to the sensor location. This is the subject of section 3.2, where we propose a Neural network based on symbolic regression to identify an analytical formula $\mathcal{F}$ that fits the observed

vibration data.

Secondly, traditional modeling approaches were based on interpreting the spectral sideband frequency structure, with amplitude information being a secondary focus. To this end, we propose an automatic modeling framework that relies only on the spectrum peaks of the vibration data in section 3.3.

Next, to address the complex structural patterns of the planetary gearbox, a transfer-function removal module is integrated into the neural network through the use of a convolution layer in 3.4. This module enables effective reconstruction of vibration data exhibiting asymmetric spectral sidebands.

The observed vibration originates from different meshing interactions. Considering possible manufacturing errors and faulty gears, finally, we propose parameterizing our neural network with respect to the number of planet gears. This approach enables the identification of an equation for each possible meshing point in the system. To achieve this, we employ in section 3.5 a stacked neural network architecture, in which separate networks are trained to derive an expression for each meshing point. This approach enables the reconstruction of each planet-related forcing function (meshing) from the overall measured vibration signal. This capability is novel and facilitates the localization of the faulty gear, thereby improving the effectiveness of fault detection.

3.2 Neural network based symbolic regression

3.2.1 Related work

Many approaches have been proposed in the literature to address the symbolic regression (SR) problem. From a traditional standpoint, genetic programming (GP) has been one of the most widely used techniques for symbolic regression, mainly due to its ability to explore the vast search space of possible analytical expressions. SR has been successfully applied in numerous real-world industrial and scientific applications involving complex system analysis, particularly through genetic algorithms. Notable examples include the discovery of Hamiltonians and Lagrangians for oscillatory systems, the identification of governing equations for gas and wind turbines, modeling

pipe failure systems, and deriving laws governing material property changes [56].

Despite their popularity, genetic algorithms have been shown to scale poorly to high-dimensional problems and are often prone to overfitting [55]. These limitations have motivated the exploration of alternative approaches.

In recent years, symbolic regression has attracted increasing interest from the deep learning (DL) community, driven by the strong ability of neural networks to capture complex, qualitative patterns in data. As a result, hybrid methods that combine neural networks with GP-based techniques have been proposed. More recently, purely neural-network-based approaches to symbolic regression have emerged [53, 55, 54, 52]. Among these, the Equation Learner (EQL) framework has gained particular attention. EQL employs a feed-forward multi-layer architecture composed of diverse functional units, including sigmoid, sine, cosine, identity, and multiplication operators. A key advantage of EQL is its ability to produce compact symbolic expressions while automatically identifying the most relevant input features.

A different class of neural-network-based symbolic regression approaches relies on deep transformer architectures, such as large language models. These methods are trained using large-scale datasets in which each training sample typically consists of a tuple of the form (formula, data sampled from the formula). While this class of approaches offers several advantages, it also presents limitations [57, 58]. These methods are beyond the scope of this work.

3.2.2 Multivariate regression

Consider a multivariate regression problem with a training set

$$\{(\mathbf{D}_t, \mathbf{y}_t)\}_{1 \leq t \leq T/N}$$

with $\mathbf{y}_t = [y((t-1)N+1), \dots, y(tN)]^T$, N being a processing window and T is the total sample size) and the dictionary $\mathbf{D}_t$ is given by We

$$D_t = [\mathbf{D}_{t,1}, \mathbf{D}_{t,2}, \mathbf{D}_{t,3}, \mathbf{D}_{t,4}] \quad (3.3)$$

where $\mathbf{D}_{t,i}$ is formed by the columns of $\mathbf{D}_i$ of indices $k = (t-1)N + 1, \dots, tN$, defined as:

$$\mathbf{D}_i(:, k) = [\sin(2\pi f_i k), \cos(2\pi f_i k), \dots, \sin(2\pi H_{max} f_i k), \cos(2\pi H_{max} f_i k)]^T \quad (3.4)$$

The i -th source signal $s_i(t)$ is a linear combination of the entries of $\mathbf{D}_{t,i}$, with an overestimated harmonics number H_{max} . The signal $\mathbf{s}$ originates from an unknown analytical function $\mathcal{F} : \mathbb{R}^{\dim(\mathbf{D}_t)} \rightarrow \mathbb{R}$ of sources $\mathbf{s}_i$, corrupted by an additive zero-mean noise, ϵ , i.e

$$\mathbf{y} = \mathcal{F}(\mathbf{s}_1, \dots, \mathbf{s}_m) + \epsilon.$$

The main task is to recover elementary sources $\mathbf{s}_i$ and learn a function ψ that approximates the true functional relation F to achieve the minimal expected error

$$\mathbf{E} \|\psi(\hat{\mathbf{s}}_1, \dots, \hat{\mathbf{s}}_m) - \mathbf{y}\|^2$$

where $\hat{\mathbf{s}}_i$ is the estimate of the i -th source, given $\mathbf{D}_i$. We will use the notation $\psi(\mathbf{D})$ instead of $\psi(\hat{\mathbf{s}}_1, \dots, \hat{\mathbf{s}}_m)$, to stress the fact that the input of the network is the dictionary $\mathbf{D}$.

3.2.3 Network architecture

Base architecture

We propose employing an *Equation Learner (EQL)* architecture [53, 55], a fully connected neural network designed to perform symbolic regression by replacing conventional nonlinear activation functions (ReLU, Sigmoid, etc) with primitive mathematical operators.

The output of the i -th symbolic layer, as illustrated in Fig. 3.2, is defined as:

$$\mathbf{g}^{(i)} = \mathbf{W}^{(i)} \mathbf{h}^{(i-1)} + \mathbf{b}^{(i)}, \quad (3.5)$$

$$\mathbf{h}^{(i)} = f(\mathbf{g}^{(i)}), \quad (3.6)$$

where $\mathbf{W}^{(i)}$ and $\mathbf{b}^{(i)}$ denote the weight matrix and bias vector of the i -th symbolic layer, respectively. The function $f(\cdot)$ represents the set of primitive (activation) functions, while $\mathbf{h}^{(0)}$ corresponds to the regression entries $\mathbf{s}_{1 \leq i \leq 4}$. The way these functions operate on $\mathbf{g}^{(i)}$ will depend on whether each function is a unary or binary operator. The activation function of the final layer is typically linear. Thus, for a network comprising L hidden layers, the output is expressed as $\hat{\mathbf{y}} = \mathbf{W}^{(L+1)} \mathbf{h}^{(L)}$.

The EQL framework utilizes a predefined set of primitive functions. In our context, given that the vibration signal exhibits a polynomial structure, the set of primitive operators is restricted to multiplication ($\times$), addition ($+$), and the identity operator ($\mathbf{1}$). Each component of $\mathbf{g}^{(i)}$ may be processed by

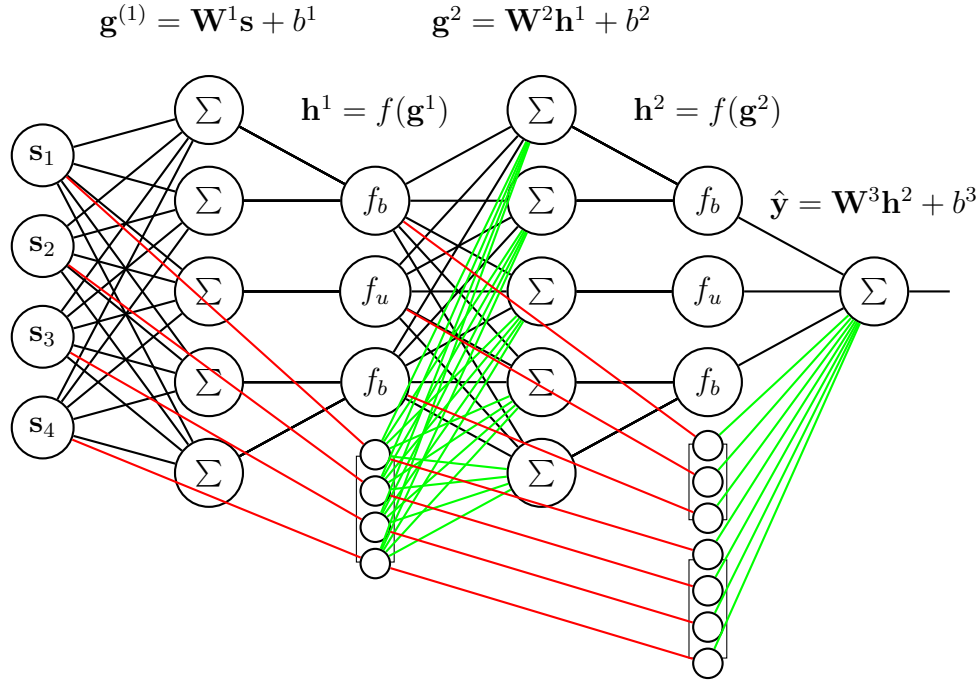


Figure 3.2: Example of EQL network for SR where f_b and f_u are the binary and unary operators. The red and green colors represent the skip connections. Bias nodes are not shown for clarity.

a distinct primitive function, and some primitives may operate on multiple inputs, enabling the network to capture complex symbolic relationships. Moreover, it is advised in [55] to repeat the same function multiple times within a layer to escape local minima during training. The number of hidden layers in the neural network is a hyperparameter that depends on the complexity of the underlying data.

In addition, the skip connections technique has been used in EQL architecture [59], consisting of concatenating the output of each hidden symbolic layer to the subsequent layer as shown in Fig. 3.2. This enforces the network to obtain simpler equations while simultaneously enabling the discovery of more complex equations. This is well known in ResNet architectures, we take the output of a layer and add it to the layer ahead with the goal of allowing gradient information to efficiently propagate through many layers and enabling extremely deep architectures [60]. In contrast, in EQL, we use the skip connection introduced in dense Nets, which concatenates rather

than sums the output of the previous layer with that of the next layer [61]. Thereby we modify the equations 3.9 as follows:

$$\mathbf{h}^{(i)} = \left[f\left(\mathbf{g}^{(i)}\right); \mathbf{h}^{(i-1)} \right], \quad (3.7)$$

Skip connections introduce a slight inductive bias toward learning simpler functions, as they allow information to bypass hidden layers and reach the output directly, without passing through the identity primitive functions of the successive layers. This also avoids exploding or vanishing as they pass through the primitive functions, since the gradient, computed over a processing window of size N , will back-propagate more easily in the network [55]. As a result, networks with more hidden layers can be trained, enabling the learning of more complex expressions.

Sinusoidal inputs

In our context, we do not have direct access to the entries $\mathbf{s}$ of the multivariate regression. However, we have some prior information about the system's dynamic motion and kinematic model. We assume that, in the case of rotating machine vibration, the sources $\mathbf{s}_i$ are periodic (or quasi-periodic) functions with known (estimated) fundamental frequencies but an unknown number of harmonics. In the Fourier representation, the sources $\mathbf{s}_i$ can be expressed as follows:

$$\begin{aligned} \mathbf{s}_i(t) &= \sum_{j=1}^{H_{max}} w_{ij} \cos(\phi_{ij}) \sin(2\pi j f_i t) + w_{ij} \sin(\phi_{ij}) \cos(2\pi j f_i t) \\ \mathbf{s}_i(t) &= \mathbf{D}_i(:, t) \mathbf{w}_i \end{aligned} \quad (3.8)$$

where w_{ij} and ϕ_{ij} are, respectively, the corresponding amplitude and phase of the j th harmonic of the frequency f_i , which are grouped into $\mathbf{w}_i$ for each source $\mathbf{s}_i$.

To address this specific input case, we modify the input layer of the EQL architecture by adding a linear layer that performs the linear transformation of the dictionary $\mathbf{D} \in \mathbb{R}^{(H_{max} \times 2) \times 4}$, which contains all the harmonics of each source's fundamental frequency. The nodes in the first EQL layer (Fig. 3.3), are grouped into m weight vectors, $\mathbf{w}_{1 \leq i \leq m}$, which are connected to each dictionary $\mathbf{D}_{1 \leq i \leq m}$. This configuration produces the inputs for the first symbolic layer $\hat{\mathbf{s}}$ whose dimensionality equals the number of sources, and is defined as

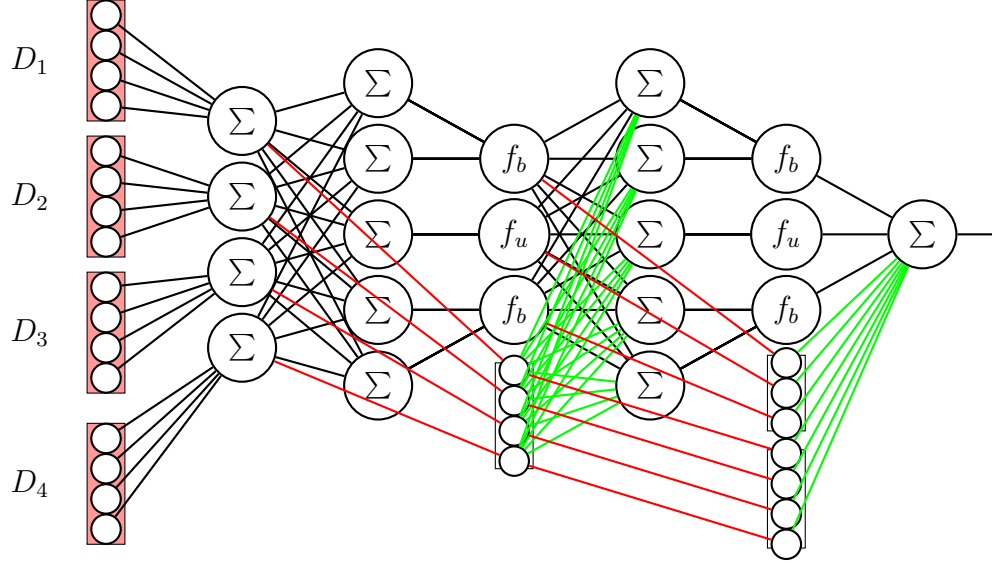


Figure 3.3: EQL network with sinusoidal inputs

$$\mathbf{h}^{(0)}(n) = \hat{\mathbf{s}} = [\mathbf{w}_1^T \mathbf{D}_{n,1}, \dots, \mathbf{w}_m^T \mathbf{D}_{n,m}]^T. \quad (3.9)$$

In summary, the proposed neural network-based symbolic regression architecture comprises an input layer for estimating dictionary weights, followed by an L -layer EQL network. The number of hidden layers L serves as a tunable hyperparameter, determined according to the complexity and nonlinearity of the modeled system.

The training of the neural network of L (a hyperparameter) symbolic layers, and parameters

$$\theta = \{\mathbf{w}_1, \dots, \mathbf{w}_m, \mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L+1)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L+1)}\} \quad (3.10)$$

is conducted using established methods akin to those used for conventional NNs, i.e., Adam [62]. Upon completion of the training process, the discovered equation can be directly extracted from the network's weights. The EQL is fully differentiable in its free parameters θ , which allows us to train it in an end-to-end fashion using back-propagation. We use the following objective function:

$$J(\theta) = \mathbf{E}(\|\psi_\theta(\mathbf{D}) - \mathbf{y}\|^2) + \lambda R_q(\theta) \quad (3.11)$$

where $\mathbf{E}$ denotes the mean value and R_q is the sparsity regularization applied to the network weights, so that the final expression contains only a few terms, and λ is the penalty parameter to control the trade-off between the reconstruction term and sparsity term.

3.2.4 Sparsity regularization

A key ingredient in making the results of SR interpretable is the sparsity regularization, which enforces the system to find the simplest possible equation that fits the data. The goal of sparsity is to set as many weight parameters as possible to 0, such that those parameters are inactive and can be removed from the final expression. A straightforward and popular way of enforcing sparsity is adding to the loss function a regularization term that is a function of the neural network weight matrices W :

$$R_q = \sum_{i=1}^{L+1} \sum_{j,k} |W_{j,k}^i|^q + \sum_{i=1}^{L+1} \sum_j |b_j^i|^q \quad (3.12)$$

where L is the number of hidden layers and j, k are the element index of weight matrices W^i and b^i .

Setting $q = 0$ in 3.12 results in L_0 regularization, which penalizes weights for being non-zero regardless of the magnitude of the weights and thus drives the solution toward sparsity. However, L_0 regularization is equivalent to a problem that is NP-hard and is not compatible with gradient descent methods commonly used for optimizing neural networks. Recent works have explored training sparse neural networks with a relaxed version of L_0 regularization through stochastic gate variables, allowing this regularization to be compatible with backpropagation [63] [64]. A much more popular and well-known sparsity technique is L_1 regularization, which was used in the original EQL network [53]. Although it does not push solutions toward sparsity as strongly as L_0 regularization, L_1 regularization is a convex optimization problem that can be solved using a wide range of optimization techniques, including gradient descent to drive the weights toward 0. However, since L_1 also penalizes the magnitude of the weights, $L_{0.5}$ has been proposed to enforce sparsity more strongly without penalizing the magnitude of the weights as much as L_1 . $L_{0.5}$ regularization is still compatible with gradient descent (although it is no longer convex) and has been applied to neural networks more strongly than L_1 , producing simpler expressions.

$L_{0.5}$ regularization has a singularity in the gradient as the weights go to 0, which can make training difficult for gradient descent-based methods. To avoid this, we use a smoothed version of $L_{0.5}$ proposed in [65] which is noted $L_{0.5}^*$ defined as follows:

$$\mathcal{L}_{0.5}^*(w) = \begin{cases} |w|^{1/2} & |w| \geq \delta \\ \left(-\frac{w^4}{8\delta^3} + \frac{3w^2}{4\delta} + \frac{3\delta}{8}\right)^{1/2} & |w| < \delta \end{cases} \quad (3.13)$$

where $\delta \in \mathbf{R}^+$ is the transition point between the standard $L_{0.5}$ function and the smoothed one.

3.2.5 Training procedure

The training scheme has three phases following the procedure in [53]. The neural network training starts with an unregularized phase, characterized by setting the regularization parameter λ to zero. This enables the optimizer to effectively search for an effective starting point without the constraint of regularization penalties. The second phase encourages the tradeoff between the sparseness regularization and the reconstruction term, where regularization is normally enabled by setting the parameter λ to a nonzero value. During the last phase, we disable $\mathcal{L}_{0.5}^*$ regularization ($\lambda = 0$ again), and we use hard thresholding instead throughout the remaining epochs. This is accomplished by setting any weight coefficient w to zero if $|w| < \gamma$, where γ is a chosen threshold. Such an approach effectively stabilizes the model's complexity, thereby ensuring a closer alignment between predicted function values and observed data points. In our implementation, we have allocated $t_1 = 1/10 \times it$ epochs (it denotes the total number of epochs) to define the duration of the first phase and left $t_2 = 9/10 \times it$ epochs for the second phase, which is large enough to ensure convergence. After convergence, we apply the hard thresholding, the third phase of our algorithm.

3.2.6 Model selection

The criteria of model selection widely used in symbolic regression is the capability of the model to extrapolate to the unseen interval of data. "Only if the right analytical formula is selected, good extrapolation capabilities can be expected" [53]. During the learning process, many NN models are generated, resulting in a family of candidate equations obtained by varying hyperparameters, particularly the sparsity regularization strength. The right

equation is the simplest one that predicts well according to the Occam’s razor principle. The selection generally is based on the tradeoff between the formula simplicity and the prediction performance on the validation data set or on labeled points in the extrapolation interval. The strong non-convexity of the problem may cause the network optimization process to become trapped in local minima or to fail in selecting the correct formula. Therefore, many independent runs with random initialization conditions are used to quantify a good performance.

3.3 Robust Modeling through Binary Neural Network-Based Symbolic Regression

One of the main focuses of phenomenological modeling is to explain the origin of the vibration spectrum peaks structure, given the kinematic model of the planetary gearbox. The sideband structure around the gear-meshing frequency may exhibit amplitude asymmetry and, in some cases, apparent frequency-structure variations. Such asymmetries can originate from transmission-path effects. Several theoretical studies have investigated the origin of sideband asymmetry, which, from an amplitude-modulation perspective, is closely related to the system transfer function. [6, 5]. In contrast, a polynomial mixture of periodic functions inherently produces symmetric spectral sidebands. Consequently, training a neural-network-based symbolic regression model without accounting for transfer-function effects may result in a poor fit with the measured data, even if the underlying periodic functions are accurately reconstructed.

In this section, we propose a robust modeling approach that focuses on the frequency structure of the vibration spectrum. First, the spectral peak locations (i.e., frequencies) of the observed periodic signal are estimated using the standard frequency detection algorithm. The extracted frequency information is then provided as input to a symbolic regression network, which reveals the underlying multi-modulation model of the signal, thus interpreting the frequency structure in terms of the characteristic frequencies of the system. The robustness of this approach arises from the fact that spectral peak locations are not affected by transfer function effects or signal phase distortions to the best of our knowledge. Also, we aim with this method to estimate the significant harmonic number related to each characteristic frequency, providing direct and informative indicators of the health condition

of the corresponding components.

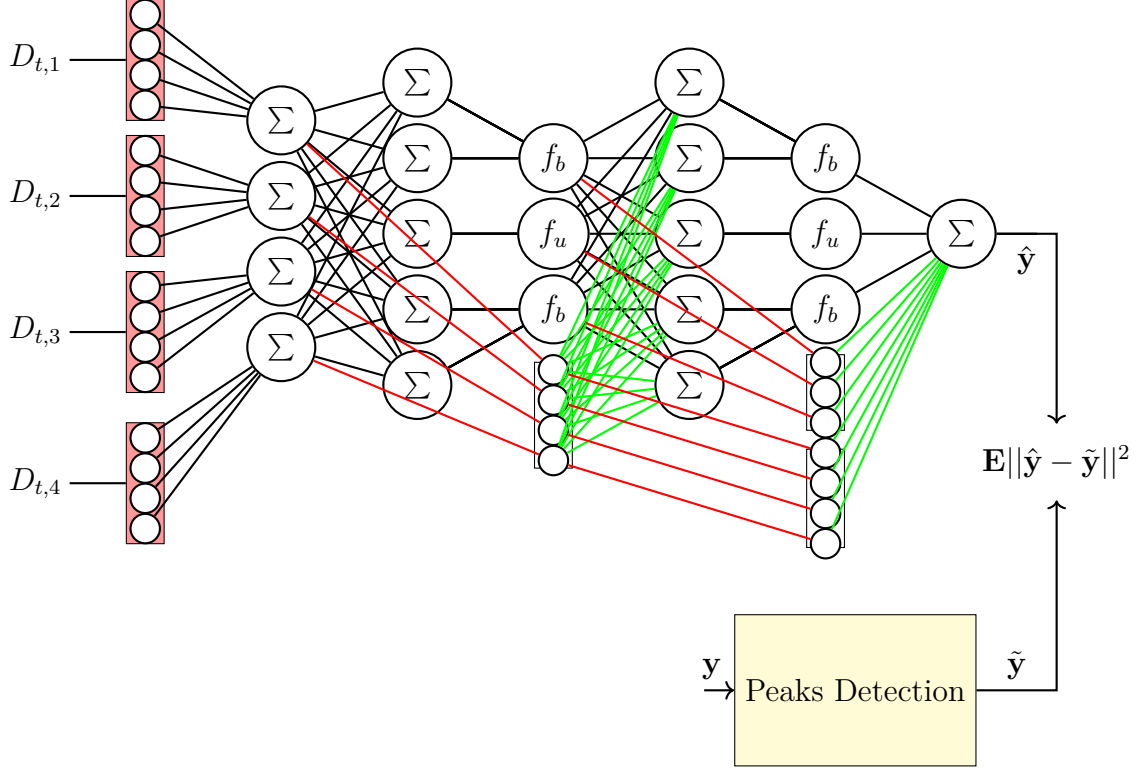


Figure 3.4: Spectrum peaks interpretation with Neural network-based symbolic regression

3.3.1 New target data

The transmission path between the meshing point and the sensor modifies the amplitude distribution of the meshing sidebands, thereby causing amplitude asymmetry. However, it is generally assumed not to modify the location of the spectral lines. Therefore, in this section, we focus on interpreting the locations of the spectral peaks using analytical expressions obtained by symbolic regression. To reduce the influence of the transfer function on spectral amplitudes, we synthesize a signal from the raw vibration measurement $y(t)$ in which all detected spectral-peak amplitudes are normalized to one.

Due to the rotation behavior, the vibration signal in our case is given by the sum of sinusoids. In the Fourier domain, the spectral content $Y(f)$ of the signal $y(t)$ can be represented as a sum of H complex sinusoids $S_k(t) = e^{j2\pi h_k t}$

and their complex conjugates $\bar{S}_k(t)$. h_k represents the unknown harmonic frequencies for $k = 1, \dots, H$. We assume that T samples are available from a noisy measurement $y(t)$ expressed as:

$$y(t) = \sum_{k=1}^H \alpha_k S_k(t) + \bar{\alpha}_k \bar{S}_k(t) + \epsilon(t), t = 1, \dots, T \quad (3.14)$$

where $\alpha_{1 \leq k \leq H}$ are the complex amplitudes of $\mathbf{S}_k$.

We introduce an auxiliary signal that is 'weights-free' computed from the raw vibration signal $\mathbf{y}$ as follows:

$$\tilde{y}(t) = \sum_{k=1}^H \cos(2\pi h_k t) + \sin(2\pi h_k t), \quad t = 1, \dots, T \quad (3.15)$$

where H is the total number of sinusoids (i.e., spectral peaks for $f > 0$) representing the harmonic frequencies, denoted h_k , of the raw signal $\mathbf{y}$. We made up the new signal with unit amplitudes for all spectrum peaks to skip the transmission path effect. The sinusoid frequencies h_k are estimated by applying the ESPRIT (Estimation of Signal Parameters via Rotational Invariance [66]) algorithm, the high-resolution frequency estimation technique, to the measured signal $\mathbf{y}$. In the sequel, we introduce the standard algorithm for spectral peaks detection as follows.

3.3.2 Spectral peaks detection algorithm

Signal Parameters via Rotational Invariance Techniques

Accurate frequency estimation is a fundamental problem in signal processing with applications in communications, radar, and sonar, etc. Among high-resolution spectral estimation techniques, the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm has gained significant attention due to its robustness and high estimation accuracy [66]. ESPRIT is a subspace-based frequency estimation algorithm that takes advantage of the rotational invariance property of signal subspaces formed by sampled signals (sensor arrays in telecommunications). Unlike classical methods such as the Fourier transform, ESPRIT does not rely on spectral peak searching, which allows it to achieve superior resolution, especially in low Signal-to-Noise Ratio (SNR) environments and with closely spaced frequency components.

Peak detection algorithm

Spectral peak detection may be performed using the "find peaks" algorithm [67]. This algorithm identifies local maxima in the discrete frequency spectrum while enforcing constraints such as minimum peak height and peak prominence to improve robustness in the presence of noise. Compared to subspace-based methods such as ESPRIT, FFT-based peak detection offers lower computational complexity and simpler implementation, but its resolution is inherently limited by the DFT grid.

3.3.3 Binary Neural Network

To approximate the mixture function $\mathcal{F}$, we use the neural network architecture described in the previous subsection fed with the same dictionary $\mathbf{D}_t$ and trained with the new target called the auxiliary signal $\tilde{\mathbf{y}}$ (instead of the raw data $\mathbf{y}$) as shown in Fig. 3.4. Herein, we seek the binarization of the neural network-based symbolic regression to provide the mathematical expression that fits well the given data with binary coefficients. Several works proposed the use of a relaxed form of L_0 regularization [63, 64]. However, in this work, we introduce a regularization term in the loss function that minimizes a function with two possible solutions: 0 or 1. The latter, an element-wise function, is defined as (see Fig. 3.5):

$$\mathcal{L}(\theta) = \sinh^2(\theta(1 - \theta)) \quad (3.16)$$

where θ represents the weight coefficients of the neural network, and $\sinh$ is the hyperbolic sine function defined as $\sinh(x) = \frac{e^x - e^{-x}}{2}$.

Thus, the objective is LASSO-like [68], defined as:

$$J(\theta) = \frac{1}{B} \sum_{n=1}^B \|\psi_{\theta}(\hat{\mathbf{s}}_1(t), \dots, \hat{\mathbf{s}}_m(t)) - \tilde{\mathbf{y}}(t)\|^2 + \lambda \sum_l^{L+1+m} |\mathcal{L}(\theta^{(l)})| \quad (3.17)$$

where λ is a penalty coefficient and $\hat{\mathbf{s}}_i(n) = \mathbf{w}_i^T \mathbf{D}_{n,i}$. The training procedure is performed in two phases. We start with a non-regularized phase, $\lambda = 0$, to give the NN a good starting point, then the regularization is enabled with a nonzero λ value. The regularizer leads to a trade-off between minimizing the reconstruction loss and enforcing sparsity by binarizing the Neural network's weights. In addition to producing an interpretable model, the proposed approach provides an estimate of the number of harmonics associated with each elementary periodic source. Such information may be valuable for future studies on rotating-machine condition monitoring, as the harmonic content of vibration signals can provide insights into the machine's health state.

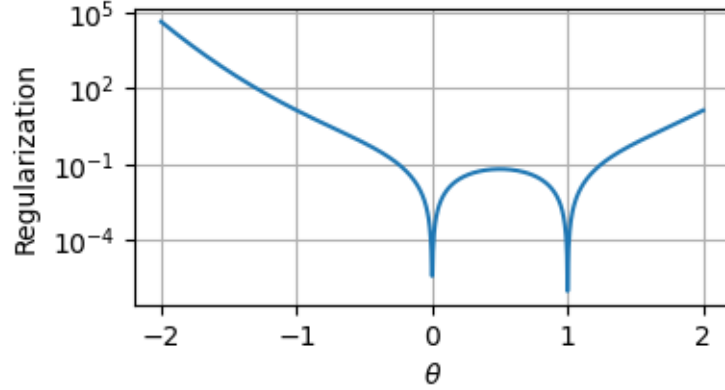


Figure 3.5: Proposed regularization with two minimas $\{0, 1\}$.

3.4 Modeling with Transfer Function effects removing

In this part, we introduce an original methodology for the automatic modelling of the observed vibration signal with transfer function effects removal. In practice, a spectral whitening step is often employed to mitigate the effects of transmission paths within the system; however, this process may not entirely eliminate these effects. As a result, the influence of the transfer function on harmonic components cannot be fully compensated, which may impact negatively the signal's modeling. In this case, we propose to rectify the previous architecture 3.2 by adding a convolution layer applied to the EQL output $\hat{\mathbf{s}}$ to reconstruct the final vibration signal $\mathbf{y}$.

3.4.1 Finite impulsive response

The objective is to derive a function ψ that approximates the true functional relationship F , and simultaneously recompensate the transfer function effects. To this end, we extend our neural network-based symbolic regression by introducing a new convolutional layer that models the system's impulse response $\mathbf{h}$. Specifically, we modify the reconstruction error in the previous objective function (3.11) as follows:

$$\mathbf{E}||\hat{\mathbf{h}} * \psi(\mathbf{D}) - \mathbf{y}||^2 \quad (3.18)$$

where $\hat{\mathbf{s}}_i$ represents the estimate of the i -th source, given $\mathbf{D}_i$. A descriptive

diagram summarizing our proposed architecture is illustrated in Fig. 3.6. For each batch of dictionaries $\mathbf{D}$ fed into the neural network, the network output $\hat{\mathbf{s}}$ is convolved with the kernel of the convolutional layer to generate the reconstructed signal $\hat{\mathbf{y}}$. We employ a one-dimensional convolutional neural network (1D-CNN) layer to learn the kernel weights automatically.

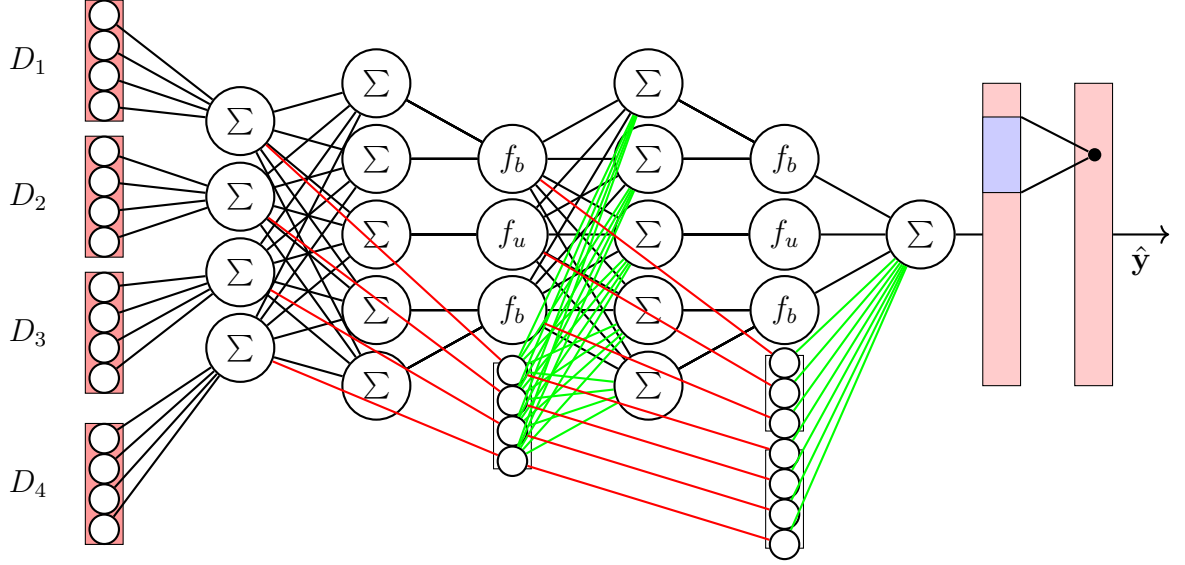


Figure 3.6: Neural network-based symbolic regression with a convolution layer used for transfer function removal.

3.4.2 Infinite impulsive response

When the expected impulse response function $\mathbf{h}$ becomes excessively long, directly modeling it in the time domain may lead to high computational cost and instability in the learning process. To address this limitation, we propose representing the system dynamics in the z -domain using a rational transfer function model. Specifically, the transfer function $H(z)$ can be expressed in its general zero-pole form as

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_{N_z} z^{-N_z}}{1 + a_1 z^{-1} + \dots + a_{N_p} z^{-N_p}}, \quad (3.19)$$

where $B(z)$ and $A(z)$ denote the numerator and denominator polynomials, parameterized respectively by the zero and pole coefficients of the system.

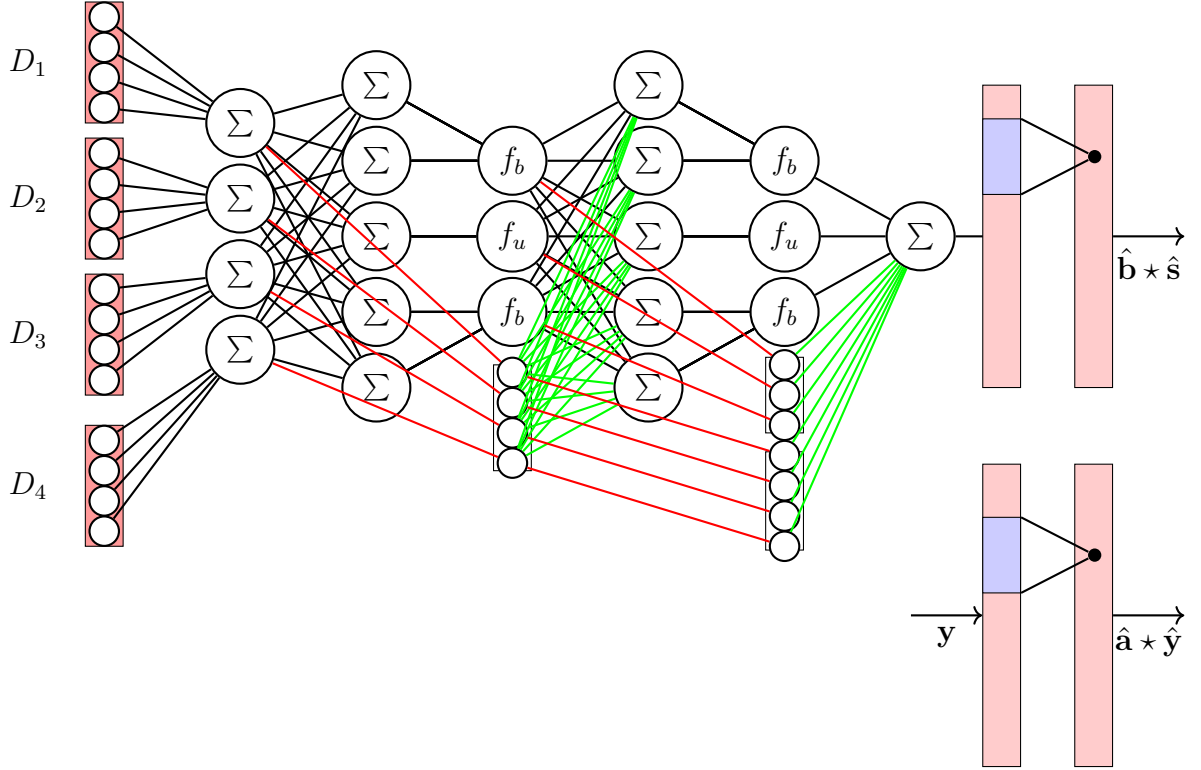


Figure 3.7: Neural network-based symbolic regression with two convolution layers used for rational transfer function removal.

In this framework, rather than estimating the full impulse response $\mathbf{h}$, we identify the coefficients of $A(z)$ and $B(z)$, which fully characterize the linear dynamics of the system. Consequently, we modify the reconstruction objective in Equation (3.18) to incorporate this parametric model as follows:

$$\mathbf{E} \|\hat{\mathbf{b}} * \psi(\mathbf{D}) - \hat{\mathbf{a}} * \mathbf{y}\|^2 \quad (3.20)$$

where $\mathbf{b} = [b_0, \dots, b_{N_z}]$ and $\mathbf{a} = [1, a_1, \dots, a_{N_p}]$ represent the estimated numerator and denominator coefficient vectors of the transfer function, respectively. For each batch of data $\{\mathbf{D}, \mathbf{y}\}^{N_t}$, the output of the symbolic regression is fed into a convolutional layer, while the vibration data $\mathbf{y}$ is passed through another convolutional layer as shown in Fig. 3.6. We aim to minimize the squared difference between the outputs of these two convolutional layers. This formulation allows the network to learn both the

nonlinear mapping $\psi(\mathbf{D})$, which models the intrinsic source relationships, and the linear dynamic behavior encoded in $H(z)$ by jointly optimizing the parameters of ψ and the coefficients $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$.

3.5 Stacked Neural Network-based Symbolic Regression

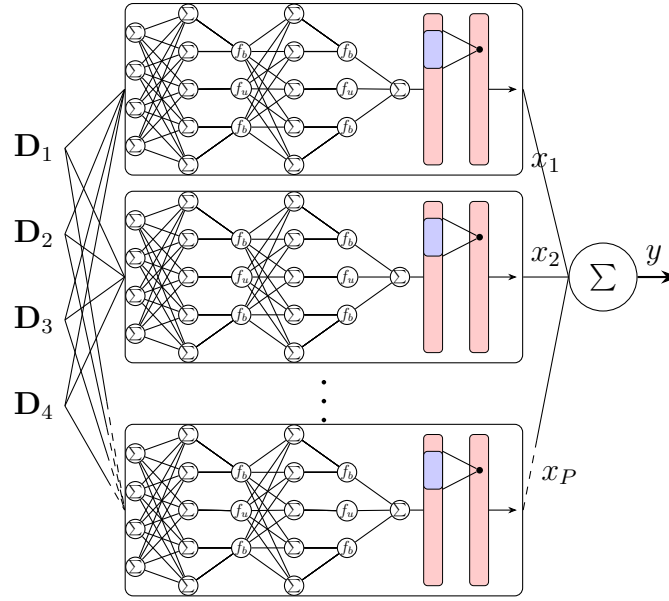


Figure 3.8: Architecture of the 3-stacked neural network-based symbolic regression. The neural network architecture in the middle box is the same as in Fig. 3.6.

3.5.1 Network architecture

The vibration signal of a planetary gearbox originates from multiple sources, mainly associated with the meshing interactions between the planet gears and the sun and ring gears. We therefore extend the proposed architecture to handle vibration signals generated by several meshing points. We assume that the vibrations (excitations) at the planet–sun and planet–ring meshing points exhibit distinct analytical models, transfer functions, and harmonic

coefficients. These differences may result from manufacturing errors, tooth-profile imperfections, unbalanced load distribution, or the presence of localized faults at specific meshing points. The architecture proposed in the previous section can be conceptualized as modeling the vibration of a single planet gear. Building on this, we introduce a new architecture that generalizes the reconstruction of vibration data, achieving a more accurate fit to the observed signals. At this end, we will have precise information on the health state of each gear in the system and interpret the data patterns with the physics motions.

We employ a separate neural network for each vibration source, resulting in a total of twice the number of planet gears, to model both meshing points. This architecture, referred to as the stacked neural network, is illustrated in Fig. 3.8 The outputs of these networks are summed to reconstruct the overall vibration signal, resulting in a more accurate fit to the observed data. Each neural network is fully connected and can be trained using backpropagation, allowing the entire system to be optimized end-to-end. The networks are trained using an objective function composed of two terms, as in the above. In this architecture, we do not enforce that the different neural network stages share the same structure or equations. Instead, each network is free to learn distinct representations, reflecting the possibility that different vibration sources may follow different behavior, particularly in the presence of faults within the system.

3.6 Simulations

For the synthesized data, we use the following model that, to our knowledge, the most fit a planetary gearbox vibration signal developed by [12] to generate a data corrupted with an additive noise ϵ as follows:

$$y(t) = \sum_{p=1}^P h_p(t) * s_{1,p}(t) (1 + s_{2,p}(t)) (1 + s_{3,p}(t) + s_{4,p}(t)) + \epsilon(t) \quad (3.21)$$

where P is the number of planets in the considered planetary gear set, while $s_{1,p}(t)$ represents the p^{th} meshing component, $s_{2,p}(t)$ the modulation at the planet carrier frequency, $s_{3,p}(t)$ the sun gear rotation contribution to planet p and $s_{4,p}(t)$ the intrinsic contribution of planet p , respectively. The fundamental frequencies for the four sources in (3.21) are set to, $\{f_1 = 99, f_2 = 1, f_3 = 2.83, f_4 = 4.67\}$ with 10 significant harmonics for all sources. The signal length is set equal to $T = 405$, 405 samples for a sampling

frequency $f_s = 11,583$. The synthesized data assume that the faulty gear is the planet gear. In this case, the sources s_3 , which are related to the planet gear rotational frequency, are activated in the model 3.21, while the source coefficients s_4 are set to zero. We consider a planetary gear set with three planet gears ($P = 3$).

The synthesized data share the same kinematic parameters as the public planetary gear set provided in the HUMS 2023 dataset challenge [69]. The synthesized excitation force is distorted by a randomly selected impulsive response of the transfer function of 30 taps, h_p , for each ring–planet mesh point. An example of the vibration spectrum around the first fundamental meshing frequency f_1 is shown in Fig. 3.9.

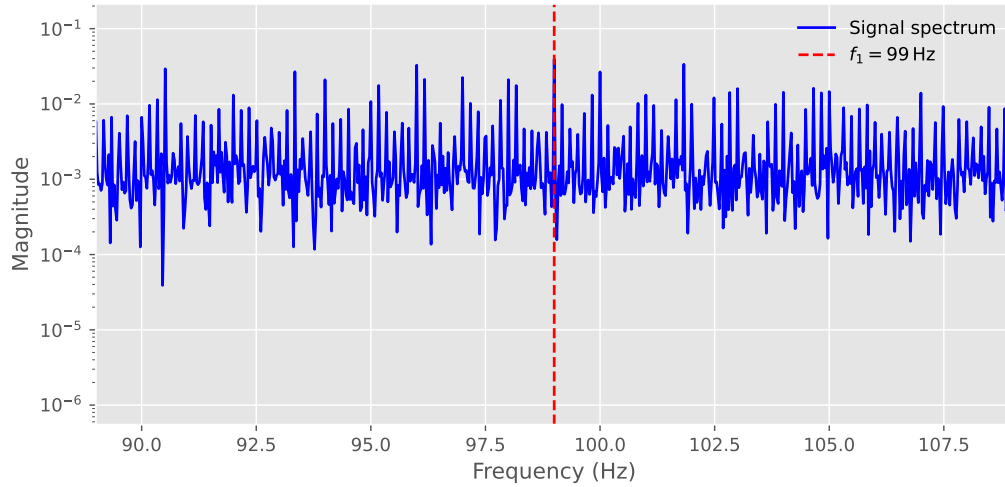


Figure 3.9: A synthesized vibration data of a three-planet gear.

The EQL network is trained using a three-phase procedure, as described earlier. In the third phase, the weight elements whose magnitudes fall below an ad-hoc threshold of $\gamma = 0.01$ are clamped to zero. An RMSProp optimizer [70] is employed, with the learning rate initially set to $l_r = 10^{-3}$ and reduced to $l_r = 10^{-4}$ in the final phase. The number of harmonics for each periodic source $\mathbf{s}_i$ is intentionally overestimated and set to 20.

The number of EQL network’s hidden layers is a hyperparameter, depending on the complexity of the expression. We chose two hidden symbolic layers in this investigation.

We train the models with the objective function in (3.11) for $it = 1000$

epochs, and the parameter a in the regularization term of $\mathcal{L}_{0.5}^*$ is set to 0.01 as in [55]. The values of λ , a , threshold γ , and EQL hidden layers are also our hyperparameters. The regularization parameter λ is selected through a research grid of value ranges, balancing between neural network weight sparsity and signal reconstruction. For the activation function used in the EQL, we consider multiplications ("×" Five times) and identity ("id" Five times) since the vibration model of the gear transmission set is considered polynomial in its arguments in the literature.

For all the experiments, we used 50% of the data for training, while the remaining 50% is used for validation, and the processing window is $N = 256$ points. We consider a Signal-to-Noise Ratio (SNR; defined as $10 \log_{10}(\mathbb{E}(s^2)/\mathbb{E}(\epsilon^2))$) equal to 0 dB.

Here, we present the simulation results for signal modeling without and then with transfer function identification. We also investigate the effectiveness of the robust solution introduced previously. Finally, we report the results of the stacked neural network used for signal reconstruction, showing separately the three terms associated with each planet gear in the synthesized data.

Signal modeling

After training our models with different regularization penalty coefficients λ , we select a value of $\lambda = 0.005$ that guarantees a good fit to the given data while providing the most compact expression of the model, as shown in figure 3.13. Moreover, one of the neural network architectures identified is:

$$\hat{s} = 60.0059 s_{11} (-0.138071 - 1.06293 s_{12}) (0.212091 - 1.27987 s_{13}) + 0.01630 \quad (3.22)$$

The connections that remain active after thresholding with $\gamma = 0.05$ are plotted in Fig. 3.11. The obtained optimal model by our framework is as follows:

$$\hat{s}(t) = \alpha \hat{s}_1(t) (a + b \hat{s}_2(t)) (c + d \hat{s}_3(t) + e \hat{s}_4(t)), \quad (3.23)$$

where α, a, b, c, d, e are constants. The source harmonics activations are shown in Fig. 3.12. The 10 harmonics used in the synthesized data are selected in the estimated sources. Source s_4 is not shown because it is not activated in the identified model formulas.

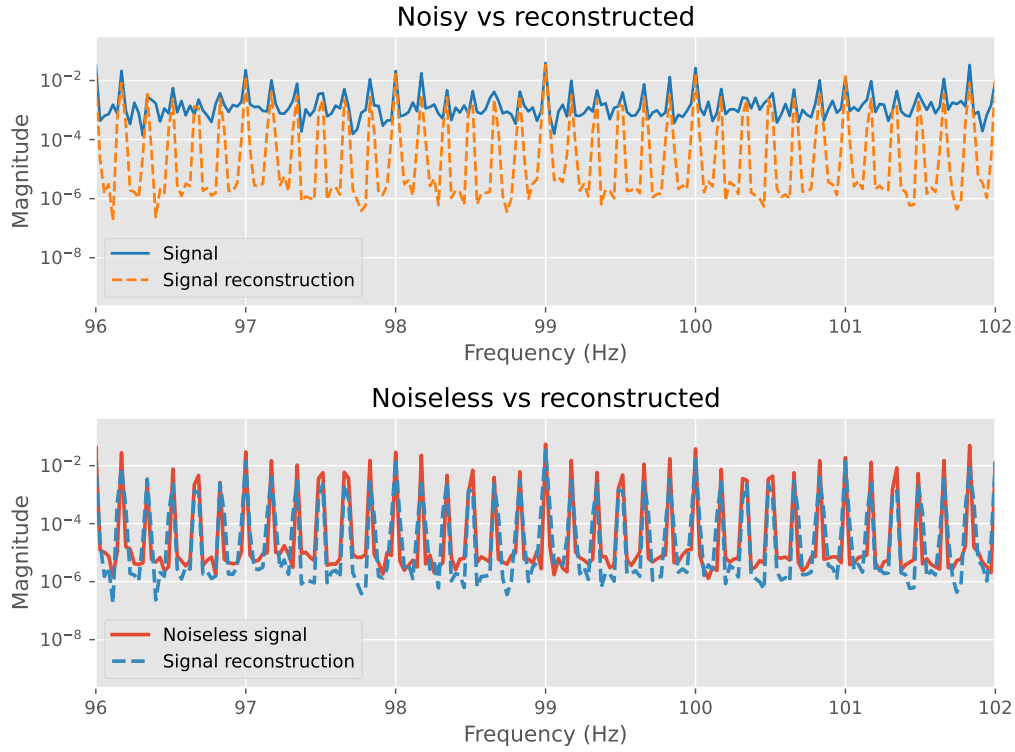


Figure 3.10: Signal reconstruction

3.6.1 Signal modeling with transfer function effects compensation

In the second part of this work, a convolution layer is employed to identify the impulse response h of the system. End-to-end optimization of the proposed architecture yields, after normalization, an accurate recovery of the model equations, even at an SNR of 0 dB.

The signal reconstruction is shown in Fig. 3.13, where a good agreement between the peaks can be observed, even at 0 dB.

The identified equation is identical to the one obtained previously, with a threshold of 0.01 applied to the network weights, and is given by

$$\hat{s} = -131.649 \cdot s_1 \cdot (-0.256499 + 1.4645 s_3) \cdot (-0.192963 - 1.58127 s_2). \quad (3.24)$$

The convolutional output is shown and compared with the three filter impulse responses used to generate the synthesized data in Fig. 3.14.

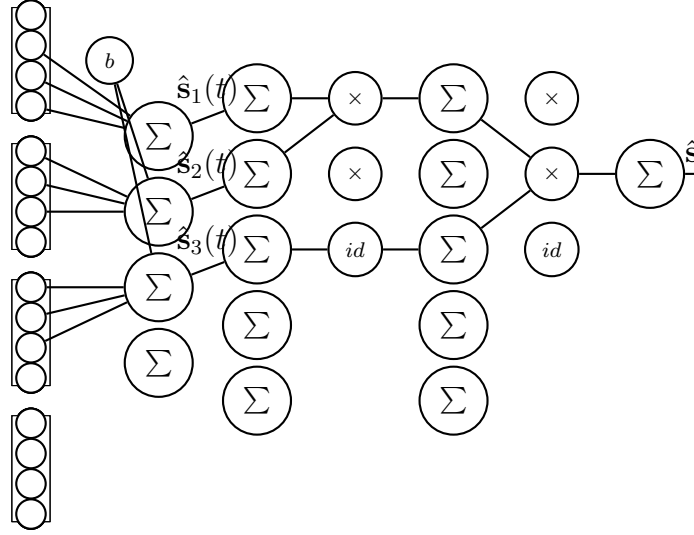


Figure 3.11: One of the final neural networks after training and thresholding, where only the desired inputs (harmonics) and a few connections are activated. For clarity, only 3 out of 4 sources are displayed as activated. Also, the exact number of harmonics per source signal is obtained after convergence, while the actual number of overestimated harmonics is 20 per signal. The first hidden layer estimates the elementary sources $s_{1 \leq i \leq 3}(t)$, where the source s_4 , associated with the sun's rotation frequency, is successively deactivated by our method.

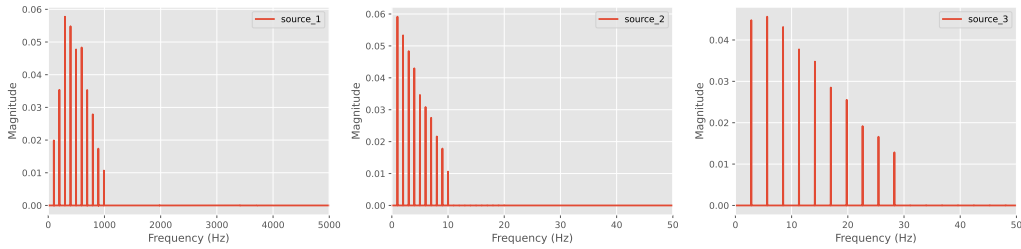


Figure 3.12: Source harmonics activation in the estimated sources.

As can be observed, none of the estimated impulse responses exactly matches any of the three filters used to generate the synthesized data. However, the predicted responses exhibit similar monotonic behavior across the three filters, providing shared qualitative information. Beyond the 10th harmonic in the frequency domain, the predicted response becomes difficult

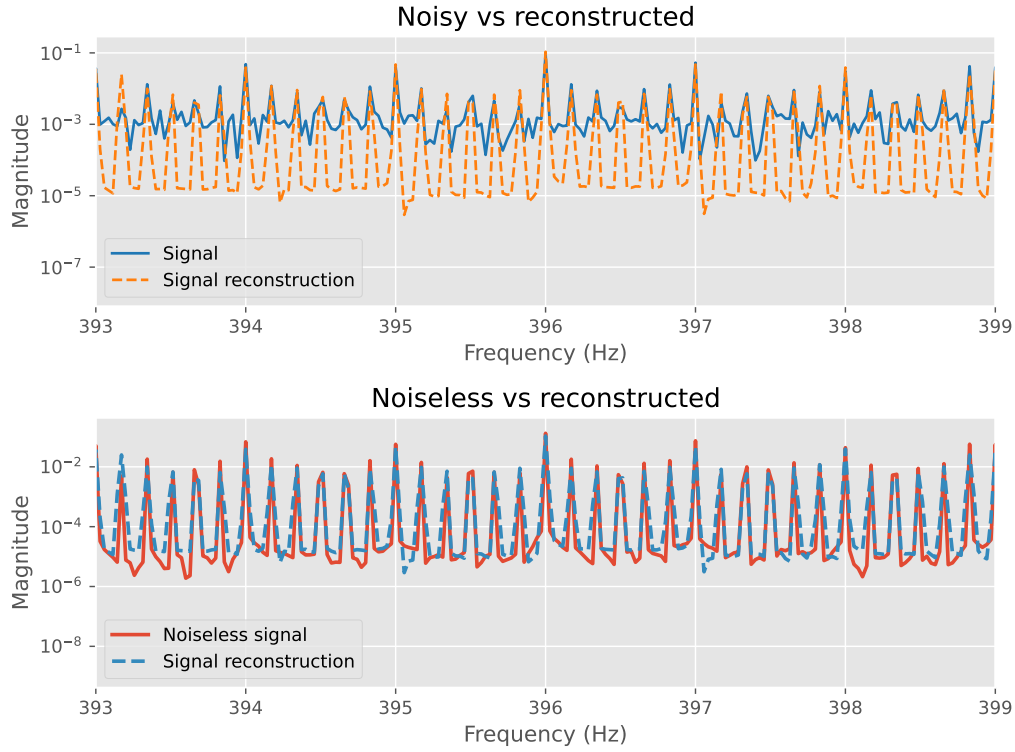


Figure 3.13: Signal reconstruction

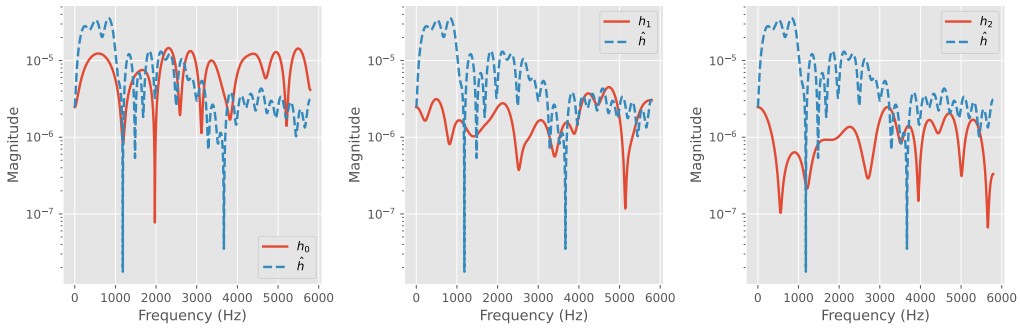


Figure 3.14: Comparison between the convolution layer output and the three filters used to generate the synthesized data.

to interpret, since only the first 10 harmonics of the carrier signal were used in

the synthesized data; therefore, no information is available in this frequency range to identify the transfer function response.

3.6.2 Robust solution for signal modeling

In this section, we present a demonstration of our alternative approach, which enables robust modeling of vibration data under specific conditions. For peak detection, we employ either the ESPRIT method or the “find peaks” algorithm based on the topology of the spectrum data. In this work, we adopt the “find peaks” algorithm to detect the highest peaks in the signal power spectrum. We use the find peaks function from the SciPy library [67], selecting prominence values that do not capture all peaks, in order to test the robustness of our modeling algorithm. From the synthesized data, the spectral peaks illustrated in Fig. 3.15 are extracted using a prominence threshold of 0.005. Owing to this relatively high prominence value, not all spectral components are detected. In particular, several low-energy sideband peaks are missed, which results in an incomplete representation of the spectral structure. As a consequence, the expected symmetry of the sidebands around certain carrier harmonics is not fully observed. This intentional limitation is introduced to evaluate the robustness of the proposed modeling approach under partial and imperfect peak detection conditions.

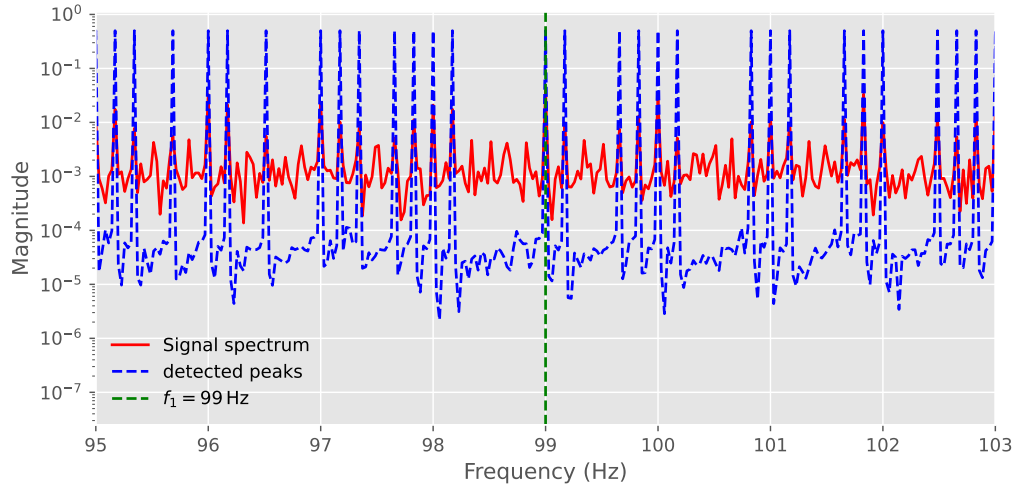


Figure 3.15: Detected spectral peaks from synthesized vibration data

Now, we optimize the EQL network by minimizing either the objective

function with the binarization penalty or the smooth $\mathcal{L}_{0.5}^*$ function, with the new signal $\tilde{\mathbf{y}}$. Then, we apply a hard threshold of 0.01 to the neural network weights to obtain the neural network. For a Signal-to-Noise Ratio (SNR) of 0 dB, The resulting expression is:

$$\hat{\tilde{\mathbf{y}}} = 0.93\mathbf{s}_1(1.01 + 0.95\mathbf{s}_2(1.01 + 0.94\mathbf{s}_3) + 0.91\mathbf{s}_3). \quad (3.25)$$

The harmonic sources s_i for $1 \leq i \leq 3$ are well selected.

3.6.3 MISO system

The purpose of this novel architecture is to improve the reconstruction fidelity of synthesized vibration signals in a planetary gear system, where multiple vibration sources originate from different gear meshing points. In this work, we assume that the number of sources is unknown, although in the synthesized data, it corresponds to the number of gear contacts present in the signal. In addition, convolutional layers are not incorporated in the initial stage, in order to evaluate the model's ability to identify the mixture model originating from different points using only a single observation. After thresholding the neural network weights under the threshold 0.01, we extract the formulas determined from each network. We obtained the following analytical equations for each sub-network: We conclude that the

Table 3.1: Equations at 0 dB.

NN	Formulas
1	0
2	$-3.382 s_{12} (0.242 - 0.79427 s_{32}) (1.516 + 6.09 s_{22})$
3	0
4	$32.24 s_{14} (0.136 + 1.089 s_{24}) (0.304 + 1.293 s_{34})$
5	$-22.44 s_{15} (0.183 - 1.42 s_{25}) (0.204 + 0.849 s_{35})$

training process achieves a balance between signal reconstruction fidelity and the simplicity of the identified equations, while using the required number of sources that contribute to the overall vibration. This is evidenced by the fact that only three neural networks are activated to reconstruct the

given signals, with only a small number of active connections within each network, resulting in compact models that accurately fit the data, as shown in Fig 3.16 and 3.17. We repeat the training process of the stacked neural

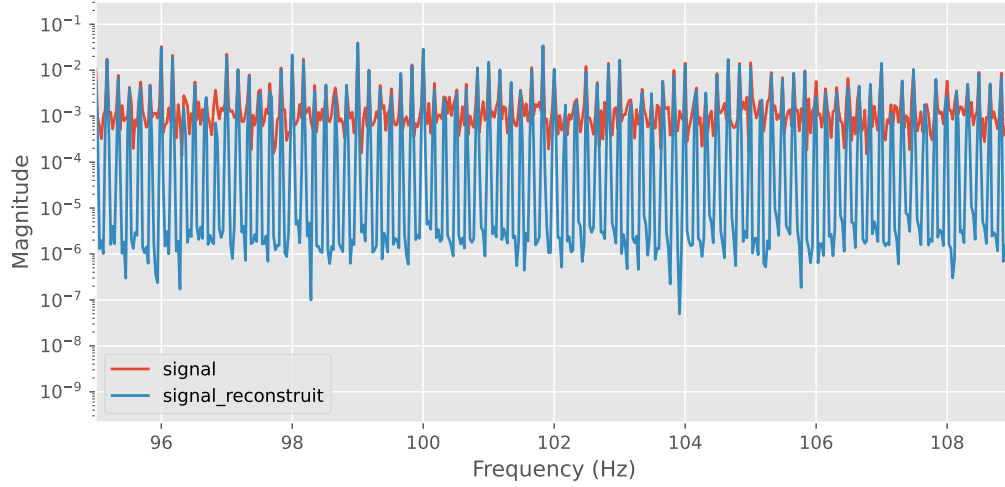


Figure 3.16: Signal reconstruction zoomed around the first meshing harmonic with a stacked Symbolic regression based-neural network

network; however, convolutional layers are introduced in each sub-network to account for the transfer function effects associated with the vibration sources originating from each contact point.

3.6.4 Real world dataset

Fixed shaft gearbox

The evaluation of our approach has been done with a simple gearbox composed of a gear and its pinion. Their meshing results in the sliding of each tooth on the other, thereby generating vibration. The test bench has been done at the GIPSA lab (Grenoble Images Parole Signal Automatique Laboratory) and instrumented by CETIM (Centre technique des industries mécaniques). The gears have $Z_1 = 20$ and Z_2 teeth, respectively, and the measurements are made at a sampling frequency of 20 kHz with $N = 60,000$ signal length. We will investigate the healthy vibration signal (first day of acquisition) to assess the efficiency of our proposed method on real world scenario. The meshing

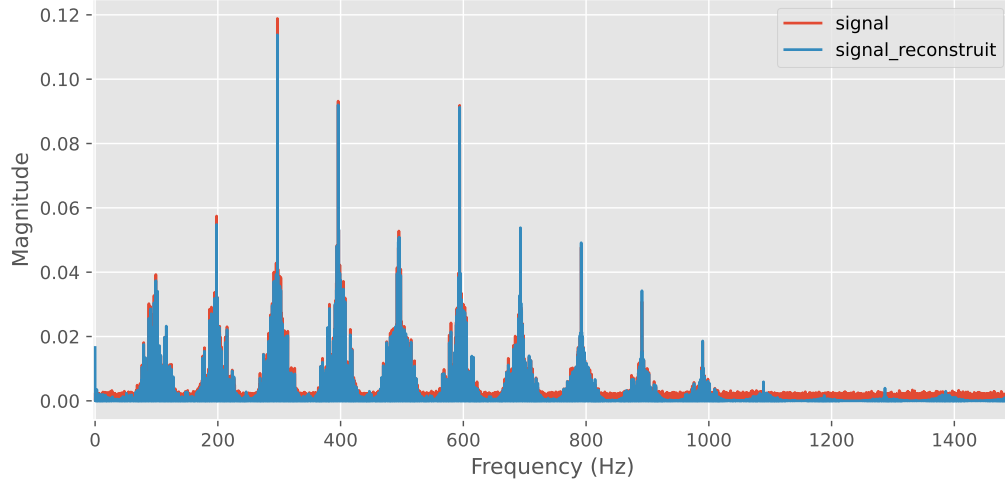


Figure 3.17: Signal reconstruction zoomed around the first meshing harmonic with stacked Symbolic regression based-neural network

frequency of the first day is $f_m = 338.7 \text{ Hz}$ from which we can deduce the rotation frequency of the gear and its pinion, respectively

$$f_{1,2} = f_m / Z_{1,2}$$

The spectrum of gearbox vibration signal and its reconstruction are shown in Fig.3.18: The incomplete reconstruction of the spectrum may be attributed to various factors, such as aliasing within the gear spectrum and the presence of additional vibration sources within the system. Addressing this issue would necessitate a more in-depth investigation into the underlying mechanical processes. Additionally, the low sampling rate, combined with the teeth numbers being so close, could make it tricky to differentiate between the two rotation frequencies. With all these factors considered, our proposed approach shows a good reconstruction of the signal peaks and accurate vibration data modeling.

The conclusive expression derived from our approach, which fitted the input vibration data and incorporated a threshold of 0.01 applied to the EQL weights, is as follows:

$$\mathbf{y} = \hat{h} * 37.84\hat{s}_m(0.226 + 1.781\hat{s}_1 + 2.096\hat{s}_2),$$

CHAPTER 3. A PHENOMENOLOGICAL MODELING VIA SYMBOLIC REGRESSION

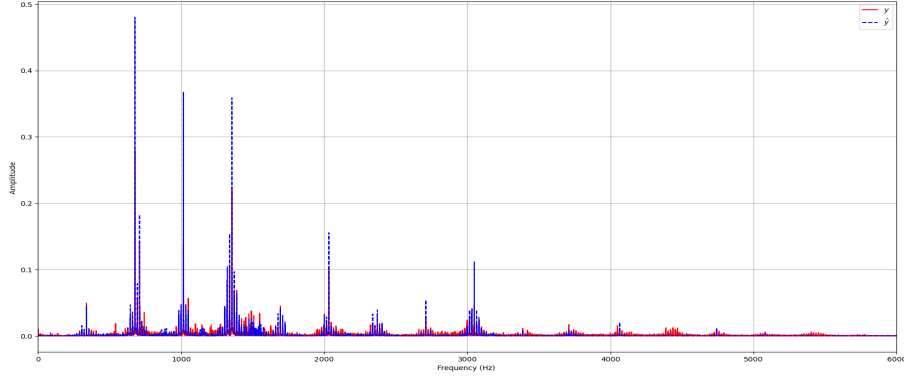


Figure 3.18: The spectra of both the original and reconstructed signals.

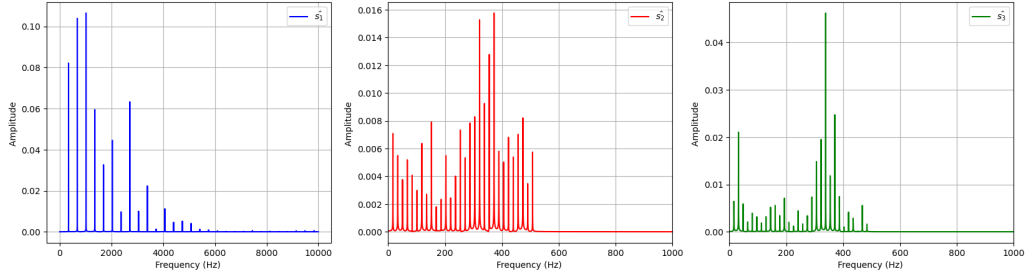


Figure 3.19: Spectral representation of the reconstructed signals

where $\hat{\mathbf{h}}$ represents the estimation of the gearbox transfer function with an initially overestimated length set to 200, and $\hat{s}_m, \hat{s}_1, \hat{s}_2$ denote the recovered meshing signal, gear signal, and pinion signal, respectively. Their spectra representations are illustrated in Fig. 3.19.

The final vibration data model identified for the fixed-axis gearbox aligns with the model defined over the last decades in the mechanical community, where the multiplicative model is widely utilized in fixed-shaft gear vibration analysis, often involving visual inspection of vibration spectra to detect frequency peaks, e.g., [71].

In future work, we aim to analyze real planetary gearbox vibration data under various scenarios, such as different load conditions. The identified model will be employed to apply a nonlinear least squares algorithm, using the source separation results as initialization. This study serves as a foundation for future research on source tracking to achieve gearbox diagnostics, particularly by monitoring the evolution of source magnitudes and

their harmonics. Additionally, our work lays the groundwork for developing demodulation techniques for vibration data, enabling practitioners to directly access elementary source separation for improved analysis

Planetary gearbox

In this experiment, we aim to develop an analytical model that accurately fits the vibration data of a planetary gearbox, captured by an accelerometer. The dataset was recorded during a fatigue test on a helicopter planetary gearbox as part of the HUMS 2023 Challenge. The test was designed to investigate the phenomenon of fatigue cracking in a planet gear. The sampling frequency of the measurements is $f_s = 11583 \text{ Hz}$. The characteristics of the planetary gearbox are as follows: the ring gear has 99 teeth, the planet gear has 35 teeth, the sun gear has 27 teeth, and the system includes five planet gears. Based on the speed measurements and the kinematic model of the system, the key rotational frequencies are determined as follows: the gear meshing frequency is $f_m = 99 \text{ Hz}$, the carrier plate rotation frequency is $f_c = 1 \text{ Hz}$, the planet gear rotation frequency is $f_p = 2.28 \text{ Hz}$, and the sun gear rotation frequency is $f_s = 4.66 \text{ Hz}$.

To obtain an analytical model, we applied our robust modeling method to a measurement from the HUMS dataset. We started the processing by detecting all harmonic locations in the vibration signal spectrum. The signal x , which will be used in the multivariate regression, is generated based on the detected peaks and is shown in Figure 3.20. It can be observed by inspecting the spectrum that some peaks of the modulation sidebands (around the fourth harmonic in Fig. 3.20) are missing, as their amplitudes are too low to be detected. This might affect the identification of the final expression that fits the data.

The expression identified after running the multivariate regression, where the regularization is empirically chosen, is as follows:

$$\begin{aligned} x(t) = & 7.3 \cdot s_m(t) (-0.35 - 2.38s_c(t)) (1.2s_p(t) - 0.4s_s(t)) \\ & \cdot (0.35 + 0.8s_c(t) + 0.4s_p(t) - 0.6s_s(t)) \end{aligned} \quad (3.26)$$

The reconstructed signal $\mathbf{x}$, zoomed in on the fourth meshing harmonic, is shown in Figure 3.21. The peaks are not all well reconstructed due to the fact that the sidebands of the meshing harmonics are not identical because of transfer function effects. As a result, our framework attempts to empirically find a model that fits the sidebands across all the meshing harmonics. Additionally, our framework is built upon prior knowledge from

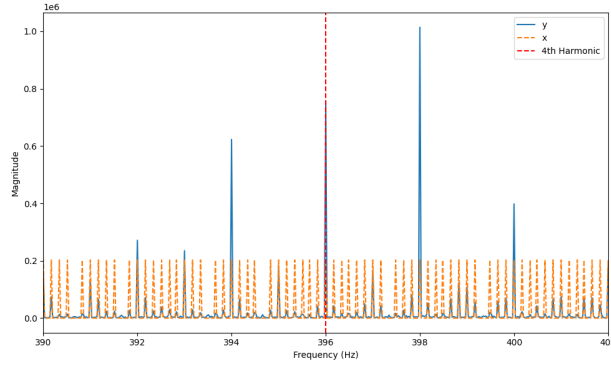


Figure 3.20: The signal $\mathbf{x}$, derived from the detected peaks of the raw signal $\mathbf{y}$, used for our multivariate regression.

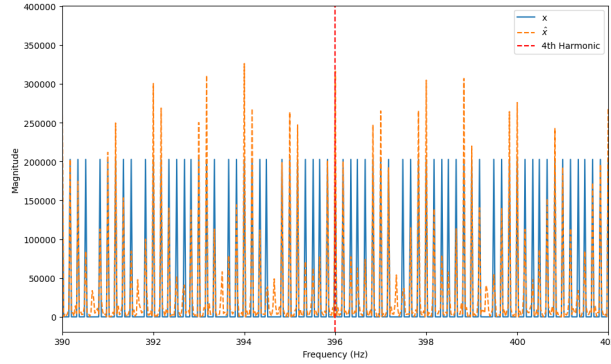


Figure 3.21: Reconstruction of the spectrum peaks in signal $\mathbf{x}$ using the proposed neural network, denoted as $\hat{\mathbf{x}}$ around the fourth meshing harmonic.

ad-hoc models, where a multi-modulation model is expected for a planetary gearbox. However, the final expression extracted from the neural network weights (Equation (3.26)) reveals interactions such as s_c^2 and s_p^2 . These interactions may stem from the physical coupling between the related planet components. In particular, this illustrates one of the key advantages of our method as it allows revealing coupling and physical interactions between source signals not taken into account by previous ad hoc modeling techniques.

This matter will be the focus of our future work, where we will extend the current work to build a digital twin model for the planetary gearbox that will decode all the meshing points of the vibration signal, including the sun/planet meshing. We seek to examine whether the results obtained here hold or if a different model expression emerges, potentially uncovering unseen

couplings and interactions between the components (planets) in the vibration signal. In this regard, we will enrich our framework with all available physical prior information about the system, including the various possible meshing points.

3.7 Conclusion

In this chapter, we developed an EQL-based architecture for modeling vibration responses in gear transmission systems with epicyclic motion. The proposed end-to-end framework not only achieves accurate signal reconstruction but also yields compact and interpretable symbolic expressions that explicitly characterize the system’s dynamic behavior. We introduced components designed to account for transmission path effects and multiple excitation mechanisms.

Convolution layers were integrated to implicitly estimate the system transfer function during the learning process. Moreover, to mitigate transmission path effects, a peak detection module was employed to estimate power spectrum harmonics, which were subsequently used for symbolic regression. This approach enables a robust understanding of the vibration spectrum structure, as the modeling process focuses only on the locations of the spectrum harmonics.

We proposed a modeling framework that treats the problem as a multi-input-single-output system, considering the system kinematic model. Multiple symbolic regression-based neural networks were concatenated to reconstruct the measured response. Each network was designed to model a distinct excitation source derived from the kinematic model, and their contributions were summed to accurately reproduce the observed vibration signal. The simulation results illustrate the effectiveness of our approach for both modeling and source separation tasks, even under low SNR. A validation study was conducted on the EQL-based architecture for planetary gear systems, serving as an initial step toward a more comprehensive validation, particularly of our MISO architecture in the future.

Chapter 4

Blind System Identification

The faulty information in the vibration recordings is often modified and attenuated by transfer function effects, in particular for local faults, such as cracks, where the duration of the excitation is short. The waveform of the excitation at the meshing point is important to detect and localize the faulty elements. Several works have been proposed recently to identify and remove the transfer functions' effects to reconstruct the force functions (excitations) at the meshing point. Recently, a cepstrum-based Modal analysis method has been proposed to identify a pole/zero model that represents the transfer function. These approaches succeed in removing the amplitude and phase effects due to the transfer function on the force functions to reconstruct the true excitation waveform in minimum-phase system conditions. They assume that the system has random broadband (smooth, flat) spectrum components occupies wider frequencies, that validate the use of the OMA cepstrum-based method [50].

In contrast, this work exploits deterministic sidebands in the vicinity of the vibration meshing (carrier) frequency and its harmonics of the measured signal of gear sets with epicyclic motion, such as a planetary gear set. The vibration signal is commonly modeled using an amplitude modulation model to describe the sidebands observed in the vibration spectrum, which generally fits the analysis of healthy gearboxes [6, 8]. However, this may be insufficient for the investigation of certain faulty gearboxes. In such situations, an Amplitude-Frequency Modulation (AM-FM) model provides a more comprehensive description of the spectral content, particularly when the gearbox is affected by manufacturing errors, gear damages, or other defects [9, 11, 10, 12, 72]. The amplitude modulation phenomena impart first-order cyclostationary characteristics to the signal. In this study, the investigation focuses on a

normal gearbox where the AM-modulation model applies. The transmission path, represented by the system transfer function (channel), alters the magnitudes of the excitation responses while preserving the structure of the vibration signal sidebands. For vibration signals, removing transfer function effects from vibration measurements has been a vital task in condition monitoring, as these effects alter the shape of weak excitation forces that occur over short durations and are more indicative of faults. In the literature, minimum entropy deconvolution-based methods (which identify a set of filter coefficients for impulsive excitation forces), exponential liftering, adaptive clutter separation, and cepstrum-based Operational Modal Analysis (OMA) methods have been widely investigated. The development of transfer-function removal techniques or modal parameter identification methods depends on several factors, particularly (i) their ability to remove the amplitude and phase effects of the transfer function in order to reconstruct the waveform of the excitations, (ii) operate under different excitations not only impulsive one (iv) handle harmonic deterministic components that can be misinterpreted as damped modes in OMA methods [73, 51, 50].

In this work, we propose a method for identifying the system transfer function by restoring the intrinsic symmetry of the spectral sidebands under an amplitude-modulation framework. This formulation leads to a nonlinear least-squares optimization problem, which is solved using the Alternating Direction Method of Multipliers (ADMM). The spectral peaks are represented through analytical expressions derived from the characteristic frequencies and a modulation model that captures the sideband structure, resulting in a nonlinear optimization problem.

4.1 Problem statement

Consider the vibration signal of a rotating machine, described by the following equation:

$$y(t) = h(t) * s(t) + \epsilon(t), \quad 0 \leq t \leq T - 1 \quad (4.1)$$

where $*$ denotes the convolution operator, $\mathbf{h}$ represents the impulse response of the system's transfer function modeling the transmission path of system excitations, and $\mathbf{s}$ is the excitation signal defined as

$$s(t) = s_1(t) \prod_{i=2}^m (1 + s_i(t)) \quad (4.2)$$

which represents amplitude modulation (AM) between m periodic source signals $\mathbf{s}_{1 \leq i \leq m}$ with estimated (or a priori known) fundamental characteristic frequencies f_i , $i = 1, \dots, m$, typically related to the machine's rotation frequencies. ϵ is an additive white Gaussian noise with zero-mean and variance σ_n^2 .

In this work, two key aspects are used to develop our method:

1. the specific multi-modulation data model in (4.2), characteristic of the vibration signal of rotating machines (e.g., [12, 74]),
2. the periodic nature of the elementary source signals arising from the rotation of the system components.

By using Fourier series decomposition, the source signals can be expressed in stationary conditions as follows:

$$\mathbf{s}_i(t) = \sum_{l=1}^{M_i} a_{i,l}^c e^{j2\pi l f_i t} + \bar{a}_{i,l}^c e^{-j2\pi l f_i t} \quad (4.3)$$

where M_i represents the number of effective (significant) harmonics of the i -th source signal and

$$a_{i,l}^c = a_{i,l} e^{j\phi_{i,l}} / (2j)$$

denotes the unknown complex amplitude of its l -th harmonic. $\bar{a}$ is the complex conjugate of a . In the sequel, for simplicity, we treat the case $m = 2$ where $s(t) = s_1(t)(1 + s_2(t))$. A similar procedure can be applied with the frequency peaks of modulations of $s_1(t)s_m(t)$, when considering $m > 2$.

For later convenience, define the vectors

$$\mathbf{a}_i^c = [a_{i,1}^c, \dots, a_{i,M_i}^c], \quad i \in \{1, 2\}.$$

Furthermore, let

$$\mathbf{a}_r = [a_{1,1}, \dots, a_{1,M_1}, a_{2,1}, \dots, a_{2,M_2}]^T, \quad (4.4)$$

$$\boldsymbol{\phi} = [\phi_{1,1}, \dots, \phi_{1,M_1}, \phi_{2,1}, \dots, \phi_{2,M_2}]^T, \quad (4.5)$$

which are $(M_1 + M_2)$ -dimensional vectors collecting all magnitude and phase parameters, respectively.

Using this mathematical formulation of the vibration signal, we develop a method tailored for rotating machines to perform blind estimation of the system's transfer function and, consequently, the source signals s_i .

4.2 Estimation Bound

In this section, we derive the expressions of the Fisher information matrix (FIM) and the Cramer-Rao Bound (CRB) associated with our estimation problem for Infinite and finite impulse responses. This derivation has in fact two objectives: (i) the first is to use the CRB to assess and benchmark the performance of our algorithm; (ii) the second is to get a basic investigation of the identifiability conditions for our problem using the FIM rank. Indeed, the dimension of the FIM Kernel reveals the number of undetermined parameters of our problem.

4.2.1 Finite Impulse Response (FIR)

Consider FIR filter $\mathbf{h} = [h(0), \dots, h(d-1)]^T$, the statistical model at hand is a deterministic Gaussian process given by:

$$\mathbf{y} = \boldsymbol{\mu} + \boldsymbol{\epsilon} = \mathbf{H}\mathbf{D}_r\tilde{\mathbf{a}}_r + \boldsymbol{\epsilon} \quad (4.6)$$

where $\boldsymbol{\epsilon}$ is the noise zero-mean Gaussian vector of covariance $\sigma^2\mathbf{I}$,

$$\mathbf{D}_r = 2[\Re(\mathbf{D}), -\Im(\mathbf{D})]$$

($\mathbf{D}$ is defined as previously but with $T+d-1$ rows, accounting for the sample size plus channel memory),

$$\tilde{\mathbf{a}}_r = [\Re(\tilde{\mathbf{a}}^c), \Im(\tilde{\mathbf{a}}^c)]^T$$

where

$$\tilde{\mathbf{a}}^c = [\mathbf{a}_1^c, \mathbf{a}_1^c \otimes \mathbf{a}_2^c, \mathbf{a}_1^c \otimes \bar{\mathbf{a}}_2^c]^T,$$

and $\mathbf{H}$ is the $T \times (T+d-1)$ Toeplitz transfer function (channel) matrix given by

$$\mathbf{H} = \begin{bmatrix} h(0) & \cdots & h(d-1) & \cdots & 0 \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & h(d-1) & \cdots & h(0) \end{bmatrix}. \quad (4.7)$$

In this case, the parameter vector of our model is $\boldsymbol{\theta} = [\mathbf{h}^T, \mathbf{a}_r^T, \phi^T, \sigma^2]^T$. The latter parameter, i.e. σ^2 can be disregarded as, in our context, its estimation has no impact on the lower bound of the other desired parameters [75].

4.2.2 Infinite Impulse Response (IIR)

Define a 1D IIR filter

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^M b_i z^{-i}}{\sum_{i=0}^N a_i z^{-i}}, \quad a_0 = 1. \quad (4.8)$$

We consider the Gaussian process model as above

$$\mathbf{y} = \boldsymbol{\mu} + \boldsymbol{\epsilon}, \quad (4.9)$$

where $\boldsymbol{\epsilon}$ is zero-mean Gaussian noise. The mean $\boldsymbol{\mu}$ is defined implicitly by the difference equation

$$\mathbf{A}\boldsymbol{\mu} = \mathbf{B}\mathbf{s}, \quad (4.10)$$

or equivalently,

$$\boldsymbol{\mu} = \mathbf{A}^{-1}\mathbf{B}\mathbf{s}. \quad (4.11)$$

The matrices $\mathbf{A}$ and $\mathbf{B}$ are circulant matrices constructed from the filter coefficients,

$$\mathbf{A} = \text{circ}([1, a_1, \dots, a_N, 0, \dots]), \quad (4.12)$$

$$\mathbf{B} = \text{circ}([b_0, b_1, \dots, b_M, 0, \dots]). \quad (4.13)$$

Since circulant matrices are diagonalized by the discrete Fourier transform (DFT) matrix $\mathbf{F}$,

$$\mathbf{A} = \mathbf{F}^{-1} \text{diag}(\hat{\mathbf{a}}) \mathbf{F}, \quad (4.14)$$

$$\mathbf{B} = \mathbf{F}^{-1} \text{diag}(\hat{\mathbf{b}}) \mathbf{F}, \quad (4.15)$$

where $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ denote the DFTs of the first columns of $\mathbf{A}$ and $\mathbf{B}$, respectively. If all entries of $\hat{\mathbf{a}}$ are nonzero, the inverse of $\mathbf{A}$ is

$$\mathbf{A}^{-1} = \mathbf{F}^{-1} \text{diag}(1/\hat{\mathbf{a}}) \mathbf{F}. \quad (4.16)$$

Therefore, the mean can be written as

$$\boldsymbol{\mu} = \mathbf{F}^{-1} \text{diag}(\hat{\mathbf{b}}/\hat{\mathbf{a}}) \mathbf{F}\mathbf{s}. \quad (4.17)$$

Differentiating $\mathbf{A}\boldsymbol{\mu} = \mathbf{B}\mathbf{s}$ yields

$$d\boldsymbol{\mu} = \mathbf{A}^{-1} \left[(d\mathbf{B})\mathbf{s} - (d\mathbf{A})\boldsymbol{\mu} \right] = \mathbf{F}^{-1} \text{diag}(1/\hat{\mathbf{a}}) \mathbf{F} \left[(d\mathbf{B})\mathbf{s} - (d\mathbf{A})\boldsymbol{\mu} \right]. \quad (4.18)$$

Remark: The circulant formulation enables efficient inversion of $\mathbf{A}$ and fast derivative computation using FFTs, unlike lower-triangular Toeplitz representations. From this expression, the partial derivatives with respect to the filter coefficients follow immediately:

$$\frac{\partial \boldsymbol{\mu}}{\partial b_i} = \mathbf{A}^{-1} \mathbf{C}_i \mathbf{s}, \quad \frac{\partial \boldsymbol{\mu}}{\partial a_i} = -\mathbf{A}^{-1} \mathbf{C}_i \boldsymbol{\mu}, \quad (4.19)$$

where

$$\mathbf{C}_i = \text{circ}(\mathbf{e}_i), \quad (4.20)$$

and $\mathbf{e}_i$ denotes the unit vector with a one at lag i .

4.2.3 Fisher matrix

The FIM matrix, denoted $\mathbf{FIM}$, for this deterministic Gaussian model is given by [76]:

$$\mathbf{FIM}_{kl} = \frac{1}{\sigma^2} \frac{\partial \boldsymbol{\mu}^T}{\partial \theta_k} \frac{\partial \boldsymbol{\mu}}{\partial \theta_l} \quad (4.21)$$

The computation of this expression is straightforward and omitted here. As mentioned previously, $\mathbf{FIM}$ will be used next for an elementary investigation (i.e., not a full proof) of the channel identifiability conditions. $\mathbf{FIM}$ is singular due to the scalar factor undeterminacy inherent to our blind identification problem, i.e., $y = h \star s = (\alpha h) \star (\frac{1}{\alpha} s)$ for any non-zero scalar factor α . Hence, we have to resort to the constrained CRB or the pseudo-inverse of the FIM $\mathbf{FIM}$ (see [75] for details),

$$CRB = \mathbf{FIM}^\#,$$

to get the desired lower bound.

4.2.4 Identifiability Analysis

Define the frequency sets based on the pre-defined model :

$$\mathbf{f}_1 := \{kf_1 : k = 1, \dots, M_1\}, \quad \mathbf{f}_{1,2} := \{kf_1 \pm mf_2 : k, m = 1, \dots, M_2\} \quad (4.22)$$

The unknown amplitudes and phases of each source is identifiable if:

- are disjoint modulo $2\pi f_s$, i.e. all the complex exponentials $e^{j2\pi f_i t}$ appearing in the signal $s_1(t)(1 + s_2(t))$ have distinct frequencies.

The filter $\mathbf{h}$ is identifiable up to a global complex scale from the observed output $y(t)$ and knowledge of $\{f_1, f_2\}$ where :

- $K^2 + K \geq L$, i.e. we have at least L independent complex exponentials in the observed signal.

Under these generic conditions, the amplitudes and phases of both $s_1(t)$, $s_2(t)$, and $\mathbf{h}$ can be uniquely recovered by analyzing the separate spectral lines at each distinct frequency.

4.3 Neural Network-based System Identification

The modeling procedure described in Section 3 yields outcomes beyond the recovery of analytical equations that describe the vibration data. The architecture of the proposed neural-network-based symbolic regression model employs convolutional layers designed to de-emphasize the effects of the transfer function. Both finite and infinite impulse responses are considered: a single convolutional layer is used for the FIR case (Fig. 3.6), while two convolutional layers were used for the IIR case (Fig. 3.7).

Following the neural network training and thresholding of all weights except those in the convolution layers, a refinement phase is performed as outlined in the training procedure above. The weights of the convolution kernels are extracted and interpreted as estimates of the FIR or IIR filters. The transfer function fit is successful when the neural-network-based symbolic regression accurately identifies the underlying equations governing the given data. This can be applied to the TF identification of single excitations(single meshing point) for a pair of gears.

Furthermore, our latest architecture, a stacked neural network-based symbolic regression (Fig. 3.8), is designed to model multiple excitations, specifically the planet-ring and planet-sun meshing points separately, given the system kinematics. Each neural network recovers the equation describing an excitation point and simultaneously learns the weights of a FIR filter representing the propagation path. The convolved outputs are then summed to reconstruct the overall response. This novel approach aims to identify the propagation path of each excitation using only a single vibration measurement. To our knowledge, this is the first method capable of identifying multiple transfer functions from a single set of observed outputs. This approach can

be regarded as an automatic modeling and system-identification framework capable of adapting to varying conditions. It simultaneously characterizes the system's interactions on the basis of its kinematics and propagation path, even as the system's response varies across operation conditions.

4.4 System identification with Structured Line Spectrum

The power spectrum of the noiseless signal in (4.1) is given by:

$$Y(f) = H(f)X(f) \quad (4.23)$$

where $S(f)$ is the Fourier transform of $\mathbf{s}$, and $H(f)$ is the frequency response

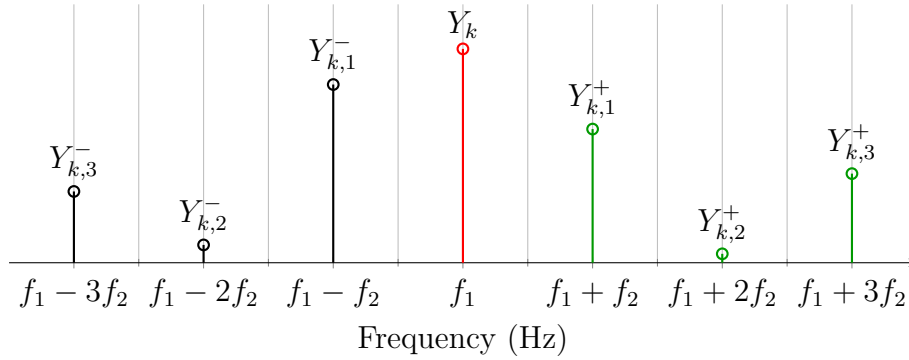


Figure 4.1: Amplitude spectrum illustration zoomed around a single carrier.

of the filter $\mathbf{h}$. The spectral representation of $\mathbf{Y}$ can be expressed as follows (see Fig.4.1) :

$$\begin{aligned} Y_{k,l}^+ &= H_{k,l}^+ a_{1,k}^c a_{2,l}^c, \\ Y_k &= H_k a_{1,k}^c, \\ Y_{k,l}^- &= H_{k,l}^- a_{1,k}^c \bar{a}_{2,l}^c \end{aligned} \quad (4.24)$$

which $(\cdot)_k$, $(\cdot)_{k,l}^+$, $(\cdot)_{k,l}^-$ represent the complex magnitudes at $k f_1$ and $k f_1 + l f_2$ and $k f_1 - l f_2$ respectively.

Afterward, based on Y_k , we can derive the expression $a_{1,k}^c = Y_k / H_k$ that can be plugged in equations $Y_{k,l}^+$ and $Y_{k,l}^-$ to obtain:

$$\begin{aligned} H_k Y_{k,l}^- &= H_{k,l}^- Y_k \bar{a}_{2,k}^c, \\ H_k Y_{k,l}^+ &= H_{k,l}^+ Y_k a_{2,k}^c \end{aligned} \quad (4.25)$$

which can be rewritten as:

$$a_{2,k}^c = \frac{H_k Y_{k,l}^+}{H_{k,l}^+ Y_k}, \quad \bar{a}_{2,k}^c = \frac{H_k Y_{k,l}^-}{H_{k,l}^- Y_k} \Rightarrow a_{2,k}^c = \frac{\bar{H}_k \bar{Y}_{k,l}^-}{\bar{H}_{k,l}^- \bar{Y}_k} \quad (4.26)$$

Consequently, for all k, l , we have the following equality:

$$\bar{Y}_k Y_{k,l}^+ H_k \bar{H}_{k,l}^- = \bar{Y}_{k,l}^- Y_k \bar{H}_k H_{k,l}^+ \quad (4.27)$$

This leads to a set of $M_1 M_2$ equations that will be used to compute the channel impulse response $\mathbf{h}$. Note that in the blind case, the channel impulse response can be estimated only up to a constant (complex) factor and a delay, representing an inherent indeterminacy [77] of the considered problem.

4.4.1 Spectrum line magnitudes estimation

The complex amplitudes of the spectrum peaks are determined on the basis of prior information on the fundamental frequencies of the elementary sources. These characteristic frequencies can be either estimated or determined on the basis of tachometer measurements (shaft rotation speed) and the machine's kinematic model. Using classical least squares estimation, we compute the vector of all complex amplitudes, denoted

$$\mathbf{a} = [\mathbf{Y}_{k,l}^-, \mathbf{Y}_k, \mathbf{Y}_{k,l}^+]^T,$$

as the solution of

$$\begin{bmatrix} \mathbf{a} \\ \bar{\mathbf{a}} \end{bmatrix} = [\mathbf{D}, \bar{\mathbf{D}}]^\# \mathbf{y} \quad (4.28)$$

where $^\#$ denotes the pseudo-inverse, $\mathbf{y} = [y(0), \dots, y(T-1)]^T$, and matrix $\mathbf{D}$ is the matrix formed by all vectors $\mathbf{d}_{k,l}$, $1 \leq k \leq M_1$ and $0 \leq |l| \leq M_2$:

$$\mathbf{d}_{k,l} = \left[e^{2j\pi(kf_1 + lf_2)} \dots, e^{2j\pi(T-1)(kf_1 + lf_2)} \right]^T. \quad (4.29)$$

4.4.2 FIR case

Let's consider an FIR system and let $\mathbf{h} = [h(0), \dots, h(d-1)]^T$ be the real-valued vector of non-zero impulse response taps, d being the filter length

(system's memory size). The frequency response of the filter can be written as:

$$\begin{aligned} H_k &= \mathbf{e}_{k,0}^T \mathbf{h}, \\ H_{k,l}^+ &= \mathbf{e}_{k,l}^T \mathbf{h}, \\ H_{k,l}^- &= \mathbf{e}_{k,-l}^T \mathbf{h} \end{aligned} \quad (4.30)$$

where $\mathbf{e}_{k,l} = [1, e^{-2j\pi(kf_1+lf_2)}, \dots, e^{-2j\pi(d-1)(kf_1+lf_2)}]^T$.

Replacing (4.30) in (4.27) leads to:

$$\bar{Y}_k Y_{k,l}^+ \mathbf{h}^T \mathbf{e}_{k,0} \mathbf{e}_{-k,l}^T \mathbf{h} - \mathbf{h}^T Y_k \bar{Y}_{k,l}^- \mathbf{e}_{-k,0} \mathbf{e}_{k,l}^T \mathbf{h} = 0 \quad (4.31)$$

Then,

$$\mathbf{h}^T (\mathbf{M}_{kl} - \mathbf{N}_{kl}) \mathbf{h} = 0 \quad (4.32)$$

where

$$\mathbf{M}_{k,l} = \bar{Y}_k Y_{k,l}^+ \mathbf{e}_{k,0} \mathbf{e}_{-k,l}^T, \quad \mathbf{N}_{k,l} = Y_k \bar{Y}_{k,l}^- \mathbf{e}_{-k,0} \mathbf{e}_{k,l}^T.$$

Our problem is formulated as minimizing the function:

$$f(\mathbf{h}) = \sum_{k,l} \|\mathbf{h}^T (\mathbf{M}_{kl} - \mathbf{N}_{kl}) \mathbf{h}\|^2 \quad (4.33)$$

which is a nonlinear least-squares (NLS) problem. To solve this NLS minimization, we propose to rewrite the problem as a constrained optimization problem, according to:

$$\begin{aligned} f(\mathbf{h}_L, \mathbf{h}_R) &= \sum_{k,l} \|\mathbf{h}_L^T (\mathbf{M}_{kl} - \mathbf{N}_{kl}) \mathbf{h}_R\|^2, \\ \text{s.t. } \mathbf{h}_R &= \mathbf{h}_L \end{aligned} \quad (4.34)$$

To achieve this constrained optimization, the ADMM is considered, as detailed next in section 4.4.5.

4.4.3 IIR case

Consider the case of a rational transfer function given by:

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=1}^{N_1} b_i z^{-i}}{\sum_{i=1}^{N_2} a_i z^{-i}} \quad (4.35)$$

where N_1 and N_2 are the degrees of the numerator and denominator polynomials, respectively. The coefficients b_i and a_i are real constants, with $a_0 = 1$

by convention. Due to the inherent scalar indeterminacy of the identification problem, we set $b_0 = 1$ to get rid of this ambiguity. The frequency response of the IIR filter is expressed as :

$$H_k = \frac{B_k}{A_k} = \frac{\mathbf{e}_{k,0}^T \mathbf{b}}{\mathbf{e}_{k,0}^T \mathbf{a}}$$

$$H_{k,l}^+ = \frac{B_{k,l}^+}{A_{k,l}^+} = \frac{\mathbf{e}_{k,l}^T \mathbf{b}}{\mathbf{e}_{k,l}^T \mathbf{a}}, \quad H_{k,l}^- = \frac{B_{k,l}^-}{A_{k,l}^-} = \frac{\mathbf{e}_{k,-l}^T \mathbf{b}}{\mathbf{e}_{k,-l}^T \mathbf{a}} \quad (4.36)$$

where $\mathbf{a} = [a_0, \dots, a_{N_2}]^T$ and $\mathbf{b} = [b_0, \dots, b_{N_1}]^T$. Plugging the transfer function expression (4.36) in (4.27) leads to:

$$\bar{Y}_k Y_{k,l}^+ B_k \bar{B}_{k,l}^- \bar{A}_k A_{k,l}^+ = \bar{Y}_{k,l}^- Y_k \bar{B}_k B_{k,l}^+ A_k \bar{A}_{k,l}^- \quad (4.37)$$

Using the Kronecker product properties, we rewrite (4.37) as:

$$\boldsymbol{\alpha}^T \mathbf{Q}_{k,l} \boldsymbol{\alpha} = \boldsymbol{\alpha}^T \mathbf{V}_{k,l} \boldsymbol{\alpha} \quad (4.38)$$

where $\boldsymbol{\alpha} = \mathbf{a} \otimes \mathbf{b}$, and

$$\mathbf{Q}_{k,l} = \bar{Y}_k Y_{k,l}^+ (\mathbf{e}_{-k,0} \otimes \mathbf{e}_{k,0}) (\mathbf{e}_{k,l} \otimes \mathbf{e}_{-k,l})^T, \quad \mathbf{V}_{k,l} = \bar{Y}_{k,l}^- Y_k (\mathbf{e}_{k,0} \otimes \mathbf{e}_{-k,0}) (\mathbf{e}_{-k,l} \otimes \mathbf{e}_{k,l})^T.$$

A solution $\boldsymbol{\alpha}$ of (4.38) is estimated by minimizing the following function:

$$g(\boldsymbol{\alpha}) = \sum_{k,l} \|\boldsymbol{\alpha}^T (\mathbf{Q}_{k,l} - \mathbf{V}_{k,l}) \boldsymbol{\alpha}\|^2 \quad (4.39)$$

As done previously, we propose to express (4.39) as a constrained optimization problem according to

$$g(\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L) = \sum_{k,l} \|\boldsymbol{\alpha}_L^T (\mathbf{Q}_{k,l} - \mathbf{V}_{k,l}) \boldsymbol{\alpha}_R\|^2, \text{ s.t. } \boldsymbol{\alpha}_L = \boldsymbol{\alpha}_R$$

Once we obtain the vector $\boldsymbol{\alpha}$, we can reshape it into an $N \times N$ matrix $\mathbf{P}$ from which, we can compute vector $\mathbf{a}$ (resp. $\mathbf{b}$) as the principal left, $\mathbf{u}_1$ (resp. right, $\mathbf{v}_1$) singular vector of $\mathbf{P}$, i.e.

$$\mathbf{a} = \mathbf{u}_1 \times \sqrt{\Sigma_{1,1}}, \quad \mathbf{b} = \mathbf{v}_1 \times \sqrt{\Sigma_{1,1}}, \quad (4.40)$$

$\Sigma_{1,1}$ being the corresponding principal singular value.

4.4.4 Least square

An alternative approach for solving this nonlinear problem has been introduced. We linearize the problem by utilizing the Kronecker product. Accordingly, both Equations (4.31) and (4.38) can be expressed in terms of the Kronecker product, respectively, as:

$$\begin{aligned} & (Y_{k,l} \bar{X}_k \mathbf{e}_{-k,l} \otimes \mathbf{e}_{k,0} - X_k \bar{Z}_k \mathbf{e}_{k,l} \otimes \mathbf{e}_{-k,0})^T \mathbf{v} = 0 \\ & \left(\bar{Y}_k Y_{k,l}^+ (\mathbf{e}_{k,l} \otimes \mathbf{e}_{-k,l}) \otimes (\mathbf{e}_{-k,0} \otimes \mathbf{e}_{k,0}) \right. \\ & \quad \left. - \bar{Y}_{k,l}^- Y_k (\mathbf{e}_{-k,l} \otimes \mathbf{e}_{k,l}) \otimes (\mathbf{e}_{k,0} \otimes \mathbf{e}_{-k,0}) \right)^T (\boldsymbol{\alpha} \otimes \boldsymbol{\alpha}) = 0 \end{aligned} \quad (4.41)$$

with $\mathbf{v} = \mathbf{h} \otimes \mathbf{h}$. Thus, the solutions $\mathbf{v}$ and $\boldsymbol{\alpha} \otimes \boldsymbol{\alpha}$ are the least singular vectors of the previous quadratic forms. As will be shown in our simulation results, this solution fails to provide the desired channel estimate when the filter size increases.

4.4.5 Alternating least square

Instead of jointly solving the nonlinear cost function w.r.t. $\mathbf{h}_r$ and $\mathbf{h}_l$ (or $\boldsymbol{\alpha}_r$ and $\boldsymbol{\alpha}_l$), we propose using alternating optimization to solve our problem more efficiently, with a closed-form solution at each step. The Alternating Direction Method of Multipliers (ADMM) [78, 79] has become a very popular approach to solving a wide variety of optimization problems in signal and image processing, prominent examples including total variation regularization and sparse representation problems. For convex problems, the convergence of ADMM has been established under various settings [78]. When the objective function is nonconvex, recent studies have focused on specific classes of nonconvex models. In our case, we consider a nonconvex fidelity term in the objective function. The convergence guarantees for this particular nonconvex scenario will be addressed in an extended version of this letter.

FIR case

The augmented Lagrangian for our constrained problem is expressed as:

$$L_\rho(\mathbf{h}_R, \mathbf{h}_L, \mathbf{w}) = f(\mathbf{h}_R, \mathbf{h}_L) + \mathbf{w}^T(\mathbf{h}_R - \mathbf{h}_L) + \frac{\rho}{2} \|\mathbf{h}_R - \mathbf{h}_L\|_2^2$$

where $\mathbf{h}_R \in R^L$ and $\mathbf{h}_L \in R^L$ are the primal variables, $\mathbf{w}$ being the dual variable respectively, and ρ a non-negative penalty parameter. ADMM alternates the variable updates (thus the alternating direction), according to

$$\begin{aligned}\mathbf{h}_R^{k+1} &= \underset{\mathbf{h}_R}{\operatorname{argmin}} L_\rho(\mathbf{h}_R, \mathbf{h}_L^k, y^k) \\ \mathbf{h}_L^{k+1} &= \underset{\mathbf{h}_L}{\operatorname{argmin}} L_\rho(\mathbf{h}_R^k, \mathbf{h}_L, y^k) \\ \mathbf{w}^{k+1} &= \mathbf{w}^k + \rho(\mathbf{h}_R^{k+1} - \mathbf{h}_L^{k+1})\end{aligned}$$

It is often more convenient to work with the scaled form of ADMM, which is obtained by the change of variable to the scaled dual variable $\mathbf{u}$. Defining the residual $\mathbf{r} = \mathbf{h}_R - \mathbf{h}_L$ and replacing $\mathbf{w}$ with $\mathbf{u}$ we obtain:

$$L_\rho(\mathbf{h}_R, \mathbf{h}_L, \mathbf{u}) = f(\mathbf{h}_R, \mathbf{h}_L) + \frac{\rho}{2} \|\mathbf{r} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2 \quad (4.42)$$

and the ADMM iterations can be written as:

$$\begin{aligned}\mathbf{h}_R^{(k+1)} &= \underset{\mathbf{h}_R}{\operatorname{argmin}} f(\mathbf{h}_R, \mathbf{h}_L^{(k)}) + \frac{\rho}{2} \|\mathbf{h}_R - \mathbf{h}_L^{(k)} + \mathbf{u}^{(k)}\|_2^2 \\ \mathbf{h}_L^{(k+1)} &= \underset{\mathbf{h}_L}{\operatorname{argmin}} f(\mathbf{h}_R^{(k+1)}, \mathbf{h}_L) + \frac{\rho}{2} \|\mathbf{h}_R^{(k+1)} - \mathbf{h}_L + \mathbf{u}^{(k)}\|_2^2 \\ \mathbf{u}^{(k+1)} &= \mathbf{u}^{(k)} + \mathbf{h}_R^{(k+1)} - \mathbf{h}_L^{(k+1)}\end{aligned} \quad (4.43)$$

IIR case

In the case of the IIR filter, by construction, there is no guarantee that the optimized parameters $\boldsymbol{\alpha}_R$ (or $\boldsymbol{\alpha}_L$) will coincide with $\mathbf{a} \otimes \mathbf{b}$, as confirmed numerically. To address this issue, we restrict the solution set by requiring the reshaped $\boldsymbol{\alpha}_R$ to form a low-rank $N \times N$ matrix. To achieve this, we introduce a nuclear-norm (denoted $\|\cdot\|_*$) penalty into the objective function. The penalty is applied to the reshaped matrix $\operatorname{unvec}(\boldsymbol{\alpha}^{(k)}) \in \mathbb{R}^{L \times L}$ to enforce it to be of a low rank (ideally of rank one). Then, the problem restatement is given by

$$\begin{aligned}\arg \min_{\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L} \quad & g(\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L) + \lambda \|\operatorname{unvec}(\boldsymbol{\alpha}_R)\|_* \\ \text{s.t.} \quad & \boldsymbol{\alpha}_R = \boldsymbol{\alpha}_L\end{aligned} \quad (4.44)$$

where $\lambda > 0$ controls the strength of the low-rank regularization. Accordingly, the augmented Lagrangian for (4.44) is

$$\begin{aligned}L_\rho(\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L, \mathbf{Z}, \mathbf{u}, \mathbf{w}) &= g(\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L) + \lambda \|\mathbf{Z}\|_* \\ &+ \rho \|\boldsymbol{\alpha}_R - \boldsymbol{\alpha}_L + \mathbf{u}\|_2^2 + \eta \|\boldsymbol{\alpha}_R - \operatorname{vec}(\mathbf{Z}) + \mathbf{w}\|_2^2,\end{aligned} \quad (4.45)$$

where $\mathbf{Z}$ is an auxiliary variable, $\mathbf{u}, \mathbf{w} \in \mathbb{R}^{nm}$ are the scaled dual variables associated with the constraints $\boldsymbol{\alpha}_R = \boldsymbol{\alpha}_L$ and $\boldsymbol{\alpha}_R = \text{vec}(\mathbf{Z})$, and $\rho, \eta > 0$ are the corresponding penalty parameters. Thus, the iterations are updated as follows:

$$\begin{aligned}\boldsymbol{\alpha}_L^{(k+1)} &= \arg \min_{\boldsymbol{\alpha}_L} L_\rho(\boldsymbol{\alpha}_R^{(k)}, \boldsymbol{\alpha}_L, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}), \\ \boldsymbol{\alpha}_R^{(k+1)} &= \arg \min_{\boldsymbol{\alpha}_R} L_\rho(\boldsymbol{\alpha}_R, \boldsymbol{\alpha}_L^{(k+1)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}), \\ \mathbf{Z}^{(k+1)} &= \text{SVT}_\tau(\text{unvec}(\boldsymbol{\alpha}_R^{(k+1)} + \mathbf{w}^{(k)})), \\ \mathbf{u}^{(k+1)} &= \mathbf{u}^{(k)} + (\boldsymbol{\alpha}_R^{(k+1)} - \boldsymbol{\alpha}_L^{(k+1)}), \\ \mathbf{w}^{(k+1)} &= \mathbf{w}^{(k)} + (\boldsymbol{\alpha}_R^{(k+1)} - \text{vec}(\mathbf{Z}^{(k+1)})).\end{aligned}$$

The $\mathbf{Z}$ -update corresponds to the solution of:

$$\mathbf{Z}^{(k+1)} = \arg \min_{\mathbf{Z}} \lambda \|\mathbf{Z}\|_* + \eta \|\mathbf{Z} - \text{unvec}(\boldsymbol{\alpha}_R^{(k+1)} + \mathbf{w}^{(k)})\|_F^2.$$

By the subgradient optimality condition of the nuclear norm, this is solved via singular value thresholding (SVT), which shrinks the singular values σ_i of $R := \text{unvec}(\boldsymbol{\alpha}_R^{(k+1)} + \mathbf{w}^{(k)})$ and promotes low-rank structure[80]. Given the SVD of $R = U\Sigma V^\top$, with $\Sigma = \text{diag}(\sigma_i)$, the proximal operator for the nuclear norm yields

$$\mathbf{Z}^{(k+1)} = U \text{diag}(\max(\sigma_i - \tau, 0)) V^\top,$$

where the threshold is $\tau = \frac{\lambda}{2\eta}$.

4.4.6 Simulations

Neural Network

For the synthesized data, we use the formula in (3.21) to generate the data, with the assumption that all the vibration sources of the ring-planet meshing are similar and interchangeable (e.g. $P = 1$). The fundamental frequencies for the four sources in (4.2) are set to $\{f_1 = 99, f_2 = 1, f_3 = 2.83, f_4 = 4.67\}$ with 10 significant harmonics for all sources. The signal length is set equal to $T = 405405$ samples for a sampling frequency $f_s = 11583$. The synthesized excitation force is distorted with the impulsive response of the transfer function, $\mathbf{h}$ shown in Fig. 4.2. The evaluation of our approach is based on

the separated sources' power spectrum correlation with the original ones. The correlation measure is defined as:

$$\text{corr}(\mathbf{s}_i, \mathbf{s}_j) = \frac{\sum_f (S_i(f) - \bar{S}_i)(S_j(f) - \bar{S}_j)}{\sqrt{\sum_f (S_i(f) - \bar{S}_i)^2} \sqrt{\sum_f (S_j(f) - \bar{S}_j)^2}}; i \neq j \quad (4.46)$$

Here, S_i represents half of the power spectrum of the source signal $\mathbf{s}_i$, defined as $S_i = \text{fft}(s_i)$, where fft denotes the Fourier transform. $\bar{S}_i$ is the mean value of $S_i(f)$.

The EQL network is trained with the procedure defined previously, where in the second phase, we clamp to zero the weight's elements under an ad-hoc threshold $\gamma = 0.01$. We use an RMS prop optimizer [70] with a learning rate that was initially set to $l_r = 1e^{-3}$. The number of EQL network's hidden layers is a hyperparameter, depending on the complexity of the expression. We chose two hidden symbolic layers in this investigation. We train the models with the objective function in (3.11) for $it = 1000$ epochs, and the parameter a in the regularization term of $\mathcal{L}_{0.5}^*$ is set to 0.01 as in [55]. The values of λ , a , threshold γ , and EQL hidden layers are also our hyperparameters. The regularization parameter λ is selected through a research grid of value ranges, balancing between neural network weight sparsity and signal reconstruction. For the activation function used in the EQL, we consider multiplications ("×" four times) and identity ("id" four times) since the vibration model of the gear transmission set is considered polynomial in its arguments in the literature. For all the experiments, we used 70% of the data for training, while the remaining 30% is used for validation, and the processing window is $N = 256$ points. Subsequently, various levels of Signal-to-Noise Ratio (SNR; defined as $10 \log 10(\mathbb{E}(\mathbf{s}^2)/\mathbb{E}(\epsilon^2))$) will be considered. The sensitivity analysis will be elaborated upon in the sequel section.

Here, we show results from our neural network-based transfer function estimation. These results are obtained by extracting the weights of the convolution layer, which represent the system's impulse response $\mathbf{h}$. The optimization of the end-to-end architecture provides, after normalization, an accurate estimation of the sources $\mathbf{s}_{1 \leq i \leq 3}$, even for low SNR level, as shown in Table 4.1.

The multi-modulation model identified based on the sparse neural network, where we threshold the network weights with a threshold of 0.001 for an

Table 4.1: Estimation performance of the sources after training the neural network.

SNR	$\text{corr}(\hat{s}_1, s_1)$	$\text{corr}(\hat{s}_2, s_2)$	$\text{corr}(\hat{s}_3, s_3)$
0 dB	0.96	0.99	0.99
-7 dB	0.90	0.99	0.98

SNR level equal to 0 dB, is:

$$\mathbf{s}(t) = 7.7\mathbf{s}_1(t)(0.01 - 1.7\mathbf{s}_3(t))(0.02 + 1.3\mathbf{s}_2(t))^1 \quad (4.47)$$

where it matches well with the ground truth model of the simulated data. For a FIR model, we use a convolution layer to estimate the filter impulse response within the EQL architecture. After optimizing the objective function, the convolution kernel weights converge to the impulse response $\mathbf{h}$, as shown in Figure 4.2 for the low-SNR. The estimation of the impulse response depends on the richness of the source signal spectra, where the number of harmonics needs to be large enough to accurately estimate the transfer function within our framework, a notion well-established in the signal processing community. In this work, the number of harmonics for elementary sources is large enough in this context (10 harmonics for every source) to ensure accurate identification of the considered system's transfer function.

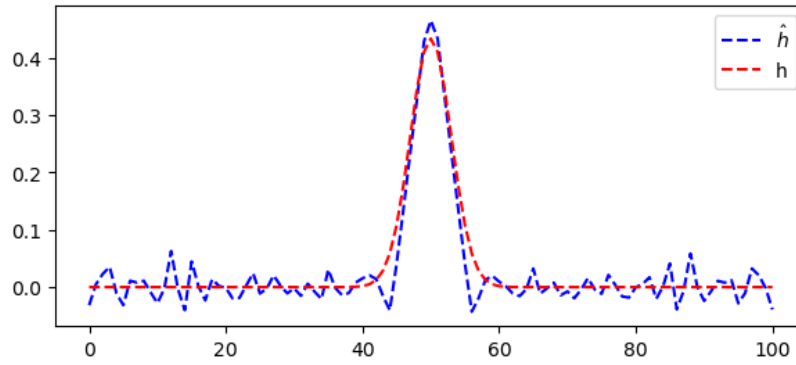


Figure 4.2: *
Estimated and true impulse responses for $SNR = 0$ dB.

¹After extracting the activated weights from the neural network, we applied the simplify function from the SymPy package to simplify the resulting expression.

For a transfer function with large memory, the use of an infinite impulse response (IIR) model is more adequate. This is particularly the case in vibration analysis, as stated by experts in the field. To ensure the effectiveness of our method, we consider a rational transfer function with $\mathbf{a} = [a_0, \dots, a_{N_p}]$, $\mathbf{b} = [b_0, \dots, b_{N_z}] \in \mathcal{R}^3$. We use two convolutional layers to estimate the rational filter, each with a length corresponding to the size of the denominator and numerator coefficient vectors. After optimizing the objective function (3.20), the estimated transfer function's impulse response, denoted as $\hat{\mathbf{h}}$ with SNR equal to $3dB$, is shown in Fig. 4.3. The estimation performance of the sources $\mathbf{s}_i$, measured with the correlation criterion 4.46, is illustrated in table 4.2. Here, sources $\mathbf{s}_2$ and $\mathbf{s}_3$ are reconstructed with high accuracy. However, the carrier signal $\mathbf{s}_1$ shows a lower correlation with the estimated source, explained by the fact that the transfer function is not accurately identified in this case, as compared to the previous FIR case. This means that, once the system model is established by our SR-based approach, a second stage of parameter estimation refinement might be needed to improve the estimation accuracy.

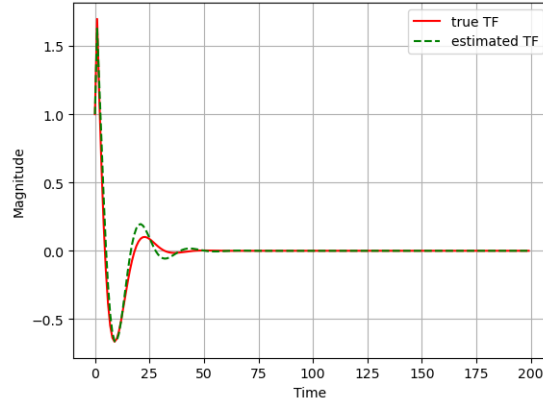


Figure 4.3: The estimated IIR filter.

Table 4.2: Estimation performance of the sources in the presence of an IIR filter.

SNR	$\text{corr}(\hat{s}_1, s_1)$	$\text{corr}(\hat{s}_2, s_2)$	$\text{corr}(\hat{s}_3, s_3)$
3 dB	0.85	0.983	0.98

The optimization of the neural network, along with the selection of the

hyperparameter λ , is crucial in this study to ensure accurate outcomes in data skeleton unveiling, source estimation, and transfer function modeling. To this end, the second phase of the training procedure is designed to achieve convergence with an initial learning rate of 10^{-3} . Subsequently, the learning rate is reduced to 10^{-4} to enable fine-tuning and further refinement of the model parameters.

Neural network: multi-input case

In this section, we generate new simulated data using the model described in 3.21 with $P = 3$. The source coefficients are randomly assigned, maintaining the same frequency parameters as before, and each term is associated with a randomly generated filter coefficient h_p . The signal-to-noise ratio (SNR) is set to 0 dB. Our aim is to validate, using simulated data, the ability to identify different transfer function impulse responses from single measurements. For each individual neural network, we use the same configuration as in previous experiments. The number of neural networks in the stacked architecture is treated as a hyperparameter; in this work, we deliberately overestimate it, setting it to 5.

Following the same training procedure as described previously, we success to activate only 3 separate neural networks as wanted with a regularization parameter $\lambda = 0.001$ for 500 epochs. The identified formulas for each activated network are as follows, which matched our formula as in 3.21.

$$\text{Network 0: } 0 \quad (4.48)$$

$$\text{Network 1: } -1.77(-1.46 - 5.66 s_{12})(-0.497931 s_{11} + 1.57 s_{11} s_{13}) \quad (4.49)$$

$$\text{Network 2: } 0 \quad (4.50)$$

$$\text{Network 3: } 27.28 s_{31}(0.158 + 1.28 s_{32})(0.316778 + 1.29944 s_{33}) \quad (4.51)$$

$$\text{Network 4: } 18.62 s_{41}(-0.174 + 1.39 s_{42})(0.25 + 1.066 s_{43}) \quad (4.52)$$

The estimated filters obtained from the convolution layers' weights are shown in Fig. 4.5. A good match between the true filters and the estimated ones can be observed within the frequency range containing harmonic components for different SNR levels, i.e., where relevant information is present. In our case, the highest carrier frequency is approximately $10f_m$, with $f_m = 99$ Hz.

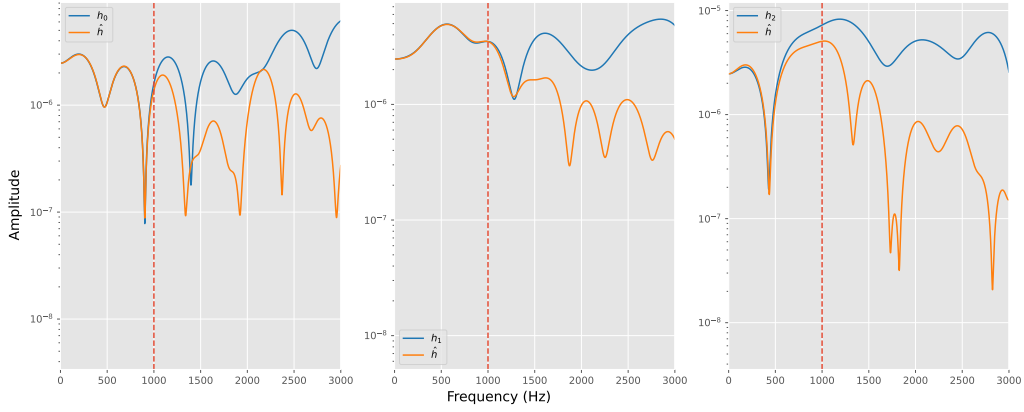


Figure 4.4: Filter estimation results for the MISO system at 20 dB.

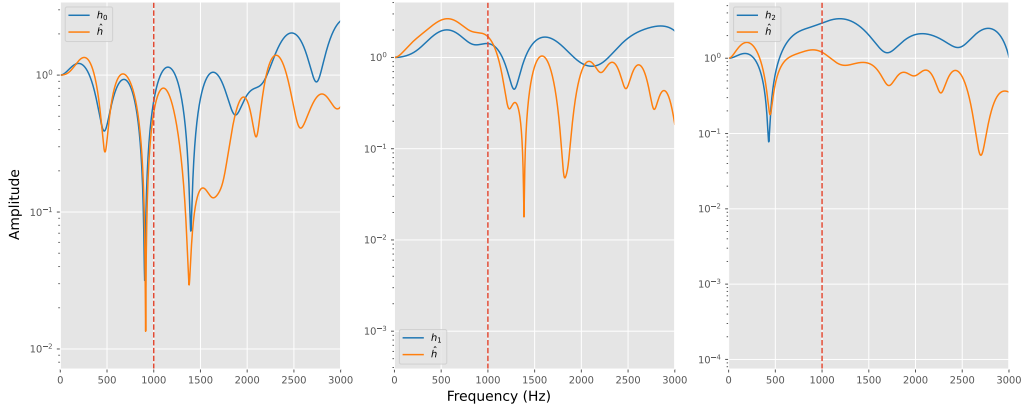


Figure 4.5: Filter estimation results for the MISO system at 0 dB.

Blind identification with modulated periodic input signals

We simulate a vibration signal with a carrier signal of fundamental frequency $f_1 = 99 \text{ Hz}$ modulated with a second source signal of fundamental frequency $f_2 = 5.83 \text{ Hz}$. The sampling frequency is equal to $f_s = 1200 \text{ Hz}$. The signal length is set to 35 times the sampling frequency. We set 10 harmonics for each source.

The number of ambiguities is related to the rank of the FIM matrix kernel, as analyzed below in Table 4.3, with different values of M_1 and M_2 . One can observe that when the number of equations is large enough (e.g.,

Table 4.3: FIM results

	Case 1	Case 2	Case 3
σ	0.01	0.01	0.01
M_1	10	1	10
M_2	1	10	10
L	10	10	10
$\dim(\text{null}(F))$	> 1	> 1	1

larger than the number of unknowns) the dimension of the FIM kernel is one. This corresponds to the scale ambiguity inherent to the blind identification problem. In other words, this elementary investigation reveals that the problem at hand is 'generically' identifiable when the number of harmonics is large enough as compared to the channel size. Otherwise, the considered blind channel estimation problem is 'unidentifiable' in which case the FIM kernel dimension becomes strictly larger than one. This identifiability problem deserves a thorough investigation, which will be provided in future works.

In this experiment, we evaluate the estimation of the FIR/IIR filter with taps of different lengths randomly generated. We run the ADMM algorithm with an empirically chosen hyperparameter ρ, η, λ (grid search).

The ADMM approach successfully achieves a low spectral residual defined as the mean square error between the FFTs of the true and estimated filters, i.e., $\|H_{\text{true}}(\omega) - H_{\text{est}}(\omega)\|_2$ at moderate and high Signal-to-Noise Ratio (SNR), while the SVD-based method in [81] fails to perform adequately when the number of taps exceeds five under the current experimental setup, averaged over 10 runs in the FIR case. This outcome is demonstrated in Fig. 4.6. Our approach also achieves good performance in terms of SNR, maintaining a low spectral residual, as shown in Fig. 4.7. An example of an FIR spectrum

Figure 4.6: Comparison of the Performance via Spectral Residual of ADDM-Based and SVD-Based Methods as a Function of Filter Tap Length

is shown in Figure 4.8, illustrating the good quality of the match. In the IIR case, the SVD-based approach fails to converge (NC) when the filter order is $N = N_1 = N_2 = 4$; it only works reliably with filters of order ≤ 2 . In contrast, the ADMM-based methods achieve low spectral residuals, as shown in Table 4.4.6. An example of the estimated parameters α is shown

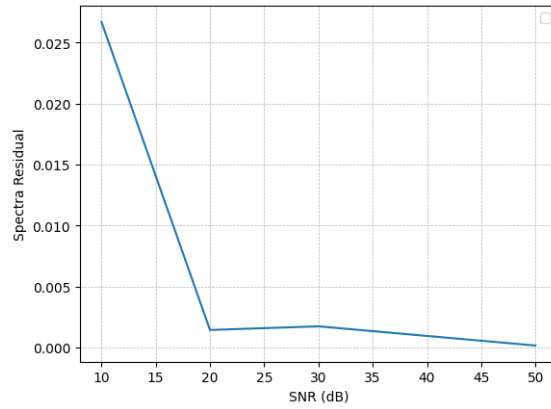


Figure 4.7: Spectral residual as a function of SNR at an FIR length of 15

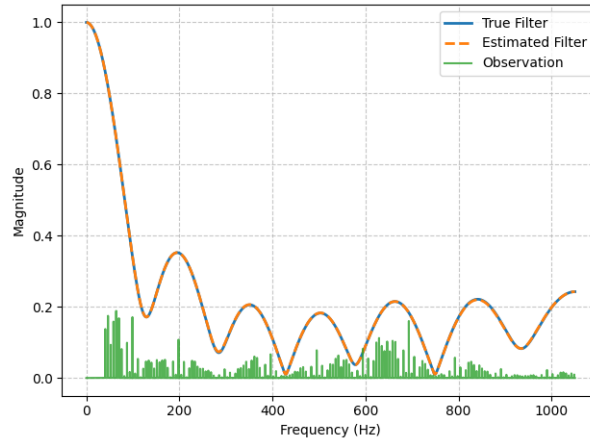


Figure 4.8: FIR example reconstruction.

in Fig. 4.9. The results demonstrate accurate estimation at SNR levels.

SNR (dB)		10	20	30
Spectra Residual (10^{-3})	ADMM	5.42	0.401	0.461
	Method[81]	NC	—	—

Table 4.4: IIR filter Spectra Residual as a function of SNR with $N = 4$

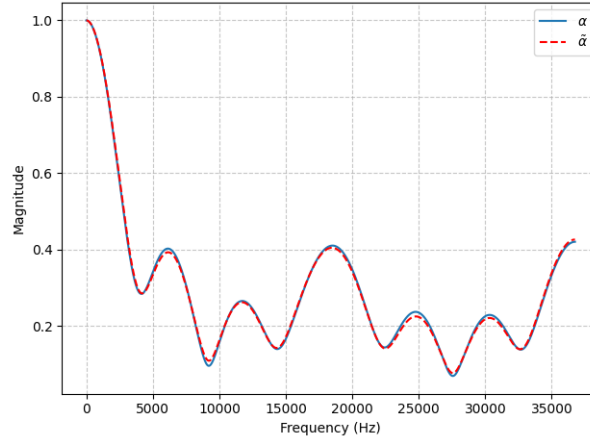


Figure 4.9: Reconstruction example of $\hat{\alpha} = \hat{\mathbf{a}} \otimes \hat{\mathbf{b}}$.

4.5 Conclusion

This chapter introduces blind channel identification methods applied to gear-box vibration analysis. A neural network-based method was introduced to estimate the impulsive response of the transfer function while modeling the vibration data using the system characteristic frequencies. In addition, leveraging a predefined analytical model and prior knowledge of the characteristic frequencies of the system, we outline a process for identifying the system's impulse response. This method lays the groundwork for future work aimed at extracting elementary sources of vibration by mitigating the influence of transfer function effects on the acquired vibration signal. Our methodology demonstrates promising estimation outcomes across various signal-to-noise ratios, underscoring its efficacy.

Chapter 5

Frequency estimation based on line spectrum structure

5.1 Introduction

In rotating machines, spectral analysis methods are typically used to identify faults occurring in the rotating components of the machinery, including the gearbox, shafts, etc. Its application is limited by the inconvenience of encoder or tachometer installation, that capture the rotational speed on running machines in variable operating conditions. Due to the potential cost savings, the use of an accelerometer instead of encoders has shown a larger interest in the condition monitoring community.

The rotational speed estimation is a critical requirement for most vibration-based condition monitoring methods, such as frequency-based methods and angular resampling methods. Vibration analysis-based methods have been explored to achieve rotational speed estimation; however, due to the presence of multiple harmonics in the vibration signals, structural resonance, and the low level of signal-to-noise ratio, the applicability of existing methods is limited and challenging, such as Bayesian estimation methods and Hilbert Transform. Furthermore, several methods have been proposed, such as Phase Demodulation, Iterative Phase Demodulation, multi-harmonic phase demodulation, Maximum tracking in combination with a Vold-Kalman filter, Multi-Order Probabilistic Approach (MOPA), among others. Readers are referred to [82] for details. The authors in [82] note that each method has its own advantages and limitations, resulting in varying levels of applicability and accuracy. They further emphasize that valuable methods for speed

estimation are those that can handle multiple harmonics rather than relying on a single dominant frequency, and that are robust to non-speed-related harmonics, structural resonances, noise, etc. Up to date, it appears that MOPA and multi-harmonic phase demodulation are among the methods that achieve the highest accuracy in speed estimation compared to others, since they leverage multiple harmonics of the frequency of interest into their estimation methodology.

Phase demodulation achieves higher accuracy when the harmonics exhibit a high signal-to-noise ratio over the entire record. A multi-harmonic phase demodulation method has been developed to improve precision by combining the advantages of MOPA, namely the use of multiple harmonics, with the high accuracy of phase demodulation applied to the harmonic exhibiting the highest signal-to-noise ratio. However, the application of multi-harmonic phase demodulation requires prior knowledge of the initial rotational speed, which increases the overall complexity of the process.

MOPA tracks harmonics in the time–frequency representation (TFR) of the vibration signal, the complexity of which may vary depending on the signal characteristics [83]. The effectiveness of MOPA is strongly dependent on the quality of the TFR. Moreover, it is assumed that frequency components with high amplitude in the TFR are the most likely to correspond to the rotational speed of the rotating component.

In short, a successful speed estimation method should be capable of incorporating all harmonics related to the rotational speed in both the estimation and tracking of the rotational frequency. However, the extensive computational cost of MOPA, its strong dependence on the quality of the time-frequency representation, and the limitation of phase demodulation methods, which require prior knowledge of the initial rotational speed, motivate the need for an alternative approach. In addition, planetary gear transmission systems in helicopters, aircraft reducers, and wind turbines exhibit unique spectral structures. Besides the harmonics of the meshing frequency, spectral lines appear in the sidebands of this frequency and its harmonics, even under healthy operating conditions. These sidebands are investigated in the phenomenological modeling step, where they reflect the modulations between the meshing frequency and the rotation frequency of the mobile elements. For more details, the reader is referred to Chapter 3. These features are not exploited in the MOPA, where only the rotation frequency and its multiples are leveraged in the estimation process. Therefore, in this work, we propose an approach to exploit the harmonics in the vicinity of the meshing frequencies, in addition to their multiple harmonics, to improve the estimation of

the rotational frequency of a component based on the machine kinematic information

5.2 Vibration spectrum structure

The gearbox vibration signal featured very structured power spectra due to the rotational motion of the system. The fault detection techniques typically use a Tacho-speed measure in order to compute the characteristic system frequencies, considering the kinematic model of the rotating machines. However, this measure is not always available. The power spectrum of the gearbox exhibits a distinct structure due to the rotation motion, where multiple harmonics appear related to the rotational speed of the input or output shaft. In the case of a healthy planetary gearbox, the gearbox vibration signal has multiple harmonic sidebands in the vicinity of the meshing frequency and its harmonics that are related to the shaft rotation frequency in terms of the kinematic parameters of the rotating machine. Consider a planetary gearbox with meshing frequency f_m and the planet carrier rotation frequency f_c , where they are related as follows:

$$f_m = f_c Z_r \quad (5.1)$$

where Z_r is the number of teeth in the ring gear.

Furthermore, the ad hoc phenomenological modeling of vibration signals under healthy operating conditions yields analytical formulas that describe the structure of spectral sidebands in the vicinity of the meshing frequency, arising from the modulation between the meshing frequency and the planet carrier rotation frequency, as shown in the Fig 5.1. Therefore, the sideband peaks around the meshing frequency and its harmonics can be expressed, for each meshing harmonic (k), as

$$\mathbf{P} = [kf_m \pm lf_c]^T, \quad k, l = 0, \dots, H. \quad (5.2)$$

Equivalently,

$$\mathbf{P} = f_m \mathbf{U}, \quad (5.3)$$

where

$$\mathbf{U} = \left[k \pm \frac{l}{Z_r} \right]^T. \quad (5.4)$$

For ($k=0$), the terms "+" and "(-)" coincide, resulting in a single spectral line.

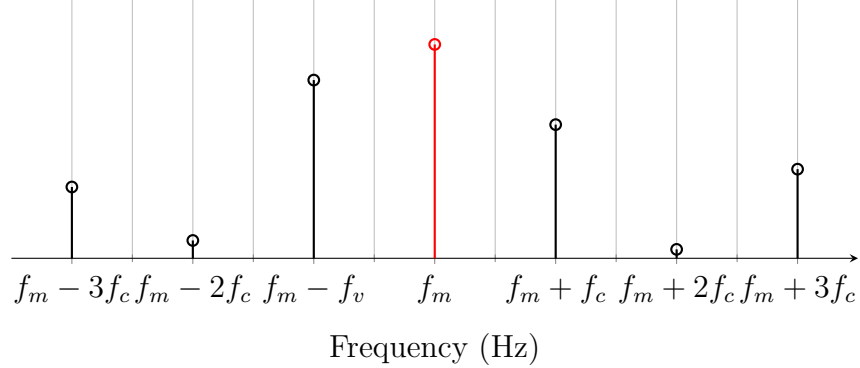


Figure 5.1: Planetary gear set vibration power spectrum illustration.

where H is the number of significant harmonics in the sidebands.

5.3 Problem statement

In gear transmission systems, vibration signals contain spectral peaks caused by the *gear meshing frequency* f_m and its sidebands. These can be modeled as:

$$P_i = f_m \left(k \pm \frac{l}{Z_r} \right), \quad k, l = 0, \dots, H, \quad (5.5)$$

where Z_r denotes the number of gear teeth, k is the meshing harmonic order, l is the sideband index, and H is the maximum harmonic order considered.

In practice, the measured spectra contain valid peaks that conform to the above model; however, these measurements are inevitably corrupted by noise. In addition, the spectra often include extraneous peaks arising from structural resonances, natural vibration modes, fault-related harmonics, bearing and shaft components, or other measurement artifacts. Such spurious components may coexist with model-consistent peaks and cannot be neglected.

The objective is to robustly estimate the single latent meshing frequency f_m from the set of detected spectral peaks, despite the presence of noise and outlier peaks that do not conform to the model.

5.4 Proposed method

The proposed meshing frequency (rotational speed) estimation relies fundamentally on the accurate detection of spectral peaks, as all subsequent modeling and estimation steps operate directly on the extracted peak frequencies $\{P_i\}$.

5.4.1 Peaks detection

Spectral peak detection is performed in the frequency domain using a prominence-based criterion, as implemented in the `find_peaks` function of the `scipy.signal` library. Prominence-based peak detection has a strong interpretation in terms of topological features of one-dimensional scalar fields (topological data analysis).

Let $S(f)$ denote the estimated power spectrum sampled on a discrete frequency grid. In this framework, a spectral peak corresponds to a local maximum whose significance is quantified by its prominence, defined as the vertical distance between the peak amplitude and the highest minimum separating it from any higher neighboring peak.

Formally, for a peak at frequency $f = P_i$, the prominence π_i is given by

$$\pi_i = S(P_i) - \max\{S(f_L), S(f_R)\},$$

where f_L and f_R denote the frequencies of the nearest local minima to the left and right of P_i that bound the peak before a higher peak is encountered. This construction is equivalent to measuring the persistence of a connected component in the superlevel-set filtration of $S(f)$.

Peaks are retained if their prominence exceeds a prescribed threshold $\pi_{\min}$, ensuring that only structurally significant spectral components are selected. This criterion provides robustness to noise, spectral leakage, and slowly varying baselines, which are common in vibration spectra.

5.4.2 Weighted Least Squares

Let P_i denote the i -th measured spectral peak, $i = 1, \dots, M$, and let U_j denote the j -th theoretical normalized frequency coefficient of the form

$$U_j = k \pm \frac{l}{Z_r},$$

with $k, l = 0, \dots, H$, where Z_r is the number of gear teeth and H is the maximum harmonic order. Let J denote the total number of such theoretical coefficients. Each measured peak P_i is associated with the closest theoretical coefficient through

$$U_i = \arg \min_{U_j} |P_i - f_m U_j|, \quad j = 1, \dots, J,$$

where U_i denotes the normalized frequency assigned to P_i .

Assume that each measured peak is associated with a positive weight $w_i > 0$ reflecting its reliability, for example, based on peak amplitude. The meshing frequency f_m is then estimated using a weighted least-squares (WLS) criterion:

$$\hat{f}_m = \arg \min_f \sum_{i=1}^M w_i (P_i - f U_i)^2. \quad (5.6)$$

Closed-Form Solution

Define the following column vectors and diagonal weighting matrix:

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_M \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_M \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_M \end{bmatrix}, \quad (5.7)$$

where N denotes the total number of detected spectral peaks.

The weighted least-squares problem in (5.6) can then be written compactly as

$$\hat{f}_m = \arg \min_f (\mathbf{P} - f \mathbf{U})^\top \mathbf{W} (\mathbf{P} - f \mathbf{U}). \quad (5.8)$$

The above minimization admits a closed-form solution given by

$$\hat{f}_m = \frac{\mathbf{U}^\top \mathbf{W} \mathbf{P}}{\mathbf{U}^\top \mathbf{W} \mathbf{U}}. \quad (5.9)$$

This formulation enables direct estimation of the latent meshing frequency f_m while accounting for the relative reliability of each measured spectral peak through the associated weights w_i .

For a fixed assignment of normalized frequencies $\mathbf{U}$, the Weighted Least-Squares (WLS) estimator admits the closed-form solution in (5.9). The computational complexity of this estimator is linear in the number of detected

spectral peaks. Specifically, the dominant operations consist of computing the inner products $\mathbf{U}^\top \mathbf{W} \mathbf{P}$ and $\mathbf{U}^\top \mathbf{W} \mathbf{U}$, each requiring $\mathcal{O}(N)$ operations, where N is the number of measured peaks.

Thus, the overall computational complexity of the WLS estimator is

$$\mathcal{O}(N),$$

making it highly efficient and well-suited for real-time or embedded implementations. However, the WLS formulation implicitly assumes that all measured peaks are generated according to the underlying model and that deviations are due solely to Gaussian noise. As a result, the estimator is sensitive to outliers, since even a small number of spurious peaks can significantly bias the estimate of f_m .

5.4.3 RANSAC-Based Estimation

To improve robustness against outliers and spurious spectral components, a RANSAC (RANdom SAmple Consensus) framework [84] is employed to estimate the meshing frequency. The algorithm iteratively generates candidate meshing frequencies using randomly selected spectral peaks as follows:

1. Randomly select a measured spectral peak P_i and a theoretical normalized frequency coefficient

$$U_j = k \pm \frac{l}{Z_r}.$$

2. Compute a candidate meshing frequency as

$$\hat{f}_m^{\text{cand}} = \frac{P_i}{U_j}.$$

3. Identify the inlier set as the collection of measured peaks that satisfy

$$|P_k - \hat{f}_m^{\text{cand}} U_l| < \varepsilon, \quad \text{for some } U_l,$$

where $\varepsilon > 0$ denotes a predefined tolerance.

4. Compute a weighted inlier score defined as

$$\text{score} = \sum_{k \in \mathcal{I}} w_k,$$

where $\mathcal{I}$ denotes the inlier set and $w_k > 0$ is a reliability weight associated with the k -th measured peak based on peak amplitude.

5. Repeat the above steps for N_{iter} iterations and select the candidate $\hat{f}_m$ that maximizes the weighted inlier score.

The output of the RANSAC procedure is a subset of inlier spectral peaks that are mutually consistent with a single latent meshing frequency, while peaks inconsistent with the model are effectively rejected as outliers.

In contrast to WLS, the RANSAC-based estimator explicitly accounts for the presence of outliers by maximizing a consensus measure rather than minimizing a quadratic error. Each RANSAC iteration generates a candidate meshing frequency from a randomly selected peak-coefficient pair and evaluates its consistency against all measured peaks.

For each iteration, inlier detection requires evaluating the residual

$$|P_k - \hat{f}_m^{\text{cand}} U_l|$$

for all $k = 1, \dots, M$ and for the set of theoretical coefficients $\{U_l\}_{l=1}^J$, resulting in a per-iteration complexity of

$$\mathcal{O}(MJ),$$

where J denotes the number of theoretical normalized frequency coefficients.

After N_{iter} iterations, the total computational complexity of RANSAC becomes

$$\mathcal{O}(N_{\text{iter}} MJ).$$

While this complexity is significantly higher than that of WLS, RANSAC provides strong robustness guarantees, as its performance depends primarily on the proportion of inliers rather than the magnitude of outliers.

The WLS and RANSAC estimators represent two complementary points on the robustness-complexity trade-off. The WLS estimator is optimal in the maximum-likelihood sense under Gaussian noise and in the absence of outliers, but its quadratic loss function leads to poor robustness when spurious spectral peaks are present. Conversely, the RANSAC-based estimator approximates an ℓ_0 -type objective by maximizing the number (or weighted sum) of inliers, thereby achieving robustness to gross outliers at the expense of increased computational cost. In practice, RANSAC is well-suited for coarse estimation and outlier rejection, while WLS can be applied subsequently to the inlier set to obtain a statistically efficient refinement of the meshing frequency estimate. This hybrid strategy combines the robustness of RANSAC with the efficiency

and optimality of WLS, and is particularly attractive in vibration-based condition monitoring scenarios where extraneous spectral components are common.

5.5 Frequency refinement

Several model-based condition monitoring methods rely on measuring the input/output speed and using the gearbox kinematic model to determine the fundamental rotational frequencies of meshing and rotating elements. Consequently, it is essential to verify how fluctuations in speed measurements affect the system's rotational characteristic frequencies and to determine how to pre-process these measurements to refine the frequencies, thereby enhancing model-based approaches.

In that context, we propose a pre-processing step that can be employed whenever there is a need to do frequency refining for vibration demodulation purposes (especially when an angular resampling over the raw measurements is not used). Consider the planetary gearbox models illustrated in chapter 3, the spectrum peaks location, for instance, for the meshing frequency first order are the following

$$\{f_1 \pm k_1 f_2 \pm k_3 f_3\},$$

where, k_1, k_2 and k_3 are the harmonics of sources s_1, s_2 and s_3 respectively. Consider that a given fundamental frequency is inaccurately estimated with an error δ as follows:

$$\hat{f} = f + \frac{\delta}{N} f_s,$$

where f is the true frequency value to be determined, N is the length of the signal (in terms of DFT), and f_s is the sampling frequency.

We propose to use an iterative frequency estimation (refinement) algorithm [85] to compute the error δ . Specifically, we aim to iterate over all 'inaccurately' known fundamental frequencies and adjust them in a way that enhances the signal demodulation quality. Given an estimated frequency $\hat{f}$ and a signal s , our goal is to refine this estimate and get closer to the true frequency f . Thus, the signal $s(k)$ focusing on the frequency f can be written as:

$$s(k) = ae^{j2\pi k \frac{f}{f_s}} + I(k) + \epsilon(k), \quad k = 0, 1, \dots, N-1,$$

where $I(k)$ represents all other harmonics or components of the signal, excluding the frequency f of interest, $\epsilon(k)$ is the additive noise term, and a

Algorithm 1 Frequency Refinement.

- 1: **Input:** Initial frequency estimate $\hat{f}$
- 2: **Output:** Refined frequency f
- 3: Initialize $\hat{\delta}_0 = 0$
- 4: **for** $i = 1$ to Q **do**
- 5: Compute

$$X_p = \sum_{k=0}^{N-1} s(k) e^{-j2\pi k(\hat{f} + \frac{\hat{\delta}_{i-1} + p}{N})}, \quad p = \pm 0.5$$

- 6: Update

$$h(\hat{\delta}_{i-1}) = \frac{1}{2} \Re \left\{ \frac{X_{0.5} + X_{-0.5}}{X_{0.5} - X_{-0.5}} \right\}$$

- 7: $\hat{\delta}_i = \hat{\delta}_{i-1} + h(\hat{\delta}_{i-1})$
 - 8: **end for**
 - 9: **return** $f = \hat{f} + \frac{\hat{\delta}_Q}{N} f_s$
-

is the corresponding amplitude of the peak at the frequency f . If needed, this process can be iterated over all the harmonics of a vibration signal, not just the system's fundamental frequencies. The algorithm is modified and written as follows in Algorithm 1 (see details in [85]):

In the algorithm, Q refers to the number of iterations (typically $Q = 2$) and $\hat{m}$ is the DFT peak index in the vicinity of frequency $\hat{f}$, computed initially using rotation speed information and kinematic model.

5.6 Simulation

In this part, we conducted simulations to compare the performance of the LASSO algorithm illustrated in chapter 6 trained with a dictionary built using perfectly estimated fundamental frequencies and their disturbed estimation. This will illustrate the essential role of the preprocessing step of dictionary refinement in ensuring a good fit for the observation signal and, consequently, a good estimation of the parameters of the signals $\mathbf{s}_i$.

To begin with, we applied LASSO algorithm using a dictionary $\mathbf{D}$ constructed with the true fundamental frequency f_i . In a subsequent run, the dictionary $\mathbf{D}$ was constructed based on distorted fundamental frequencies $\tilde{f}_i = \frac{(m+\delta)f_s}{N}$, where m represents the DFT index of f_i . Afterward, we re-ran

Table 5.1: Performance of LASSO reconstruction before and after dictionary refinement for different perturbation values δ at $SNR = 0$ dB.

δ	0	0.1	0.2	0.3	0.5
D without refinement	0.0204	1.02	1.29	1.11	1.57
D with refinement	–	0.253	0.253	0.253	0.25

the second case with refining the fundamental frequency $\tilde{f}_i$ with Algorithm 1. The reconstruction performance between the true input signal and reconstructed one is defined as $\text{error}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{\sum_{k=0}^{N-1} (\mathbf{y}(k) - \hat{\mathbf{y}}(k))^2}{\sum_{k=0}^{N-1} \mathbf{y}(k)^2}$. For both cases, the results are detailed in Table 5.1. It is clear that, as the perturbation δ increases, the quality of signal reconstruction degrades significantly. Specifically, when $\delta = 0.5$, the reconstruction error rises to approximately 1.3, indicating a notable decrease in the accuracy of the LASSO fit. Conversely, the fundamental frequency refinement preprocessing step proves to be essential in maintaining a reduced reconstruction error around 0.34, even under conditions of high perturbation in frequency estimates. This suggests a high fidelity in signal reconstruction when this preprocessing step is employed.

A demonstration of the reconstruction performance and the estimation of signal $s_{1 \leq i \leq 3}$ (amplitudes and phases) is presented in Figure 5.2 for a fundamental frequency perturbation set to $\delta = 0.5$ and $SNR = 0\text{dB}$. Without refining the input dictionary, the LASSO algorithm encounters difficulties in accurately reconstructing the signal, leading to poor estimation of signal components. However, figure 5.3 illustrates the results after tuning the dictionary, showing promising signal estimation. This highlights the importance of preprocessing steps such as frequency refinement to achieve a precise fit for input signals and consequently enhance the separation of elementary sources.

5.7 Application: planetary gearbox meshing frequency estimation

In this section, we employ the HUMS 2023 Challenge dataset [69] to evaluate the proposed estimation of the fundamental carrier (meshing) frequency using vibration data acquired from a planetary gearbox with four planets and known kinematic parameters (e.g., the number of gear teeth).

CHAPTER 5. FREQUENCY ESTIMATION BASED ON LINE SPECTRUM STRUCTURE

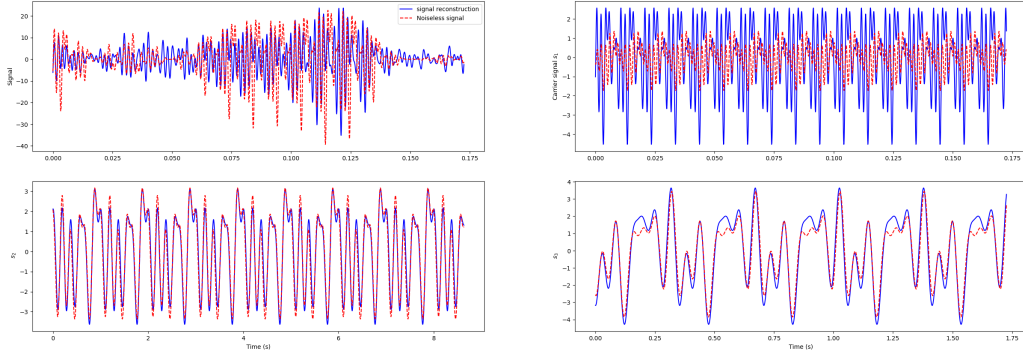


Figure 5.2: Signal reconstruction without dictionary refinement for fundamental frequency perturbation $\delta = 0.5$ with $SNR = 0$ dB.

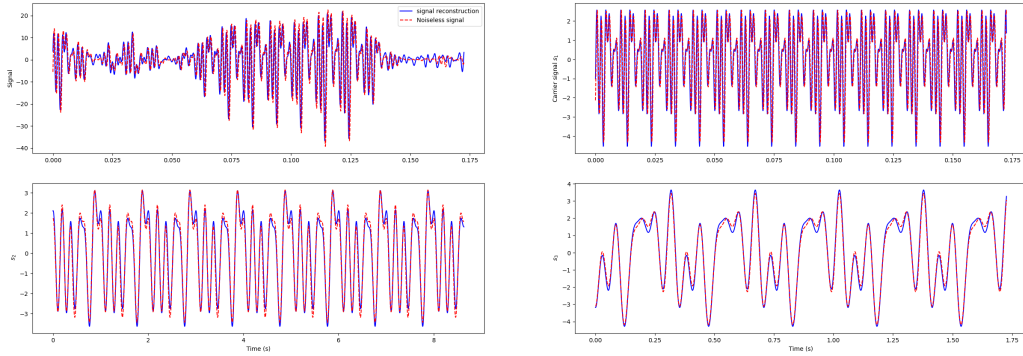


Figure 5.3: Signal reconstruction with dictionary refinement for fundamental frequency perturbation $\delta = 0.5$ with $SNR = 0$ dB.

The analysis begins with the application of a peak-finding algorithm to the vibration power spectrum, using a default peak height threshold of 0.005. It should be noted that the selection of peak height and prominence parameters requires careful consideration, particularly in noisy environments, and warrants dedicated investigation beyond the scope of the present study. The results of the harmonic detection step are shown in Fig. 5.4. The height parameter of the detection algorithm is selected such that only a subset of prominent peaks is extracted from the power spectrum, rather than all possible spectral peaks. This choice is intended to evaluate the robustness of the proposed frequency estimation method in the absence of equally spaced

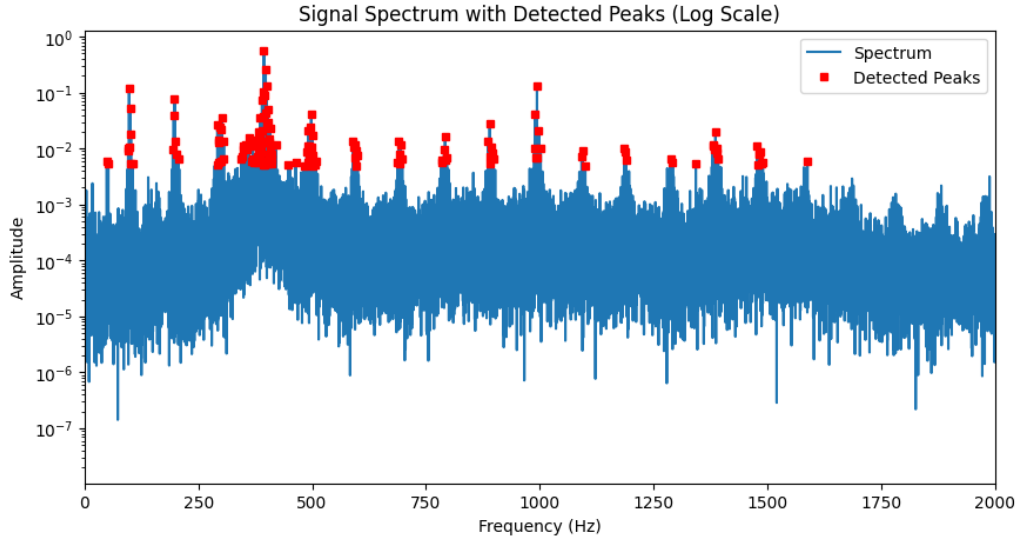


Figure 5.4: Power spectrum peaks detection.

detected harmonics. Moreover, some detected harmonics originate from other modulation processes that are not included in the set of normalized frequencies used in the matching procedure and are therefore treated as outliers. After running our matching algorithm on the detected harmonic values and the set of normalised harmonics, we have obtained the following results

- True frequency $f_m : 99Hz$
- Total peaks: 69
- Estimated frequency $\hat{f}_m : 98.999 Hz$
- Inliers found: 53

Regarding the estimated results, we demonstrate that with the existence of outliers in the detected peaks, we arrive at a fine estimation of the fundamental carrier frequency $\hat{f}_m = 98.999 Hz$. Work on applying our approach in a variable regime on batched data is planned in the near-term

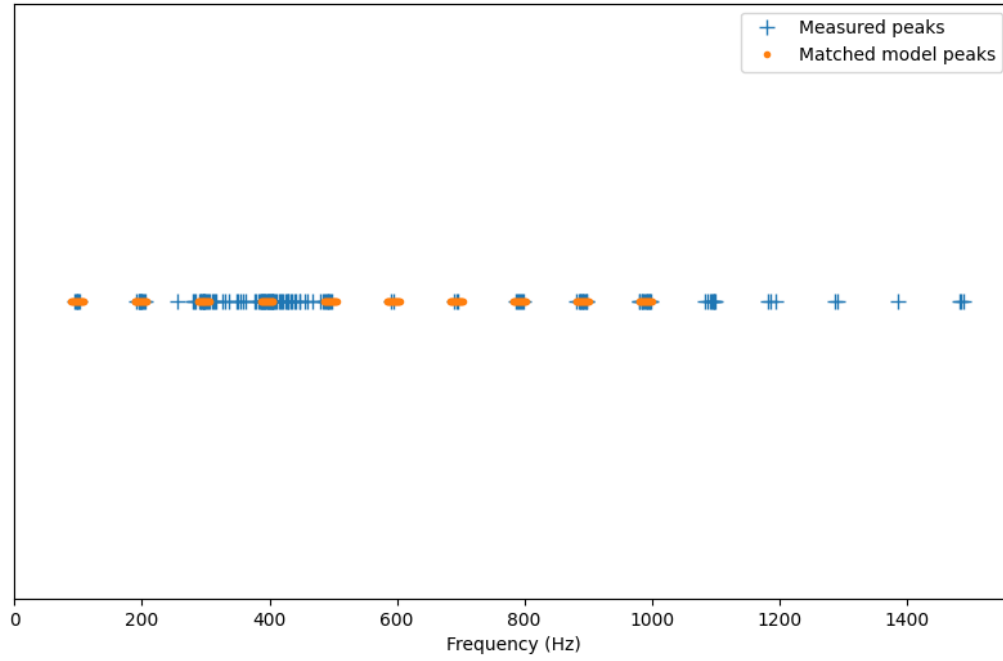


Figure 5.5: Measured and Model-Predicted Spectral Peaks

perspective, to evaluate the ability of this method to track the instantaneous frequency.

5.8 Conclusion

In this Chapter, we have presented approaches for either frequency estimation or refinement based on a predefined analytical model that describes the relationship between power-spectrum harmonics. This is crucial for model-based estimation methods in order to have fine reconstruction and separation accuracy. Our approach is strongly based on the harmonics peaks relationship model, which is very well studied within the phenomenological modeling process of gear transmission sets.

Chapter 6

Source Separation and Tracking of Periodic Signals

6.1 Introduction

Vibration analysis methods have shown good performance in detecting various gearbox failures, as the vibration signals contain critical information regarding the gearbox's health. Rotating machines, such as planetary gear sets, have vibration signals that exhibit strong modulation characteristics due to the amplitude modulation effect caused by the dynamic behavior and the gear faults [19, 86, 71]. The modulating frequencies are closely associated with the characteristic rotating frequencies. Moreover, modulation phenomena can be observed in vibration signals under both healthy and faulty conditions. This is due to manufacturing errors, time-varying vibration transmission paths, and unequally distributed loads, among other factors. These issues make it difficult to distinguish fault-related modulations from unrelated ones. Failure to address such challenges can often mislead the diagnosis and localization of faulty components. In rotating machine vibration analysis, the main challenge is isolating a set of signals (known as source separation or source extraction) related to the system's moving parts from the global vibration signal. This challenge has been widely investigated for decades in fields such as audio processing and telecommunications. Intensive research has been conducted to address this issue for linear mixtures, quadratic mixtures, and polynomial mixtures [39, 43, 42]. In health monitoring, accurately distinguishing fault-related modulation components from those unrelated to faults is essential for effective fault diagnosis and localization in rotating gearboxes. Traditional envelope analysis, Hilbert demodulation, and empirical mode decomposition

(EMD) methods often struggle to separate modulation contributions from different system components in gearbox vibration signals, which can hinder the monitoring of faulty components [19, 86]. In [87], a source separation method, using a tachometer and the prior knowledge of the kinematic gearbox model, has been introduced to estimate and separate the periodic sources. This method relies on a linearization step, which results in the estimation of a large and redundant set of parameters. As a consequence, the method in [87] is inefficient, with a large loss of performance as compared to the Cramer Rao Bound (CRB).

In this chapter, we propose an improved solution that solves the previous issue and meets the CRB. Both batch and adaptive implementations are proposed using the alternating least squares optimization technique.

6.2 Problem Setup

Without loss of generality, we consider the vibration signal of a planetary gearbox system with T samples, described by the following equation [12, 74]:

$$x(t) = s(t) + n(t) \quad t = 1, \dots, T \quad (6.1)$$

$$s(t) = s_1(t)(1 + s_2(t))(1 + s_3(t)) \quad (6.2)$$

which represents multi-amplitude modulations (AM) between three periodic source signals $\mathbf{s}_{1 \leq i \leq 3}$ with estimated (or a priori known) fundamental frequencies f_i , $i = 1, \dots, 3$, typically related to the machine's rotation frequencies. $s_1(t)$ represents the meshing component, $s_2(t)$ the modulation at the carrier plate frequency, and $s_3(t)$ the sun gear rotation contribution. $n(t)$ is an additive white Gaussian noise with zero-mean and variance σ_n^2 . Consider a stationary regime (i.e., frequencies f_i are invariant) and let H_i be the number of 'active' harmonics¹ of the i -th source, so that it can be written as:

$$s_i(t) = \sum_{l=1}^{H_i} \sin(2\pi l f_i t) w_{i,l} + \cos(2\pi l f_i t) \tilde{w}_{i,l} \quad (6.3)$$

where $w_{i,l} = a_{i,l} \cos(\phi_{i,l})$ and $\tilde{w}_{i,l} = a_{i,l} \sin(\phi_{i,l})$ are the harmonic amplitudes (and phases) to be estimated. Given a set of observations for $t = 1, \dots, T$, one can rewrite equation (6.3) in vector form as:

$$\mathbf{s}_i = \mathbf{D}(f_i) \mathbf{w}_i + \tilde{\mathbf{D}}(f_i) \tilde{\mathbf{w}}_i \quad (6.4)$$

¹In practice, the vibration signal periodic sources have a finite number of harmonics, effectively contributing to the considered signal.

where $\mathbf{s}_i = [s_i(1), \dots, s_i(T)]^T$, $\mathbf{w}_i = [w_{i,1}, \dots, w_{i,H_i}]^T$, $\tilde{\mathbf{w}}_i = [\tilde{w}_{i,1}, \dots, \tilde{w}_{i,H_i}]^T$, and

$$\mathbf{D}(f_i) = \begin{bmatrix} \sin(2\pi f_i) & \cdots & \sin(2\pi H_i f_i) \\ \vdots & & \vdots \\ \sin(2\pi T f_i) & \cdots & \sin(2\pi T H_i f_i) \end{bmatrix}$$

$$\tilde{\mathbf{D}}(f_i) = \begin{bmatrix} \cos(2\pi f_i) & \cdots & \cos(2\pi H_i f_i) \\ \vdots & & \vdots \\ \cos(2\pi T f_i) & \cdots & \cos(2\pi T H_i f_i) \end{bmatrix}$$

are the source harmonics dictionaries in a stationary condition. The main focus then is estimating the Fourier coefficients $\{\mathbf{w}_i, \tilde{\mathbf{w}}_i\}$ of each source and tracking their variations, given the signal $x(t)$ and the sources' fundamental frequencies². Note that, in practice, the number of active harmonics H_i is unknown and is replaced at first by an overestimation, H_{max} , as shown next.

6.3 Neural network-Based Source Separation

Under constant operating conditions, we assume that the mixture model generating the vibration signal is unknown. To address this scenario, we propose a symbolic-regression-based neural network. The proposed architecture comprises two main stages: source estimation followed by mixture-model identification as stated in chapter 3. Specifically, during the neural network training process, the Fourier coefficients of each source are first estimated and then combined through successive symbolic layers, which progressively unveil an analytical expression that fits the observed data, as illustrated in Fig. 3.2. In the context of source separation in a MISO system, we employ the neural network architecture depicted in Fig. 3.8, where multiple neural networks are concatenated to generate the output.

After training, the estimated Fourier coefficients are extracted from the first layer of the architecture. The learning process follows a LASSO-type formulation, in which a trade-off is enforced between sparsity-promoting regularization and a reconstruction error term. This approach enables the selection of the most significant source harmonics while simultaneously identifying a simple mixture model.

²The frequencies are determined based on a measure of shaft rotation speed and the machine's kinematic model.

6.4 Signal Harmonic Selection

In Chapter 3, which addresses the modeling process of vibration data, we propose an automatic and robust modeling framework based on the harmonic peaks of the signal power spectrum, rather than on the raw time-domain data, which are strongly affected by internal and external propagation-path effects. The proposed methodology is primarily based on estimating the harmonic components of the vibration spectrum and using them to generate an artificial signal in which all harmonics are assigned unit magnitudes. As stated previously, this approach allows the modeling process to focus on the distribution of the frequency structure rather than on amplitude variations. The overall methodology is illustrated in Fig. 6.1.

We employ a neural-network-based symbolic regression approach, as introduced earlier, in which the network estimates the Fourier weights and identifies a simple analytical expression that fits the generated artificial data. The neural network is trained using LASSO-type objective functions, which promote convergence toward a compact and parsimonious representation by activating only the most significant harmonic sources. In addition, the incorporated sparsity constraint encourages a binary network, where the optimized weights tend toward values of either 0 or 1. Readers are referred to Chapter 3 for further details.

6.5 Maximum Likelihood (ML) Solution with ALS Optimization

Cramer Rao Bound

Under the assumption that $n(t)$ is Gaussian, it is straightforward to write the likelihood function of the data and to derive an explicit expression for the Cramer-Rao Lower Bound.³ The estimation error of the source's parameter vectors, denoted $\boldsymbol{\theta}$, is measured via its variance⁴ [88]. Let's denote the parameter vector as follows:

$$\begin{aligned}\boldsymbol{\theta} &= [\mathbf{w}_1^T, \tilde{\mathbf{w}}_1^T, \mathbf{w}_2^T, \tilde{\mathbf{w}}_2^T, \mathbf{w}_3^T, \tilde{\mathbf{w}}_3^T]^T \\ &= [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, \boldsymbol{\theta}_3^T]^T\end{aligned}\tag{6.5}$$

³When the Gaussian hypothesis is not valid, the proposed algorithm is still applicable, but it will no longer provide the Maximum Likelihood estimate.

⁴Here we consider only unbiased estimators.

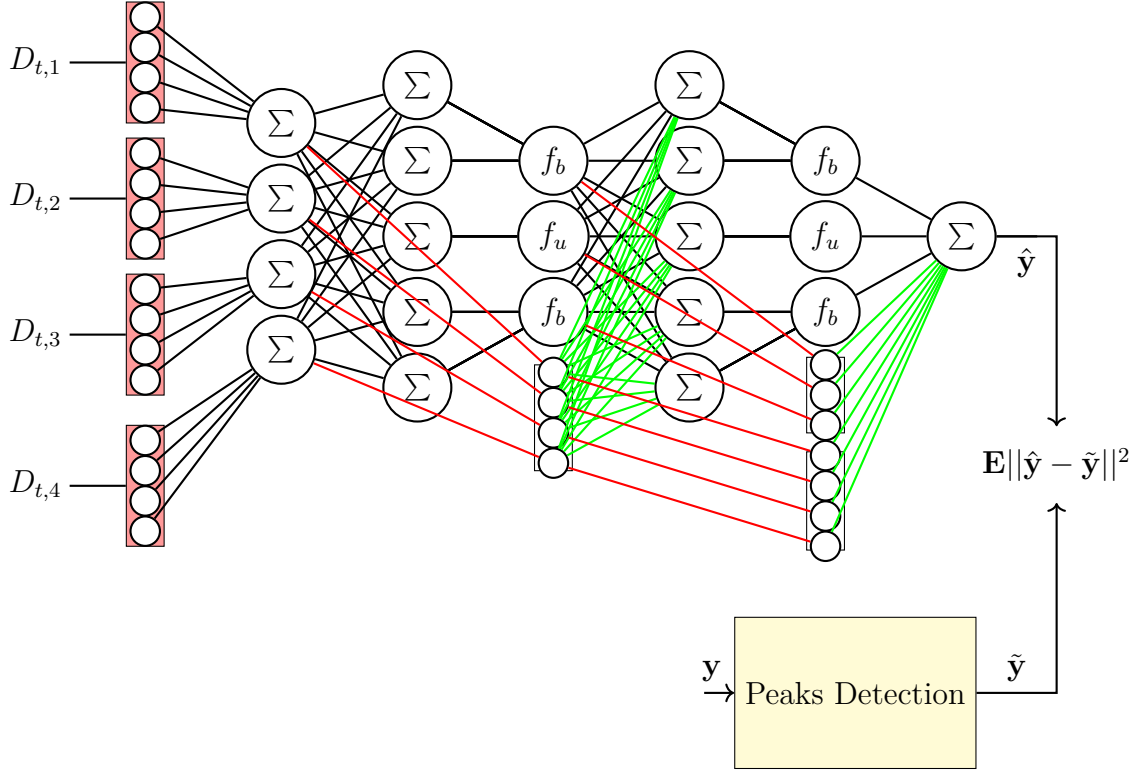


Figure 6.1: Harmonic selection and robust modeling

The problem at hand is fully identifiable, thereby the variance of any unbiased estimator $\hat{\theta}$ of θ has a theoretical limit called the Cramer-Rao Bound (CRB):

$$VAR(\hat{\theta}) \geq CRB(\theta) = F(\theta)^{-1} \quad (6.6)$$

where $F(\theta)$ is the Fisher Information Matrix (FIM), whose (i, j) -th entry, in this Gaussian noise case, is given by [76]:

$$F_{i,j}(\theta) = \frac{1}{\sigma_n^2} \left[\frac{\partial \mu(\theta)}{\partial \theta_i}^T \frac{\partial \mu(\theta)}{\partial \theta_j} \right] \quad (6.7)$$

where $\mu(\theta) = \mathbf{s} = [s(1), \dots, s(T)]^T$ is the T dimensional mean vector. The derivatives involved in (6.7) are straightforward and are omitted in this presentation. In the sequel, the CRB is used as a benchmark to illustrate the efficiency of our estimator.

6.6 Alternating Least Squares

We propose handling the nonlinear model in (4.2) by leveraging an alternating least squares (ALS) approach to minimize the objective function:

$$\arg \min_{\boldsymbol{\theta}} \sum_{t=1}^T \|x(t) - \hat{s}_1(t)(1 + \hat{s}_2(t))(1 + \hat{s}_3(t))\|_2^2 \quad (6.8)$$

where $\hat{s}_i = \mathbf{D}_i \hat{\mathbf{w}}_i + \tilde{\mathbf{D}}_i \hat{\mathbf{w}}_i$. Note that (6.8) represents the ML estimator in the white Gaussian noise case.

Rather than solving for all $\boldsymbol{\theta}_i$ jointly, the ALS method alternates between minimizing the least squares error for each source's parameters individually. By doing so, one reduces the non-linear optimization problem into a set of quadratic least squares (LS) problems (with closed form solution) [89]. For an observed data vector $\mathbf{x} = [x(1), \dots, x(T)]^T$, the ALS algorithm proceeds iteratively, updating the parameters $\boldsymbol{\theta}_{1 \leq i \leq 3}$ at each step using the closed-form analytical solution of the LS problem, as follows:

Algorithm 2 ALS (Alternating Least Squares).

- 1: **Input:** Initial data $\mathbf{x}$, number of iterations *epochs*
- 2: **Initialize:** Randomly initialize parameters $\boldsymbol{\theta}$
- 3: $k \leftarrow 0$
- 4: **while** $k < \text{epochs}$ **do**
- 5: $k \leftarrow k + 1$
- 6: Update parameters:

$$\boldsymbol{\theta}_1^{(k)} \leftarrow \mathbf{A}_1^\# \mathbf{b}_1, \quad \hat{\mathbf{s}}_1 = [\mathbf{D}_1, \tilde{\mathbf{D}}_1] \boldsymbol{\theta}_1^{(k)}$$

$$\boldsymbol{\theta}_2^{(k)} \leftarrow \mathbf{A}_2^\# \mathbf{b}_2, \quad \hat{\mathbf{s}}_2 = [\mathbf{D}_2, \tilde{\mathbf{D}}_2] \boldsymbol{\theta}_2^{(k)}$$

$$\boldsymbol{\theta}_3^{(k)} \leftarrow \mathbf{A}_3^\# \mathbf{b}_3, \quad \hat{\mathbf{s}}_3 = [\mathbf{D}_3, \tilde{\mathbf{D}}_3] \boldsymbol{\theta}_3^{(k)}$$

- 7: **end while**
 - 8: **Output:** $\boldsymbol{\theta}^{(k)}$
-

where $()^\#$ denotes the Moore-Penrose pseudo inverse and:

$$\mathbf{A}_1 = \text{diag}((1 + \hat{\mathbf{s}}_2) \odot (1 + \hat{\mathbf{s}}_3)) [\mathbf{D}_1, \tilde{\mathbf{D}}_1], \quad \mathbf{b}_1 = \mathbf{x} \quad (6.9)$$

$$\mathbf{A}_2 = \text{diag}((1 + \hat{\mathbf{s}}_3) \odot \hat{\mathbf{s}}_1) [\mathbf{D}_2, \tilde{\mathbf{D}}_2], \quad \mathbf{b}_2 = \mathbf{x} - \hat{\mathbf{s}}_1 \odot (1 + \hat{\mathbf{s}}_3) \quad (6.10)$$

$$\mathbf{A}_3 = \text{diag}((1 + \hat{\mathbf{s}}_2) \odot \hat{\mathbf{s}}_1) [\mathbf{D}_3, \tilde{\mathbf{D}}_3], \quad \mathbf{b}_3 = \mathbf{x} - \hat{\mathbf{s}}_1 \odot (1 + \hat{\mathbf{s}}_2) \quad (6.11)$$

where $\text{diag}(\mathbf{x})$ is the diagonal matrix with vector $\mathbf{x}$ as main diagonal and $\odot$ refers to the elementwise multiplication.

6.6.1 Algorithm's initialization

Random initialization can occasionally lead to poor convergence, i.e., convergence to local minima. Hence, to overcome this issue and to enhance the convergence rate, we replaced the random initialization with the weights given by LASSO applied to a linearized form of (4.2). The LASSO-like objective function is defined as follows:

$$\arg \min_{\mathbf{w}} \frac{1}{2} \sum_t \|x(t) - \mathbf{D}\mathbf{w}(t)\|_2^2 + \lambda \|\mathbf{w}\|_1 \quad (6.12)$$

where $\mathbf{D}$ represents the dictionary containing all possible combinations of the sources' harmonics according to the model (4.2) (see [87] for details), $\mathbf{w}$ denotes the modulated harmonic weights of the sources, and λ is the ℓ_1 -norm regularization penalty coefficient, which ensures the selection of the most significant source harmonics. Indeed, in practice, the number of harmonics is unknown, and so we build the dictionary $\mathbf{D}$ with an over-estimated number of harmonics for each source. However, the authors of [87] demonstrate that this approach does not achieve the theoretical Cramér-Rao bound, exhibiting a significant performance gap with the CRB, which will also be shown later. This result is not surprising, as this method estimates a quite large and redundant number of parameters, i.e., a total of $2H_1(1 + H_2)(1 + H_3)$ weight coefficients instead of $2(H_1 + H_2 + H_3)$ weight coefficients for our method.

6.7 Adaptive algorithm

When material degradations start occurring, the number of harmonics and their amplitudes and phases vary over time, in a continuous pattern [12]. Health monitoring and abnormalities detection require considering an adaptive process to estimate these time-varying parameters, initialized with the closed-form solution provided by LASSO or ALS at the initial time step.

Many possible solutions can be chosen for the adaptive algorithm. In this work, we have considered using a gradient-like method, namely ADAM optimizer⁵ (see [90] for the algorithm's details), applied on a mini-batch

⁵ADAM stands for Adaptive Moment Estimation.

of observations of size N , i.e. $\mathbf{x}_t = [x((tN + 1), \dots, x((t + 1)N))]^T$, and initialized with the parameter value at the previous iteration.

Our adaptive algorithm is summarized in the following table:

Algorithm 3 Time-varying Parameters Estimation.

- 1: **Input:** Observations $\mathbf{x}_0, \mathbf{x}_1, \dots$
 - 2: **Initialize:** $\boldsymbol{\theta}_0 = \text{ALS}(\mathbf{D}, \mathbf{x}_0)$
 - 3: **for** $t = 1, 2, \dots$ **do**
 - 4: $\boldsymbol{\theta}_t = \text{ADAM}(\boldsymbol{\theta}_{t-1}, \mathbf{D}, \mathbf{x}_t)$
 - 5: **end for**
 - 6: **Output:** $\boldsymbol{\theta}_t$
-

6.8 Simulation results

6.8.1 Neural network

The synthesized signal is generated according to the following model, which, as discussed in the previous chapter and to the best of our knowledge, provides the most appropriate representation of a planetary gearbox vibration signal, as developed by [12]:

$$y(t) = \sum_{p=1}^P h_p(t) * s_{1,p}(t) (1 + s_{2,p}(t)) (1 + s_{3,p}(t) + s_{4,p}(t)) \quad (6.13)$$

where P denotes the number of planets in the considered planetary gear set. The term $s_{1,p}(t)$ represents the p^{th} meshing component, $s_{2,p}(t)$ the modulation at the carrier plate frequency, $s_{3,p}(t)$ the contribution of the sun gear rotation to planet p , and $s_{4,p}(t)$ the intrinsic contribution of planet p .

The fundamental frequencies of the sources s_i in (3.21) are defined as $\{f_1 = 99, f_2 = 1, f_3 = 2.83, f_4 = 4.67\}$, with 10 significant harmonics considered for each source. The signal length is set to $T = 405,405$ samples, with a sampling frequency $f_s = 11583$. The simulated data are generated with $P = 3$. The source coefficients are randomly assigned, and each term is associated with a randomly generated filter coefficient h_p . The signal-to-noise ratio (SNR) is fixed at 0 dB. The objective is to validate, using simulated data, the ability to isolate the elementary source associated with each polynomial term from a single measurement.

For each neural network, we adopt the same configuration as in the previous

chapters. The number of neural networks in the stacked architecture is treated as a hyperparameter; in this study, it is intentionally overestimated and set to 5. The (overestimated) number of source harmonics is set to 20. Following the same training procedure described in the previous chapter 3, we successfully activate only three distinct neural networks, as expected, using a regularization parameter $\lambda = 0.001$ for 500 epochs. A satisfactory reconstruction is obtained, as shown in Fig. 6.2, demonstrating good matching of the sideband harmonics. The selected harmonics of the sources for each

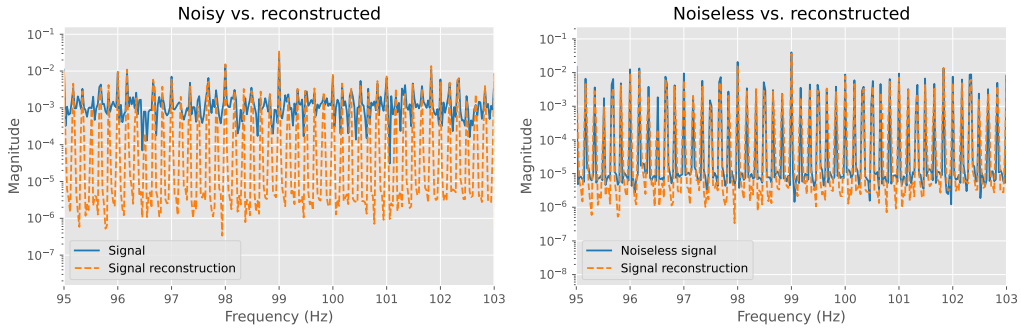


Figure 6.2: Zoomed portion of the signal reconstruction at the first carrier harmonic.

polynomial term are shown in Fig. 6.3 and Fig. 6.4. Each row represents the power spectrum of the sources activated within each network at different SNR levels (0 dB and 20 dB). The ten most significant harmonics are clearly identifiable.

Table 6.1: Correlation between true and predicted signals for different networks at various SNR.

Active network	$\text{corr}(s_{1,p}, \hat{s}_{1,p})$		$\text{corr}(s_{2,p}, \hat{s}_{2,p})$		$\text{corr}(s_{3,p}, \hat{s}_{3,p})$	
	0 dB	20 dB	0 dB	20 dB	0 dB	20 dB
NN 1	-	-	-	-	-	-
NN 2	0.96	0.99	0.99	0.99	0.99	0.99
NN 3	-	-	-	-	-	-
NN 4	0.99	0.99	0.99	0.99	0.99	0.99
NN 5	0.938	0.99	0.99	0.99	0.99	0.99

Moreover, to evaluate the source separation task, i.e., the recovery of the original sources $s_{i,p}$ from the mixed data, we compute the correlation between the recovered sources and the original ones in the frequency domain. The source separation results at 20 dB and 0 dB are presented in Table 6.1. After thresholding the neural-network weights below 0.01, we extract the analytical expressions identified by each sub-network and present the resulting equations in Table 4.4.6. These results demonstrate strong correlation performance, where the ten most significant harmonics are selected, and the source s_4 is not selected as intended. In addition, the analytical expressions extracted from the thresholded sub-networks closely correspond to the true equations used in the simulated data. In the time domain, the sources are separated up to a scaling factor, which is an inherent ambiguity in BSS. Only in the case of the sources $s_{1,p}$, they are separated up to both a phase and a scaling factor, which constitutes a standard ambiguity in BSS.

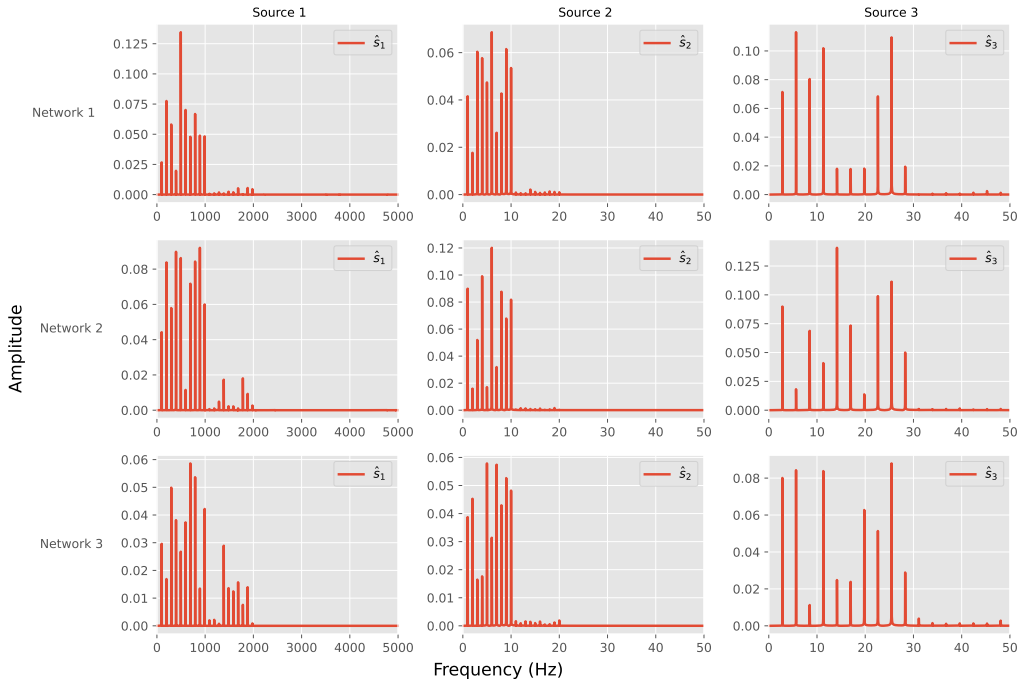


Figure 6.3: Source separation results at 0 dB.

CHAPTER 6. SOURCE SEPARATION AND TRACKING OF PERIODIC SIGNALS

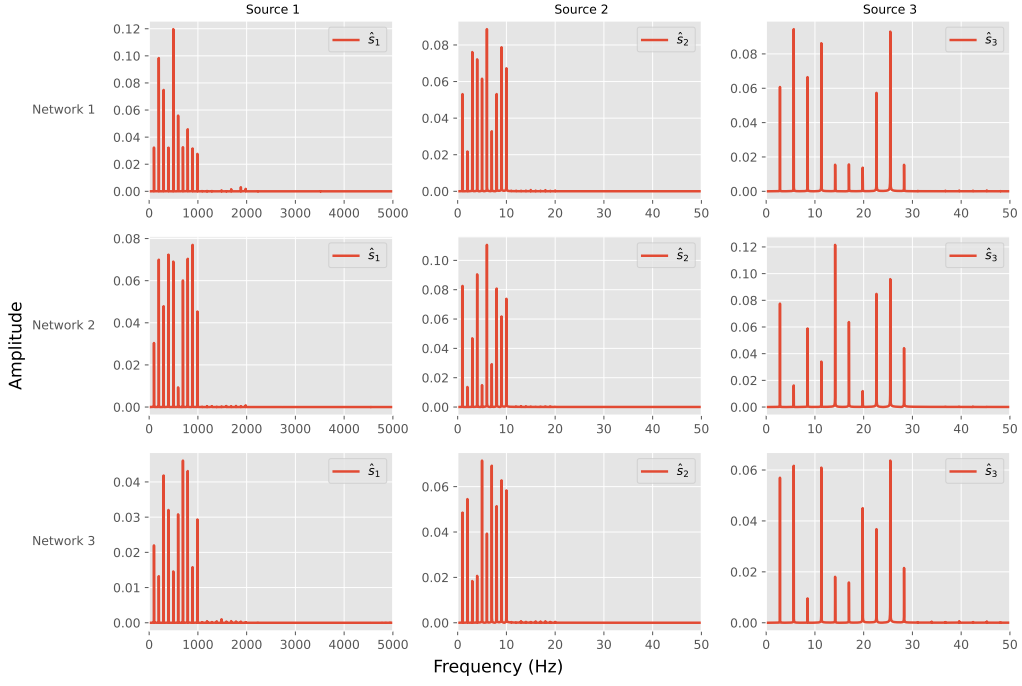


Figure 6.4: Source separation results at 20 dB.

Table 6.2: Neural Network equations at 0 dB.

NN	Formulas
1	0
2	$-3.382 s_{12} (0.242 - 0.79427 s_{32}) (1.516 + 6.09 s_{22})$
3	0
4	$32.24 s_{14} (0.136 + 1.089 s_{24}) (0.304 + 1.293 s_{34})$
5	$-22.44 s_{15} (0.183 - 1.42 s_{25}) (0.204 + 0.849 s_{35})$ height

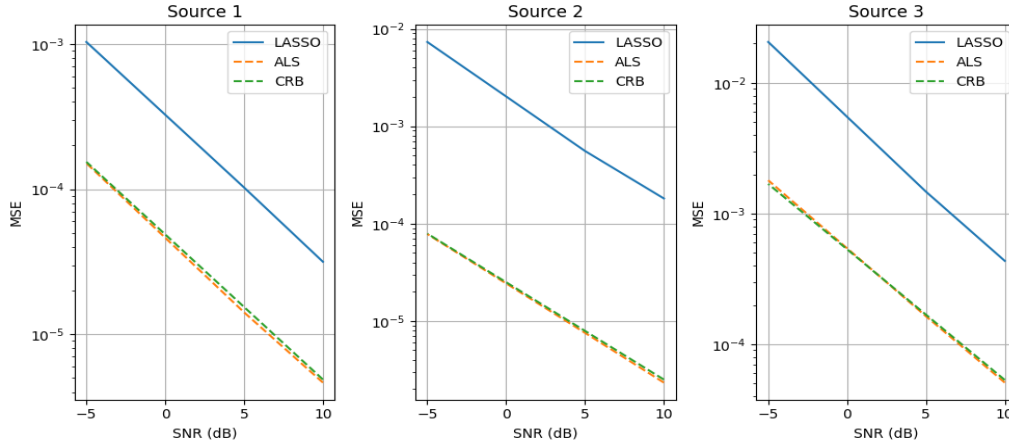


Figure 6.5: Our approach-based ALS performance against the CRB with a low level of SNR over 100 runs.

6.8.2 ALS

Synthesized data

To assess the performance of the ALS-based proposed approach and compare it against the Cramer-Rao theoretical limit, we consider a synthetic signal $\mathbf{x}$ sampled at $f_s = 1000$ Hz with $T = 405,405$ samples. The signal consists of $M_1 = 3$ carrier harmonics with a fundamental frequency $f_1 = 99$ Hz. The first modulated source, $\mathbf{s}_2$, contains $M_2 = 3$ harmonics with a fundamental frequency $f_2 = 1$ Hz, and the second modulated source, $\mathbf{s}_3$, has a fundamental frequency $f_3 = 2.83$ Hz and contains also $M_3 = 3$ harmonics. To evaluate the robustness of the approach against the noise, white Gaussian noise has been added to the simulated signal $\mathbf{s}$ such that the input signal-to-noise ratio varies from -5 dB to 10 dB.

Figure 6.5 shows the results of running LASSO on our model with a sparsity penalty. The mean square error (MSE) of the source parameters exhibits a large gap from the CRB. However, using LASSO as an initialization for ALS later leads to a good match with the CRB. In addition to the statistical efficiency of our solution, it also demonstrates robustness to harmonic number overestimation errors. Indeed, the plots in Figure 6.5 have been obtained with an overestimated number of harmonics $M = 5$ (for all sources) while the exact number was $M = 3$.

In our second experiment, we generate data with variable harmonic

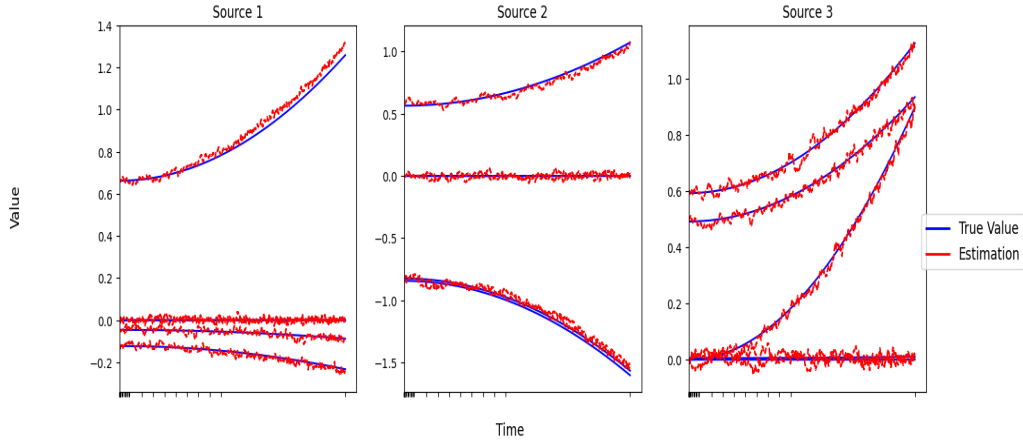


Figure 6.6: Tracking of source parameters over time with $SNR = 10$ dB. Each plot shows the tracking of five harmonics of the considered source.

amplitudes and phases for both the carrier and the modulations, then run our tracking (adaptive) algorithm. ALS is first applied to an initial batch of data before attempting to track the parameters. The performance evaluation for $SNR = 10$ is shown in Figure 6.6, where we present the results over time. We conclude that starting with an optimal initialization using ALS allows for an easy gradient descent. In the figure, we observe that some parameters are initially set to zero during the first time steps (source 3), illustrating a situation of harmonic number overestimation. However, over time, they gradually increase due to degradation, and our tracking algorithm captures this behavior. This is crucial to system health monitoring.

6.8.3 Application: Planetary gearbox

In this section, we examine the ALS approach to analyze the vibration signal recorded during a fatigue test on a helicopter planetary gearbox. The test was designed to investigate the phenomenon of fatigue cracking in a planet gear. The sampling frequency is $f_s = 11583$ Hz. The characteristics of the planetary gearbox are as follows: the ring gear has 99 teeth, the planet gear has 35 teeth, the sun gear has 27 teeth, and there are 4 planet gears. Only the last 7 days have been loaded, during which we have 526 files of 100-second data acquisitions each. For more details on the dataset, refer to [69]. Our focus is to individually estimate the meshing signal s_1 , the planet carrier-

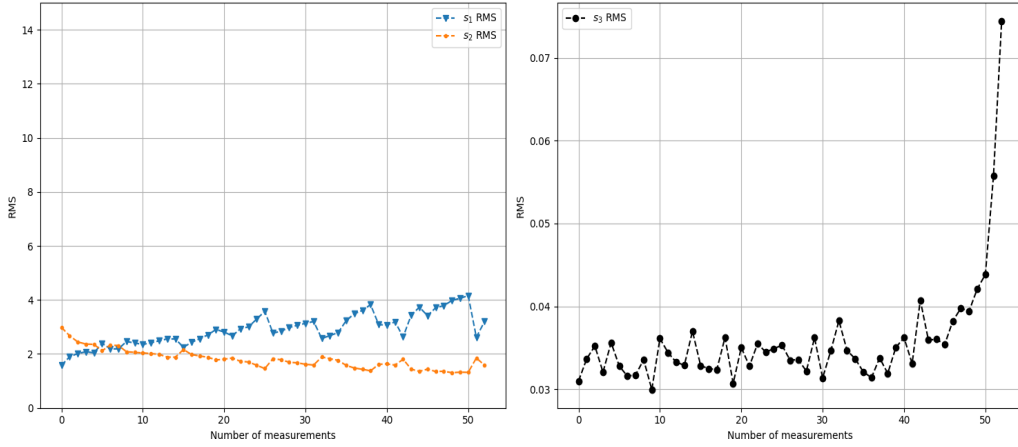


Figure 6.7: The root-mean-square (RMS) of the estimated sources s_1, s_2, s_3 . The modulation signal s_1 exhibits a monotonic trend correlated with the increasing planetary gear degradation, especially in the final measurements, where an artificial crack is introduced along the planetary gear. In contrast, the meshing and modulation signal s_2 fluctuates over time without a clear monotonic pattern.

related signal s_2 , and the planet gear-related signal s_3 . We set the number of overestimated harmonics for each source s_i to 15. The vibration signal model of such planetary gearbox is the same given by (4.2). We performed ALS on our dataset, analyzing the measurements from 1 to 526 with an increment of 10. We then calculated the root mean squares (RMS) of the sources $s_{i \leq i \leq 3}$, as shown in Figure 6.7. The RMS of the meshing source s_1 and the carrier plate signal s_2 fluctuates. However, the RMS of the planet signal exhibits a monotonic behavior, which indicates the occurrence of a fault in the planet gear that matches the planet-introduced crack. The crack increases drastically in the last days, as the HUMS23 Team [69] made the crack more severe during that period. With our simple proposed method, one can easily track the defective part while monitoring its degradation over time.

6.9 Conclusion

In this chapter, we presented two approaches for source separation. First, we proposed the use of a symbolic regression-based neural network to isolate the elementary sources of each polynomial term from single measurements. This method demonstrated good accuracy in recovering the desired sources; however, the carrier signal showed poor estimation, potentially due to effects from the transfer function. In future work, this approach may be followed by an ALS algorithm to refine the carrier signal estimation.

Second, we introduced an approach of a multi-modulation decomposition using an alternating least squares approach, providing a vibration signal and a set of system rotation frequencies. The extracted sources have crucial information about the related components, which could aid in early fault detection and localization. A synthetic signal was used to illustrate the statistical efficiency of the proposed approach, i.e., it attains the theoretical CRB limit.

Additionally, we ran a use-case application with a planetary gearbox, where we successfully identified the defective part as the planet gear and tracked its degradation over time. However, further development is needed to extend this method to non-stationary vibration signals (e.g., variable speeds and loads). This will require incorporating a frequency estimation and tracking step alongside amplitude estimation in the nonlinear model to dynamically adapt to changing conditions.

Chapter 7

Conclusion

Effective vibration-based condition monitoring of rotating machinery, such as gear transmission systems, relies on a comprehensive understanding of vibration generation mechanisms and the reliable extraction of informative features. In rotating machines, the acquired vibration signal typically exhibits a very structured spectrum composed of gear meshing frequencies, their harmonics, and other dominant components in their sidebands. Interpreting the relationships between these spectral components and their physical origins, particularly their association with the rotational speeds of moving elements, has been a major focus of condition and health monitoring research for decades. Despite the powerful conventional vibration separation and analysis techniques designed for fault detection, their applicability remains limited when dealing with complex mechanical architectures. Epicyclic transmission systems, in particular, are characterized by multiple moving excitation sources and restricted measurement accessibility, which significantly complicate the computation of elementary vibration components and the identification of fault locations. Compared with fixed-shaft systems, where such techniques are generally effective, conventional methods do not appear suitable for feature extraction in epicyclic gear systems due to the moving excitation sources. Moreover, the multiple excitation sources are modified by their propagation paths through the system structure before reaching the sensor point. As a result, direct measurement of the excitations at the meshing points is generally infeasible. This observation motivates the identification of the impulse responses of the system transfer functions, which enables the reconstruction of the original excitation sources by compensating for the transmission path effects.

Following this direction, this PhD thesis has started developing new tools

for fault detection of rotation machine that exhibits a complex mechanism.

7.1 Contributions

In this thesis, we investigated the phenomenological modeling of vibration data from rotating machines, with a particular focus on gear transmission systems with epicyclic motion. This included the interpretation of spectrum carrier harmonics and the dominant harmonics in their vicinity in relation to system dynamics. An automatic method based on symbolic analysis was proposed to describe the vibration signal using the characteristic frequencies of the machine. Additionally, the identification of transfer function effects was examined both during the modeling process and as a post-modeling step, based on a predefined vibration signal model. A fundamental carrier frequency estimation method, based on sideband structure analysis, was also introduced. Finally, a model-based source separation approach was proposed to recover the elementary vibration components and to track their variations.

Vibration phenomenological modeling

The vibration signal captured from rotating machines exhibits complex characteristics due to their dynamic behavior, structural imperfections, manufacturing errors, and the presence of potential faulty elements. Consequently, an automatic approach for deriving models that represent vibration data in terms of characteristic rotational frequencies is highly desirable. Traditional analysis methods typically require substantial domain expertise and are time-consuming, as different machine kinematic configurations and operating regimes produce distinct vibration spectrum structures. These structures often contain numerous dominant harmonics that must be accurately interpreted to reliably assess the machine's health condition.

- We leverage a symbolic regression-based neural network framework to fit the vibration data using the characteristic frequencies of the machine. A fully connected neural network is employed in which typical activation functions are replaced by mathematical operators representing the primitive functions of the recovered analytical formulas. In addition, a convolution layer is introduced following of the neural network to compensate for the effects of the system transfer functions.
- To account for the fact that vibration signals originate from multiple excitation sources, we further propose a stacked neural network-based

symbolic regression approach. In this framework, the outputs of an overestimated number of neural networks are concatenated to reconstruct the vibration signal. This formulation enables the treatment of a multiple-input single-output system, allowing the separate reconstruction of each excitation source along with its corresponding transmission path effects. The learning process of the proposed networks is governed by a trade-off between a reconstruction error term and a sparsity penalty term, ensuring that the identified formulas are interpretable and compact.

- Since vibration signal modeling relies on the interpretation of dominant harmonics in the vicinity of the carrier frequency and its harmonics, we propose a robust data modeling approach based on the same neural network-based symbolic regression framework. In this approach, the neural network is trained using artificially generated vibration signals. These signals are constructed following a harmonic detection step and are synthesized as a Fourier series composed of the detected harmonics with unit magnitudes.

Transfer function identification

Accurate reconstruction of the true excitation source is essential for reliable assessment of system health. This process requires the removal of the effects introduced by the system's transfer functions. Recently, transfer function identification has been performed using the vibration stochastic component, with cepstrum-based modal analysis demonstrating effectiveness in estimating the impulse response. However, in this thesis, we focus on the deterministic component of the vibration signal to recover the impulse response of the transfer function, since the vibration data exhibit an extremely rich harmonic frequency content.

- The transfer function identification is performed downstream of our neural network-based symbolic regression framework. Within the network's forward pass, Fourier coefficients of characteristic frequencies and their coupling analytical expressions are estimated through hidden symbolic layers. In addition, a dedicated convolution layer is employed to compensate for the effects of the transfer function by estimating either FIR or IIR parameters.
- We have proposed an extended modeling framework that treats each excitation source separately with the stacked neural network, thereby

enabling the identification of the transfer function of each vibration component from a single measurement.

- In the post-processing stage following modeling, a predefined model describing the power-spectrum sideband structure and the estimated fundamental characteristic frequencies is employed. Based on this model, we propose a new approach for recovering transfer-function parameters. In rotating machinery, particularly systems with epicyclic motion, carrier-frequency sidebands often exhibit asymmetric magnitudes, which may result from transfer-function effects. By reformulating these magnitude asymmetries into a mathematical representation based on the fundamental characteristic frequencies, we formulate a nonlinear minimization problem to estimate the coefficients of FIR and IIR transfer function impulsive response. This approach yields good results for filters with low memory taps; however, further work on convergence and hyperparameter selection is needed to demonstrate its applicability to real-world datasets.

Frequency estimation

Based on a predefined model of the sideband harmonic structure, we propose a peak-matching method between the detected spectral peaks and the normalized spectral peaks derived from the model. This approach enables effective estimation of the fundamental carrier frequency. In addition, an iterative refinement algorithm is proposed to adjust the harmonic frequencies of the spectrum, as small frequency variations can lead to inaccuracies in source separation methods. The estimation algorithm is applied to a planetary gear set, where sidebands around the meshing frequency and its harmonics often appear even under healthy operating conditions, owing to the system's unique dynamic behavior. The proposed approach provides an accurate estimation for the planetary gear set.

Source separation with alternating least squares

An alternating least squares method was implemented to obtain a maximum likelihood estimator based on a nonlinear model, which achieves the theoretical Cramér-Rao bound in simulations, unlike the linearized model. Furthermore, a gradient descent step was employed to track variations in vibration components, assuming slow element degradation and gradual regime changes. This approach provides a continuous method for monitoring the

fundamental vibration components of the system. The method was initially validated on a planetary gear set with a cracked planet gear.

7.2 Perspectives

This PhD thesis opens several perspectives for improving the proposed methodologies, from application viewpoints, and for suggesting new promising ideas that further enhance the applicability of our methodologies to real-world datasets.

7.2.1 Near-Term

Parametric phenomenological modeling

Developing a vibration data modeling approach for variable operating regimes is crucial. Under such regimes, models may change in response to high-speed or load conditions, potentially leading to new system interactions. There is a potential possibility of learning parametric analytical expressions that describe non-stationary data, which could provide a framework for the source separation of non-stationary polynomial mixtures. Continual learning of model parameters can be achieved using batched datasets, where, as is well known in the deep learning community, hypernetworks, a network that generates the weights of a target model based on time steps, can be employed [91].

Frequency Overlapping

From our perspective, the overlap of meshing harmonic sidebands, which is commonly observed in real-world data, may affect the modeling process. We will begin an investigation using simulated data to examine how such overlap in the frequency domain influences the identified model.

7.2.2 Future works

Cyclic transfer function

The unique dynamic behavior of an epicyclic gear transmission exhibits time-varying transmission paths due to the cyclic motion of the meshing point between the planet and ring gears around the sun gear. Consequently, the system's transfer function demonstrates a periodic profile, which can be

systematically incorporated into the modeling and analysis of the transmission process. In state-of-the-art methods, researchers often use a Hann window to model the amplitude variations of excitation sources from the meshing point to the sensor location [6], or a periodic modulation function based on the rotational frequency of the planet carrier [13]. We aim to investigate the identification of cyclic transfer functions, which appear to be novel in the context of current system identification methods, for broader applications.

Line structured estimation

The vibration signal exhibits a highly structured power spectrum, with multiple harmonics arising from different machine components and vibration sources. Extracting groups of characteristic frequencies from this spectrum, among all existing harmonics, represents a promising direction for analysis. Unlike classical frequency and peak detection methods, such as ESPRIT or topology-based peak-finding algorithms, these approaches treat harmonics solely, without considering the relation among them. We propose investigating the estimation of characteristic parameters (frequencies) for groups of harmonics in the spectrum. In this context, sparse Bayesian learning offers a potential research direction, as it allows the leveraging of prior knowledge of the mechanical system into the Bayesian formulation [92].

For larger input dictionary with autoencoders

In this thesis, we represent the elementary vibration components as ideal periodic sources with a Fourier representation. This assumption, however, constrains the input space and limits the accurate reconstruction of the vibration signal. Autoencoders and variational autoencoders have been used for blind source separation of independent sources, where an encoder–multi-decoder framework is exploited to separate a linear mixture by enforcing independence criteria among the latent representations associated with each decoder. The output of each decoder corresponds to an estimated source, and the sum of these outputs reconstructs the observed data. This approach has been applied in the context of music source separation [93]. However, this approach has not been explored for nonlinear polynomial mixtures, such as in mechanical vibration measurements. In future work, we propose investigating an encoder–multi-decoder architecture to reconstruct the elementary sources, which are subsequently entered into a neural network-based symbolic regression to learn the nonlinear mixing process at the decoder outputs and reconstruct the measured data. The learning process must ensure the

uniqueness of the decoder outputs, either by imposing constraints on the latent space or by enforcing correlations in the power spectra of the decoder outputs that reflect the underlying mechanical kinematic model.

Integrating Dynamic Equations with Symbolic Regression

A gearbox dynamic model, based on the numerical solution of the system's equations of motion, has commonly been used to validate and compare with analytical phenomenological models, as both approaches describe the system's vibrations. Each model offers distinct advantages for condition monitoring under variable operating speed and load [7]. In future work, we plan to combine the dynamic equations with symbolic regression to enhance source estimation and prediction. This approach will also ensure that the analytical models identified through symbolic regression remain consistent with the underlying dynamic equations. Our motivation stems from recent advances in physics-informed neural networks, where neural networks are trained in conjunction with system dynamic equations to improve model accuracy and physical consistency.

Bibliography

- [1] J. Lin and R. G. Parker, “Analytical characterization of the unique properties of planetary gear free vibration,” 1999.
- [2] P. McFadden and J. Smith, “An explanation for the asymmetry of the modulation sidebands about the tooth meshing frequency in epicyclic gear vibration,” *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 199, no. 1, pp. 65–70, 1985.
- [3] J. McNames, “Fourier series analysis of epicyclic gearbox vibration,” *J. Vib. Acoust.*, vol. 124, no. 1, pp. 150–153, 2002.
- [4] R. G. Parker and J. Lin, “Mesh phasing relationships in planetary and epicyclic gears,” *J. Mech. Des.*, vol. 126, no. 2, pp. 365–370, 2004.
- [5] C. Molina Vicuña, “Theoretical frequency analysis of vibrations from planetary gearboxes,” *Forschung im Ingenieurwesen*, vol. 76, no. 1, pp. 15–31, 2012.
- [6] M. Inalpolat and A. Kahraman, “A theoretical and experimental investigation of modulation sidebands of planetary gear sets,” *Journal of sound and vibration*, vol. 323, no. 3-5, pp. 677–696, 2009.
- [7] J. Parra and C. M. Vicuña, “Two methods for modeling vibrations of planetary gearboxes including faults: Comparison and validation,” *Mechanical Systems and Signal Processing*, vol. 92, pp. 213–225, 2017.
- [8] M. Inalpolat and A. Kahraman, “A dynamic model to predict modulation sidebands of a planetary gear set having manufacturing errors,” *Journal of Sound and Vibration*, vol. 329, no. 4, pp. 371–393, 2010.

- [9] Z. Feng and M. J. Zuo, "Vibration signal models for fault diagnosis of planetary gearboxes," *Journal of Sound and Vibration*, vol. 331, no. 22, pp. 4919–4939, 2012.
- [10] L. Hong, J. S. Dhupia, and S. Sheng, "An explanation of frequency features enabling detection of faults in equally spaced planetary gearbox," *Mechanism and Machine Theory*, vol. 73, pp. 169–183, 2014.
- [11] M. Zhang, K. Wang, D. Wei, and M. J. Zuo, "Amplitudes of characteristic frequencies for fault diagnosis of planetary gearbox," *Journal of Sound and Vibration*, vol. 432, pp. 119–132, 2018.
- [12] E. Hubert, *Vibration monitoring of an aeronautic power transmission system*. PhD thesis, Université de Lyon, 2019.
- [13] M. Zhang, D. Li, M. J. Zuo, J. Liu, H. Xiang, Y. Song, and K. Wang, "An improved phenomenological model of vibrations for planetary gearboxes," *Journal of Sound and Vibration*, vol. 496, p. 115919, 2021.
- [14] V. Sharma and A. Parey, "A review of gear fault diagnosis using various condition indicators," *Procedia engineering*, vol. 144, pp. 253–263, 2016.
- [15] Z. Chen and Y. Shao, "Dynamic features of a planetary gear system with tooth crack under different sizes and inclination angles," *Journal of Vibration and Acoustics*, vol. 135, no. 3, p. 031004, 2013.
- [16] P. McFadden, "A technique for calculating the time domain averages of the vibration of the individual planet gears and the sun gear in an epicyclic gearbox," *Journal of Sound and vibration*, vol. 144, no. 1, pp. 163–172, 1991.
- [17] D. G. Lewicki, K. E. LaBerge, R. T. Ehinger, and J. Fetty, "Planetary gearbox fault detection using vibration separation techniques," in *67th Annual Forum and Technology Display (Forum 67)*, no. E-17816, 2011.
- [18] J. M. Ha, B. D. Youn, H. Oh, B. Han, Y. Jung, and J. Park, "Autocorrelation-based time synchronous averaging for condition monitoring of planetary gearboxes in wind turbines," *Mechanical Systems and Signal Processing*, vol. 70, pp. 161–175, 2016.
- [19] P. McFadden, "Detecting fatigue cracks in gears by amplitude and phase demodulation of the meshing vibration," *Journal of Vibration and Acoustics*, vol. 108, no. 2, pp. 165–170, 1986.

- [20] Y. Lei, D. Han, J. Lin, and Z. He, “Planetary gearbox fault diagnosis using an adaptive stochastic resonance method,” *Mechanical systems and signal processing*, vol. 38, no. 1, pp. 113–124, 2013.
- [21] Z. Liu, X. Zhao, M. J. Zuo, and H. Xu, “Feature selection for fault level diagnosis of planetary gearboxes,” *Advances in Data Analysis and Classification*, vol. 8, no. 4, pp. 377–401, 2014.
- [22] Z. Feng and M. Liang, “Fault diagnosis of wind turbine planetary gearbox under nonstationary conditions via adaptive optimal kernel time–frequency analysis,” *Renewable Energy*, vol. 66, pp. 468–477, 2014.
- [23] Z. Feng, X. Chen, M. Liang, and F. Ma, “Time–frequency demodulation analysis based on iterative generalized demodulation for fault diagnosis of planetary gearbox under nonstationary conditions,” *Mechanical Systems and Signal Processing*, vol. 62, pp. 54–74, 2015.
- [24] G. Cheng, X. Chen, H. Li, P. Li, and H. Liu, “Study on planetary gear fault diagnosis based on entropy feature fusion of ensemble empirical mode decomposition,” *Measurement*, vol. 91, pp. 140–154, 2016.
- [25] M. Lebold, K. McClintic, R. Campbell, C. Byington, and K. Maynard, “Review of vibration analysis methods for gearbox diagnostics and prognostics,” in *Proceedings of the 54th meeting of the society for machinery failure prevention technology*, vol. 634, p. 16, Virginia Beach, VA, 2000.
- [26] J. Shen, L. Zhang, and N. Hu, “Fault diagnosis of planet gear using continuous vibration separation and minimum entropy deconvolution,” *Applied Sciences*, vol. 10, no. 22, p. 8062, 2020.
- [27] Y. Guo, L. Zhao, X. Wu, and J. Na, “Vibration separation technique based localized tooth fault detection of planetary gear sets: A tutorial,” *Mechanical Systems and Signal Processing*, vol. 129, pp. 130–147, 2019.
- [28] P. McFadden, “A revised model for the extraction of periodic waveforms by time domain averaging,” *Mechanical systems and signal processing*, vol. 1, no. 1, pp. 83–95, 1987.
- [29] P. D. Samuel and D. J. Pines, “Vibration separation methodology for planetary gear health monitoring,” in *Smart Structures and Materials 2000: Smart Structures and Integrated Systems*, vol. 3985, pp. 250–260, SPIE, 2000.

- [30] Y. Guo, “Gear fault diagnosis based on narrowband demodulation with frequency shift and spectrum edit,” *International Journal of Engineering and Technology Innovation*, vol. 6, no. 4, p. 243, 2016.
- [31] Y.-r. Wang, Q. Jin, G.-d. Sun, and C.-f. Sun, “Planetary gearbox fault feature learning using conditional variational neural networks under noise environment,” *Knowledge-Based Systems*, vol. 163, pp. 438–449, 2019.
- [32] S. Ma, F. Chu, and Q. Han, “Deep residual learning with demodulated time-frequency features for fault diagnosis of planetary gearbox under nonstationary running conditions,” *Mechanical Systems and Signal Processing*, vol. 127, pp. 190–201, 2019.
- [33] H. Chen, N. Hu, Z. Cheng, L. Zhang, and Y. Zhang, “A deep convolutional neural network based fusion method of two-direction vibration signal data for health state identification of planetary gearboxes,” *Measurement*, vol. 146, pp. 268–278, 2019.
- [34] M. Roan, J. G. Erling, and L. H. Sibul, “A new, non-linear, adaptive, blind source separation approach to gear tooth failure detection and analysis,” *Mechanical Systems and Signal Processing*, vol. 16, no. 5, pp. 719–740, 2002.
- [35] G. Elia, M. Cocconcelli, E. Mucchi, and G. Dalpiaz, “Combining blind separation and cyclostationary techniques for monitoring distributed wear in gearbox rolling bearings,” *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 231, no. 6, pp. 1113–1128, 2017.
- [36] P. McFadden and J. Smith, “A signal processing technique for detecting local defects in a gear from the signal average of the vibration,” *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 199, no. 4, pp. 287–292, 1985.
- [37] K. Sabri, M. El Badaoui, F. Guillet, A. Adib, and D. Aboutajdine, “Gear signal separation by exploiting the spectral diversity and cyclostationarity,” in *2006 IEEE International Conference on Acoustics Speech and Signal Processing Proceedings*, vol. 3, pp. III–III, IEEE, 2006.

- [38] Y. Yang, R. Xie, M. Li, and W. Cheng, “A review on the application of blind source separation in vibration analysis of mechanical systems,” *Measurement*, vol. 227, p. 114241, 2024.
- [39] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso, and E. Moulines, “A blind source separation technique using second-order statistics,” *IEEE Transactions on signal processing*, vol. 45, no. 2, pp. 434–444, 1997.
- [40] C. Serviere and P. Fabry, “Blind source separation of noisy harmonic signals for rotating machine diagnosis,” *Journal of Sound and Vibration*, vol. 272, no. 1-2, pp. 317–339, 2004.
- [41] F. Duan, M. Corsar, L. Zhou, and D. Mba, “Using independent component analysis scheme for helicopter main gearbox bearing defect identification,” in *2017 IEEE International Conference on Prognostics and Health Management (ICPHM)*, pp. 252–259, IEEE, 2017.
- [42] Y. Deville, L. T. Duarte, and S. Hosseini, *Nonlinear Blind Source Separation and Blind Mixture Identification: Methods for Bilinear, Linear-quadratic and Polynomial Mixtures*. Springer Nature, 2021.
- [43] Y. Deville and L. T. Duarte, “An overview of blind source separation methods for linear-quadratic and post-nonlinear mixtures,” in *International Conference on Latent Variable Analysis and Signal Separation*, pp. 155–167, Springer, 2015.
- [44] R. Randall, J. Antoni, and W. Smith, “A survey of the application of the cepstrum to structural modal analysis,” *Mechanical Systems and Signal Processing*, vol. 118, pp. 716–741, 2019.
- [45] G. L. McDonald and Q. Zhao, “Multipoint optimal minimum entropy deconvolution and convolution fix: application to vibration fault detection,” *Mechanical Systems and Signal Processing*, vol. 82, pp. 461–477, 2017.
- [46] K. Liang, M. Zhao, J. Lin, J. Jiao, and C. Ding, “Maximum average kurtosis deconvolution and its application for the impulsive fault feature enhancement of rotating machinery,” *Mechanical Systems and Signal Processing*, vol. 149, p. 107323, 2021.
- [47] R. Lu, M. R. Shahriar, P. Borghesani, R. B. Randall, and Z. Peng, “Removal of transfer function effects from transmission error measurements

- using cepstrum-based operational modal analysis,” *Mechanical Systems and Signal Processing*, vol. 165, p. 108324, 2022.
- [48] R. B. Randall, “A history of cepstrum analysis and its application to mechanical problems,” *Mechanical Systems and Signal Processing*, vol. 97, pp. 3–19, 2017.
- [49] O. Matania, R. Klein, and J. Bortman, “Algorithms for spectrum background estimation of non-stationary signals,” *Mechanical Systems and Signal Processing*, vol. 167, p. 108551, 2022.
- [50] R. Lu, P. Borghesani, R. B. Randall, W. A. Smith, and Z. Peng, “Removal of transfer function effects from gear vibration signals under constant and variable speed conditions,” *Mechanical Systems and Signal Processing*, vol. 184, p. 109714, 2023.
- [51] W. A. Smith and R. B. Randall, “Cepstrum-based operational modal analysis revisited: a discussion on pole–zero models and the regeneration of frequency response functions,” *Mechanical systems and signal processing*, vol. 79, pp. 30–46, 2016.
- [52] W. La Cava, B. Burlacu, M. Virgolin, M. Kommenda, P. Orzechowski, F. De França, Y. Jin, and J. Moore, “Contemporary symbolic regression methods and their relative performance,” *Advances in neural information processing systems*, vol. 2021, pp. 1–16, 12 2021.
- [53] G. S. Martius and C. Lampert, “Extrapolation and learning equations,” in *5th International Conference on Learning Representations, ICLR 2017-Workshop Track Proceedings*, 2017.
- [54] S. Sahoo, C. Lampert, and G. Martius, “Learning equations for extrapolation and control,” in *International Conference on Machine Learning*, pp. 4442–4450, PMLR, 2018.
- [55] S. Kim, P. Y. Lu, S. Mukherjee, M. Gilbert, L. Jing, V. vCeperić, and M. Soljavčić, “Integration of neural network-based symbolic regression in deep learning for scientific discovery,” *IEEE transactions on neural networks and learning systems*, vol. 32, no. 9, pp. 4166–4177, 2020.
- [56] S.-M. Udrescu and M. Tegmark, “Ai feynman: A physics-inspired method for symbolic regression,” *Science advances*, vol. 6, no. 16, p. eaay2631, 2020.

- [57] M. Valipour, B. You, M. Panju, and A. Ghodsi, “SymbolicGPT: A generative transformer model for symbolic regression,” in *NeurIPS 2022 Workshop on Efficient Natural Language and Speech Processing (ENLSP-II)*, 2022.
- [58] P.-A. Kamienny, S. d’Ascoli, G. Lample, and F. Charton, “End-to-end symbolic regression with transformers,” *Advances in Neural Information Processing Systems*, vol. 35, pp. 10269–10281, 2022.
- [59] A. Costa, R. Dangovski, O. Dugan, S. Kim, P. Goyal, M. Soljavčić, and J. Jacobson, “Fast neural models for symbolic regression at scale,” *arXiv preprint arXiv:2007.10784*, 2020.
- [60] K. He, X. Zhang, S. Ren, and J. Sun, “Deep residual learning for image recognition,” in *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770–778, 2016.
- [61] G. Huang, Z. Liu, L. Van Der Maaten, and K. Q. Weinberger, “Densely connected convolutional networks,” in *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 4700–4708, 2017.
- [62] P. K. Diederik, “Adam: A method for stochastic optimization,” (*No Title*), 2014.
- [63] C. Louizos, M. Welling, and D. P. Kingma, “Learning sparse neural networks through l_0 regularization,” in *International Conference on Learning Representations (ICLR)*, 2018.
- [64] S. Srinivas, A. Subramanya, and R. Venkatesh Babu, “Training sparse neural networks,” in *Proceedings of the IEEE conference on computer vision and pattern recognition workshops*, pp. 138–145, 2017.
- [65] W. Wu, Q. Fan, J. M. Zurada, J. Wang, D. Yang, and Y. Liu, “Batch gradient method with smoothing $l_{1/2}$ regularization for training of feedforward neural networks,” *Neural Networks*, vol. 50, pp. 72–78, 2014.
- [66] A. Paulraj, R. Roy, and T. Kailath, “Estimation of signal parameters via rotational invariance techniques- esprit,” in *Nineteenth Asilomar Conference on Circuits, Systems and Computers, 1985.*, pp. 83–89, 1985.
- [67] S. Developers, “`scipy.signal.find_peaks` – find peaks in a 1-d array.” https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.find_peaks.html, 2024. Accessed: 2026-01-18.

- [68] R. Tibshirani, "Regression shrinkage and selection via the lasso," *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 58, no. 1, pp. 267–288, 1996.
- [69] HUMS, "Hums2023_data_challenge_dataset," 2023.
- [70] T. Tieleman and G. Hinton, "Rmsprop: Divide the gradient by a running average of its recent magnitude. coursera: Neural networks for machine learning," *COURSERA Neural Networks Mach. Learn*, vol. 17, 2012.
- [71] E. Hubert, A. Barrau, and M. El Badaoui, "New multi-carrier demodulation method applied to gearbox vibration analysis," in *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 2141–2145, IEEE, 2018.
- [72] X. Yang, L. Wang, K. Ding, and X. Ding, "Vibration am-fm sidebands mechanism of planetary gearbox with tooth root cracked planet gear," *Engineering Failure Analysis*, vol. 137, p. 106353, 2022.
- [73] A. Sadhu, S. Narasimhan, and J. Antoni, "A review of output-only structural mode identification literature employing blind source separation methods," *Mechanical Systems and Signal Processing*, vol. 94, pp. 415–431, 2017.
- [74] N. Yousfi, K. Abed-Meraim, Y. Marnissi, M. Leiber, and M. EL Badaoui, "Neural network-based symbolic regression for empirical modeling of the behavior of a planetary gearbox," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*., IEEE, 2024.
- [75] M. Nait Meziane, K. Abed-Meraim, N. Zhao, and L. T. Nguyen, "On the Gaussian Cramer-Rao Bound for blind single-input multiple-output system identification: Fast and asymptotic computations," *IEEE Access*, vol. 8, September, 2020.
- [76] S. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory*. 1st Edition, Pearson, 1993.
- [77] K. Abed-Meraim, W. Qiu, and Y. Hua, "Blind system identification," *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1310–1322, 1997.
- [78] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein, *et al.*, "Distributed optimization and statistical learning via the alternating direction method

- of multipliers,” *Foundations and Trends® in Machine learning*, vol. 3, no. 1, pp. 1–122, 2011.
- [79] E. Ghadimi, A. Teixeira, I. Shames, and M. Johansson, “Optimal parameter selection for the alternating direction method of multipliers (admm): Quadratic problems,” *IEEE Transactions on Automatic Control*, vol. 60, no. 3, pp. 644–658, 2014.
- [80] J.-F. Cai, E. J. Candès, and Z. Shen, “A singular value thresholding algorithm for matrix completion,” *SIAM Journal on optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.
- [81] N. Yousfi, K. Abed-Meraim, M. El-Badaoui, and Y. Marnissi, “Blind identification of mechanical systems with modulated periodic input signals,” in *2024 32nd European Signal Processing Conference (EUSIPCO)*, pp. 2577–2581, 2024.
- [82] C. Peeters, Q. Leclere, J. Antoni, P. Lindahl, J. Donnal, S. Leeb, and J. Helsen, “Review and comparison of tachless instantaneous speed estimation methods on experimental vibration data,” *Mechanical Systems and Signal Processing*, vol. 129, pp. 407–436, 2019.
- [83] Q. Leclère, H. André, and J. Antoni, “A multi-order probabilistic approach for instantaneous angular speed tracking debriefing of the cmmno 14 diagnosis contest,” *Mechanical Systems and Signal Processing*, vol. 81, pp. 375–386, 2016.
- [84] R. Raguram, O. Chum, M. Pollefeys, J. Matas, and J.-M. Frahm, “Usac: A universal framework for random sample consensus,” *IEEE transactions on pattern analysis and machine intelligence*, vol. 35, no. 8, pp. 2022–2038, 2012.
- [85] E. Aboutanios and B. Mulgrew, “Iterative frequency estimation by interpolation on fourier coefficients,” *IEEE Transactions on Signal Processing*, vol. 53, no. 4, pp. 1237–1242, 2005.
- [86] D. Yu, J. Cheng, and Y. Yang, “Application of emd method and hilbert spectrum to the fault diagnosis of roller bearings,” *Mechanical systems and signal processing*, vol. 19, no. 2, pp. 259–270, 2005.
- [87] N. Yousfi, K. Abed-Meraim, Y. Marnissi, and M. EL Badaoui, “A sparse coding approach for planetary gearbox vibration signal decomposition,” in *The Int. Society for Air Breathing Engines, ISABE*, 2024.

BIBLIOGRAPHY

- [88] W. Q. K. Abed-Meraim and Y. Hua, “Blind system identification,” *Proceedings of the IEEE*, vol. 85, no. 18, pp. 1310–1322, August, 1997.
- [89] K. Astrom and B. Wittenmark, “Adaptive control, (2nd edition) addison wesley,” *Reading*, 1995.
- [90] D. P. Kingma and J. Ba, “Adam: A method for stochastic optimization,” in *International Conference on Learning Representations (ICLR)*, 2015.
- [91] D. Ha, A. Dai, and Q. V. Le, “Hypernetworks,” in *International Conference on Learning Representations (ICLR)*, 2017.
- [92] J. Möderl, E. Leitinger, F. Pernkopf, and K. Witrisal, “Variational inference of structured line spectra exploiting group-sparsity,” *IEEE Transactions on Signal Processing*, 2024.
- [93] J. Neri, R. Badeau, and P. Depalle, “Unsupervised blind source separation with variational auto-encoders,” in *2021 29th European Signal Processing Conference (EUSIPCO)*, pp. 311–315, IEEE, 2021.